Modelling the Effect of Orientation on the Shock Response of a Damageable Composite Material

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Abstract. The purpose of this paper is the investigation of the effect of fibre orientation on the shock response of a damageable carbon fibre-epoxy composite (CFEC). A carbon fibre-epoxy composite (CFEC) shock response in the through-thickness orientation and in one of the fibre directions is significantly different. Modelling the effect of fibre orientation on the shock response of a CFEC has been performed using a generalised decomposition of the stress tensor [A.A. Lukyanov, Int. J. Plasticity 24, 140 (2008)] and an accurate extrapolation of high-pressure shock Hugoniot states to other thermodynamics states for shocked CFEC materials. The analysis of the experimental data subject to the linear relation between shock velocities and particle velocities has shown that damage softening process produces discontinuities both in value and slope in the generalized bulk shock velocity U_S^A and particle velocity u_p relation [A.A. Lukyanov, Eur Phys J B 74, 35 (2010)]. Therefore, in order to remove these discontinuities, the three-wave structure (non-linear anisotropic, fracture and isotropic elastic waves) that accompanies damage softening process is proposed in this work for describing CFEC behavior under shock loading. A numerical calculation shows that Hugoniot Stress Levels (HELs) agree with the experimental data for selected CFEC material in different directions at low and at high intensities. In the through-thickness orientation, the material behaves similar to a simple polymer. In the fibre direction, the proposed model explains a pronounced ramp, before at sufficiently high stresses, and a much faster rising shock above it. The results are presented and discussed, and future studies are outlined.

Introduction

There are several types of composite materials, e.g., heterogeneous and layered composites. Investigation of composite material behaviour has found significant interest in the research community due to the widespread application of anisotropic materials in aerospace, military and civil engineering problems. Despite this, mechanical behaviour of composite materials in shock waves due to its complexities is still not fully understood, e.g. formation and propagation of the shock waves, initialization and propagation of micro- and macro- cracks. The purpose of this paper is the investigation of the effect of fibre orientation on the shock response of a damageable carbon fibre-epoxy composite (CFEC). In general, a single theoretical and numerical framework is not applicable to different composite materials; however, it is possible to use in most of the cases a thermodynamically consistent framework for modelling the response of these materials. This framework is building on the non-linear continuum mechanics formalism, couples non-linear elasticity with appropriate inelastic model within a thermodynamically consent numerical incremental formalism. Due to the experimentally observed behavior of anisotropic materials, a new orthotropic hydrocode model was developed from the theoretical basis. This was necessary because hydrostatic pressure inside these anisotropic materials depends on deviatoric strain components as well as volumetric strain. Non-linear effects, such as shock effects, can be incorporated through the volumetric straining in the material. This article presents these constitutive equations for the shock waves modelling of damageable carbon fibre-epoxy composite. The developed model includes orthotropic material stiffness, a non-linear anisotropic equation of state and material damage softening within a unified formulation.

Mathematical model of damageable composite

The composite materials response under shock loading leads to a nonlinear behaviour (i.e., large compressions, damage softening), therefore, the mathematically and thermodynamically consistent constitutive equations in order to distinguish between thermodynamic compressibility effect (equation of state) response and the ability of the material to carry shear loading are required [1], [3-5], [7-9], [11-16], [20-21]. It is a well known fact that for isotropic material the stress tensor can be described in terms of two quantities: the hydrostatic stress (or pressure), which only induces a change of scale, and the deviatoric stress, which only induces a change of shape. However, for anisotropic materials (e.g. for an orthotropic material), the decomposition of the stress and strain tensors into spherical and deviatoric parts in stress space and strain space results in stress and strain components which do not correspond to each other due to the material properties' anisotropy. For an anisotropic materials, the mean stress depends on the deviatoric strains, therefore, the decomposition used for isotropic materials is not applicable [7-9], [14-16], [20-21]. The generalised decomposition of the stress tensor is defined as:

$$\sigma_{ij} = -p^{*}\alpha_{ij}(\omega) + \tilde{S}_{ij}\,,\; \tilde{S}_{ij} = \sigma_{ij} - \alpha_{ij}(\omega)\cdot\frac{1}{\alpha_{mn}(\omega)\alpha_{mn}(\omega)}\sigma_{kl}\alpha_{kl}(\omega) \quad (1)$$

$$p^{*} = p^{EOS} + \frac{\beta_{ij}(\omega)\tilde{S}_{ij}}{\alpha_{kl}(\omega)\beta_{kl}(\omega)},\; \tilde{S}_{ij}^{\nabla} = C_{ijkl}^{\alpha}(\omega)d_{kl} \quad (2)$$

$$C_{ijkl}^{\alpha}(\omega) = \left(C_{ijkl}(\omega) - \frac{\alpha_{ij}(\omega)\cdot\alpha_{pq}(\omega)C_{pqkl}(\omega)}{\alpha_{mn}(\omega)\alpha_{mn}(\omega)} \right),\; d_{kl} = \mathbf{D}_{kl} - \delta_{kl}\mathbf{D}_{ii}/3 \quad (3)$$

where $p^{*}\alpha_{ij}(\omega)$ is the generalised spherical part of the stress tensor, $\tilde{S}_{ij}$ is the generalised deviatoric stress tensor ($\alpha_{ij}(\omega)\tilde{S}_{ij} = 0$), p^{*} is the total generalised "pressure", ω is the damage parameter, $\alpha_{ij}(\omega)$ is the first generalisation of the Kronecker delta symbol at a given damage state ω, p^{EOS} is the pressure related to an Equation of State (EOS) and $\beta_{ij}(\omega)$ is the second generalisation of the Kronecker delta symbol at a given damage state ω, $\mathbf{D}_{kl}$ is the strain rate (symmetric part of the velocity gradient), d_{kl} is the deviator rate of deformation, and symbol ∇ denotes a frame-invariant Jaumann (objective) rates. The summation convention is implied by the repeated indices. The elements of the tensor $\alpha_{ij}(\omega)$ are

$$\alpha_{ii}(\omega) = \left(\sum_{k=1}^{3} C_{ik}(\omega) \right)\cdot 3\bar{K}_{C}(\omega),\; i = 1,2,3;\; \alpha_{ij}(\omega)\alpha_{ij}(\omega) = 3 \quad (4)$$

$$K_{C}(\omega) = \frac{1}{3\sqrt{3}}\sqrt{\left(\sum_{k=1}^{3} C_{1k}(\omega) \right)^{2} + \left(\sum_{k=1}^{3} C_{2k}(\omega) \right)^{2} + \left(\sum_{k=1}^{3} C_{3k}(\omega) \right)^{2}} \quad (5)$$

$$K_C(\omega)=\frac{1}{9\bar{K}_C(\omega)} \tag{6}$$

$$C_{ij}(\omega)=(1-\omega)\cdot C_{ij}^A+\omega\cdot C_{ij}^I \tag{7}$$

where $C_{ij}(\omega)$ is the positive-definite stiffness matrix (written in Voigt notation) at a given damage state ω, C_{ij}^A is the stiffness matrix (written in Voigt notation) for intact material (i.e., for $\omega=0$), C_{ij}^I is the stiffness matrix (written in Voigt notation) for fully damaged material (i.e., for $\omega=1$). The elements of the tensor $\beta_{ij}(\omega)$ are

$$\beta_{ii}(\omega)=\left(\sum_{k=1}^{3}J_{ki}(\omega)\right)\cdot 3K_S(\omega),\, i=1,2,3;\;\; \beta_{ij}(\omega)\beta_{ij}(\omega)=3 \tag{8}$$

$$\frac{1}{K_S(\omega)}=\sqrt{3}\sqrt{\left(\sum_{k=1}^{3}J_{1k}(\omega)\right)^2+\left(\sum_{k=1}^{3}J_{2k}(\omega)\right)^2+\left(\sum_{k=1}^{3}J_{3k}(\omega)\right)^2} \tag{9}$$

where $J_{ij}(\omega)=\left[C_{ij}(\omega)\right]^{-1}$ are elements of compliance matrix (written in Voigt notation) at a given damage state ω, $K_S(\omega)$ represents the second generalised bulk modulus at a given damage state ω. Note that the generalised decomposition of the stress tensor can be applied for all composite materials of any symmetry and represents a mathematically consistent generalisation of the conventional isotropic case. In the limit of isotropy, the proposed generalisation returns to the traditional classical case where tensors $\beta_{ij}(\omega)$, $\alpha_{ij}(\omega)$ equal δ_{ij} which is independent of damage parameter and parameters $K_C(\omega)$ and $K_S(\omega)$ reduce to the well-know expression for conventional isotropic bulk modulus.

The extrapolation has been done by using a very popular form of equation of state p^{EOS} that is used extensively for isotropic solid continua is the Mie- Grüneisen EOS. The most commonly used form of the Mie-Grüneisen equation of state for solid materials which uses shock Hugoniot as the reference curve is given below:

$$p^{EOS}=f(\rho,e)=P_H^A(\omega)\cdot\left(1-\frac{\Gamma(\omega)}{2}\cdot\mu\right)+\rho\Gamma(\omega)\cdot e,\, P_H^A(\omega)=\rho_0^*(\omega)U_H^A(\omega)u_p \tag{10}$$

where $P_H^A(\omega)$ is the Hugoniot pressure at a given damage state ω, μ is relative change of volume, $\Gamma(\omega)$ is the Grüneisen parameter at a given damage state ω, and e is the specific internal energy. The Rankine-Hugoniot equations for the shock jump conditions can be regarded as defining a relation between any pair of the variables ρ, p, e, u_p and U_S^A [22]. Generally, the Hugoniot pressure and a shock velocity U_S^A is a non-linear function of particle velocity u_p and damage parameter ω. It is given by the following relation [23]:

$$U_S^A(\omega)=c(\omega)+S_1(\omega)\cdot u_p+S_2(\omega)\cdot\left(\frac{u_p}{U_S^A}\right)\cdot u_p+S_2(\omega)\cdot\left(\frac{u_p}{U_S^A}\right)^2\cdot u_p \tag{11}$$

where U_S^A represents the generalised shock velocity in the directions of anisotropic bulk orientation. The corresponding Mie-Grüneisen EOS defines pressure as:

$$p^{EOS}=\begin{cases}\dfrac{\rho_0^* c^2(\omega)\mu\left[1+\left(1-\dfrac{\Gamma(\omega)}{2}\right)\mu-\dfrac{\Gamma(\omega)}{2}\mu^2\right]}{\left[1-\left(S_1(\omega)-1\right)\mu-S_2(\omega)\dfrac{\mu^2}{\mu+1}-S_3(\omega)\dfrac{\mu^3}{(\mu+1)^2}\right]^2}+(1+\mu)\cdot\Gamma(\omega)\cdot E,\ \mu>0\\ \rho_0^* c^2(\omega)\mu+(1+\mu)\cdot\Gamma(\omega)\cdot E,\quad \mu<0\end{cases} \tag{12}$$

$$c(\omega)=\left[\sqrt{\frac{K_S(\omega)}{\rho_0^*(\omega)}},\sqrt{\frac{K_C(\omega)}{\rho_0^*(\omega)}}\right],e=\frac{E}{\rho_0^*(\omega)},\ \Gamma(\omega)=\frac{\gamma_0(\omega)+a(\omega)\mu}{1+\mu},\ \mu=\frac{\rho}{\rho_0^*(\omega)}-1 \tag{13}$$

where $\rho_0^*=\rho_0^*(\omega)$ is the initial (released) density for a given damage state ω, E is the internal energy per initial specific volume, $c(\omega)$ is the intercept of the U_S^A-u_p curve at a given damage state ω, $S_1(\omega)$, $S_2(\omega)$, $S_3(\omega)$ are the coefficients of the slope of the U_S^A-u_p curve Eq. 11 at a given damage state ω, $\gamma_0(\omega)$ is Grüneisen gamma for undeformed material at a given damage state ω, and $a(\omega)$ is the first order volume correction to γ_0 at a given damage state ω. Furthermore, the EOS properties similar to elastic properties can be described by the following relations:

$$c(\omega)=(1-\omega)\cdot c^A+\omega\cdot c^I\ ,S_i(\omega)=(1-\omega)\cdot S_i^A+\omega\cdot S_i^I,\ \ i=1,2,3; \tag{14}$$

$$\gamma_0(\omega)=(1-\omega)\cdot\gamma_0^A+\omega\cdot\gamma_0^I, a(\omega)=(1-\omega)\cdot a^A+\omega\cdot a^I \tag{15}$$

$$\rho_0^*(\omega)=(1-\omega)\cdot\rho_0^A+\omega\cdot\rho_0^I \tag{16}$$

Note that $c(\omega)=c^A$, $S_1(\omega)=S_1^A$, $S_2(\omega)=S_2^A$, $S_3(\omega)=S_3^A$, $\gamma_0(\omega)=\gamma_0^A$, $a(\omega)=a_0^A$, $\rho_0^*(\omega)=\rho_0^A$ for intact material (i.e., for $\omega=0$) and $c(\omega)=c^I$, $S_1(\omega)=S_1^I$, $S_2(\omega)=S_2^I$, $S_3(\omega)=S_3^I$, $\gamma_0(\omega)=\gamma_0^I$, $a(\omega)=a_0^I$, $\rho_0^*(\omega)=\rho_0^I$ for fully damaged material (i.e., for $\omega=1$). Parameters $\rho_0^{A,I}$, $c^{A,I}$, $S_1^{A,I}$, $S_2^{A,I}$, $S_3^{A,I}$, $\gamma_0^{A,I}$, $a^{A,I}$ represent material properties which define it's EOS [8] and have the following values: $\rho_0^A=1500\,\mathrm{kg/m^3}$, $\rho_0^I=1400\,\mathrm{kg/m^3}$, $c^A=3590.6\,\mathrm{m/s}$, $c^I=2745.7\,\mathrm{m/s}$, $S_1^A=10.755$, $S_1^I=2.9119$, $S_2^{A,I}=0$, $S_3^{A,I}=0$, $\gamma_0^{A,I}=0.85$, $a^{A,I}=0.5$. In this work, it assumed three-wave structure (non-linear anisotropic, fracture and isotropic elastic waves), the relation Eqs. 14 - 16, and the following damage model:

$$\omega = \begin{cases} 0, & \left\|u_p\right\|_2 \le \left\|u_p^{**}\right\|_2 \\ \left(\dfrac{\left\|u_p\right\|_2 - \left\|u_p^{**}\right\|_2}{\left\|u_p^{*}\right\|_2 - \left\|u_p^{**}\right\|_2} \right)^N, & \left\|u_p\right\|_2 \in \left[\left\|u_p^{**}\right\|_2, \left\|u_p^{*}\right\|_2 \right] \\ 1, & \left\|u_p\right\|_2 > \left\|u_p^{*}\right\|_2 \end{cases} \tag{17}$$

where u_p^{**} is the particle velocity at the beginning of damage softening process, N is the material parameter, $\left\|\bullet\right\|_2$ is the Euclidean norm. Note these parameters are functions of orientation. Also, using the experimental data [24], the following data is defined: $u_p^{**} = u_L^{**} = 53.8\,\mathrm{m/s}$, $u_p^{*} = u_L^{*} = 538.5\,\mathrm{m/s}$, $N = N_L = \dfrac{1}{16}$ (through the thickness), and $u_p^{**} = u_F^{**} = 58.9\,\mathrm{m/s}$, $u_p^{*} = u_F^{*} = 566.1\,\mathrm{m/s}$, $N = N_F = \dfrac{1}{25}$ (along the fibre 0^0), and $u_p^{**} = u_W^{**} = 57.3\,\mathrm{m/s}$, $u_p^{*} = u_W^{*} = 550.1\,\mathrm{m/s}$, $N = N_W = \dfrac{1}{25}$ (along the fibre 90^0). It is assumed that the locus of critical particle velocities is an ellipse:

$$\frac{\left(u_p^x\right)^2}{\left(u_L^{*,**}\right)^2} + \frac{\left(u_p^y\right)^2}{\left(u_F^{*,**}\right)^2} + \frac{\left(u_p^z\right)^2}{\left(u_W^{*,**}\right)^2} = 1,\; u_p^{*,**} = \sqrt{\left(u_p^x\right)^2 + \left(u_p^y\right)^2 + \left(u_p^z\right)^2} \tag{18}$$

where $u_p^{*,**}$ represents critical velocities u_p^{*} and u_p^{**} in the Eq. 18. Velocities (u_p^x, u_p^y, u_p^z) are computed using transformation matrix $\mathbf{Q}$ between orthogonal Cartesian coordinate (x', y', z') and orthogonal material Cartesian coordinate (x, y, z) in $\mathbf{R}^3$. The orthogonal Cartesian coordinate (x', y', z') is defined by generalized bulk shock velocity U_S^A such as the vector U_S^A aligns with x'-axis. The experimental data [24] were obtained for longitudinal (through thickness $x' \leftrightarrow x$) orientation and along the fibre 0^0 ($x' \leftrightarrow y$) orientation shock wave propagation in the selected CFC material. The transformation matrix $\mathbf{Q}$ can be defined by the rotations Euler's angles (ϕ, θ, ψ) as:

$$\mathbf{Q} = \left(q_{ij}\right),\; i = 1,2,3;\;\; j = 1,2,3 \tag{19}$$

where

$$q_{11} = \cos\psi\cos\varphi - \cos\theta\sin\varphi\sin\psi,\; q_{13} = \sin\psi\sin\theta,\; q_{33} = \cos\theta$$

$$q_{12} = \cos\psi\sin\varphi + \cos\theta\cos\phi\sin\psi,\; q_{23} = \cos\psi\sin\theta$$

$$q_{21} = -\sin\psi\cos\varphi - \cos\theta\sin\varphi\cos\psi,\; q_{31} = \sin\theta\sin\phi$$

$$q_{22} = -\sin\psi\sin\varphi + \cos\theta\cos\phi\cos\psi,\; q_{32} = -\sin\theta\cos\phi$$

Note that Euler's angles are $\phi = 0$, $\theta = 0$, $\psi = 0$ for through the thickness orientation, $\phi = -\pi/2$, $\theta = 0$, $\psi = 0$ for along the fibre 0^0 orientation, and $\phi = \pi/2$, $\theta = \pi/2$, $\psi = 0$ for along the fibre 90^0 orientation.

Numerical simulation of anisotropic shock wave propagation in CFEC

The work discussed below concerns the modelling of shock response of a carbon-fibre epoxy composite in different directions. The experiment was done by the technique of plate impact and details can be found in [8, 20-21, 24]. The plate-impact numerical simulations were performed by solving conventional conservation laws (dealing with mass, linear momentum and energy) for monopolar media, these coupled with the appropriate constitutive equations [7, 14-16, 20-21]. The plate impact test was modelled using a uniaxial strain state (one-dimensional mathematical formulation in strain space) and the adiabatic approximation [7, 14-16, 20-21]. The process of validation and verification of the non-damageable model (anisotropic and isotropic), numerical code and simulation of shock wave propagation in anisotropic materials was presented previously [7, 14-16, 20-21]. Material properties of anisotropic undamaged CFEC material (x- direction corresponds to the through the thickness direction, y-direction corresponds to the fill direction, and z- direction to the wrap direction) are $E_x = 13.678\,\text{GPa}$, $E_y = 68.467\,\text{GPa}$, $E_z = 66.537\,\text{GPa}$, $\nu_{xz} = 0.0044$, $\nu_{xy} = 0.0045$, $\nu_{zy} = 0.04$ [8]. Material properties of isotropic damaged CFEC material are: $\lambda = 10.434\,\text{GPa}$ (first parameter Lame), $G = 0.18\,\text{GPa}$ (second parameter Lame) [8].

A fundamental character of the material parameters specified in previous sections must be validated in different directions. Numerical computed properties are in good agreement with the experimental data for selected CFEC material in different directions at low and at high intensities.
In fig. 1, the data is depicted where Shock Hugoniot stress levels data as a function of orientation of the shock axis (in through thickness and fibre 0^0 orientations) is examined. Although, the results show good agreement in the stress magnitude between experimental data and numerical simulations for both orientations, there is a lack of experimental data below particle velocity of $u_F^{**} = 58.9\,\text{m/s}$ for making a final conclusion.

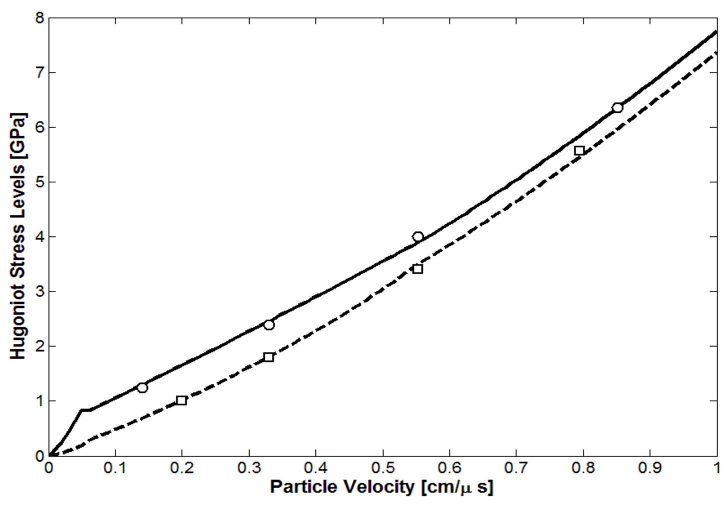

Fig. 1. Shock Hugoniot stress levels of carbon fibre epoxy composite in stress-particle velocity space for through thickness (dashed line) and fibre 0^0 (solid line) orientations. Experimental data for through thickness and fibre 0^0 orientations are depicted by squares and circles respectively.

In the through-thickness orientation, the material behaves similar to a simple polymer. In the fibre direction, the proposed model explains a pronounced ramp, before at sufficiently high stresses, and a much faster rising shock above it. The ramped nature of the shock back surface gauge tracers allowed determining two wave velocities, a velocity on the ramped part c_{toe} and a velocity on the main part c_{head} of the signal respectively. The velocities c_{toe} and c_{head} were computed at the beginning of the rises of the stress pulses and at the maximum stress amplitude [24]. The numerical simulation results in the fibre 0^0 orientation lead to the average value of *ca.* $c_{toe} = 6900\,\mathrm{m/s}$ over the following particle velocity range $\left[0; u_F^{**}\right]$, where $u_F^{**} = 58.9\,\mathrm{m/s}$ is the particle velocity at the beginning of damage softening process. There is a quite a high degree of scatter in this experimental data, but it was reported a value of *ca.* $7000\,\mathrm{m/s}$ [24]. Note that the longitudinal speed of sound in the fibre 0^0 orientation computed using homogeneous Hooke's law is *ca.* $6666\,\mathrm{m/s}$, further suggesting that if this wave has been transmitted along 0^0 fibre orientation, it is for undamaged anisotropic structure. In contrast, the second wave velocity c_{head} in the fibre 0^0 orientation, whilst initially greater (for low intensities of shock waves and damage distribution) than the shock velocity in the through thickness orientation, eventually converges with that data set for fully damaged state.

The good agreement between the results can be observed and leads to the conclusion that constitutive equation presented in this paper can be used for the simulation of shock wave propagation within damageable CFEC material. Reduction of the model to the conventional constitutive equations in the limit of isotropy allows for its use in modelling wide range of materials.

Summary

In this paper, thermodynamically and mathematically consistent constitutive equations suitable for characterising shock wave propagation in a damageable anisotropic composite (CFEC) material are presented. An accurate extrapolation of high-pressure shock Hugoniot states to other thermodynamics states for shocked Carbon-Epoxy Fibre Composite (CFEC) materials was presented. A generalised decomposition for separation of material volumetric compression (compressibility effects equation of state) from deviatoric strain effects is formulated, which allows for the consistent calculation of stresses in different regions of material's behaviour (i.e. in the non-linear anisotropic, fracture and isotropic regions). Numerical simulations of the behaviour of the CFEC material under shock loading conditions based on the new system of constitutive equations were performed. The plate impact experiments on CFEC material were carried out by Millett et al. [24]. A comparison of the experimentally obtained general pulse shape and Hugoniot stress level in different directions with numerical simulation shows a good agreement and suggests that the EOS is performing satisfactorily. In addition, an analytical calculation showed that the Hugoniot Stress Levels (HSLs) in different directions, for a CFEC composite subject to the three-wave structure (non-linear anisotropic, fracture and isotropic elastic waves), agree with experimental measurements at both low shock intensities (where the 0^0 orientation was significantly stiffer than the through-thickness orientation) and at high shock intensities (where the HSLs of the two orientations converged due to the presence of damage softening), this also in agreement with the stability requirements formulated by Bethe [8].

References

[1] J. K. Chen, A. Allahdadi, T. Carney, Compos. Sci. Technol. 57 (1997), p. 1268.

[2] J. P.G Sawyer, R. Jones, P. R. McKinlay, Comp. Struct. 42 **(**1998), p. 93.
[3] S. J. Hiermaier, W. Riedel, C. J. Hayhurst, R. A. Clegg, C. M. Wentzel, Report No. E 43/99; Ernst Mach Institut: Freiburg im Breisgau, Germany, 1999.
[4] C. J. Hayhurst, S. J. Hiermaier, R. A. Clegg, W. Riedel, M. Lambert, Int. J. Impact Eng. 23 (1999), p. 365.
[5] C. J. Hayhurst, I. H. G. Livingstone, R. A. Clegg, R. Destefanis, M. Faraud, Int. J. Impact Eng. 26 (2001), p. 309.
[6] H. Espinosa, H.C. Lu, P.Zavattieri, S. Dwivedi, J. Comp. Mater. 35 (2001), p. 369.
[7] A. A. Lukyanov, Mech. Adv. Mat. & Struct., 17 **(**2010**)**, p. 304.
[8] A. A. Lukyanov, Eur. Phys. J B 74 **(**2010), p. 35.
[9] A. A. Lukyanov, J. Appl. Phys. 108 **(**2010**)**, 066102.
[10] M. Pankow, A. M. Waas, Ch.-F. Yen, S. Ghiorse, Comp. Struct. 93 (2011), p. 1347.
[11] S. B. Segletes, U.S. Army Ballistic Research Laboratory Report BRL-TR-2825, Aberdeen Proving Ground, MD, 1987.
[12] P. E. O'Donoghue, C. E. Jr. Anderson, G. J. Friesenhahn, C. H. Parr, J. Comp. Mater. 26 (1992), p. 1860.
[13] C. E. Anderson, P. A. Cox, G. R. Johnson, P. J. Maudlin, Comp. Mech. 15 (1994), p. 201.
[14] A. A. Lukyanov, Int. J. Plast. 24 (2008), p. 140.
[15] A. A. Lukyanov, Eur. Phys. J B 64 (2008), p. 159.
[16] A. A. Lukyanov, J. Appl. Mech. 76 (2009), 061012.
[17] J. M. Winey, Y. M. Gupta, J. Appl. Phys. 96 (2004), p. 1993.
[18] J. M. Winey, Y. M. Gupta, J. Appl. Phys. 99 (2006), 023510.
[19] J. M. Winey, Y. M. Gupta, J. Appl. Phys. 107 (2010) 103505.
[20] A. A. Lukyanov, V. B. Pen'kov, J. Appl. Math. Mech. 73 (2009), p. 457.
[21] A. A. Lukyanov, Vestnik Samarskogo Gosudarstvennogo Universiteta Estestvennonauchnaya seria 2 (2010), p. 105.
[22] M. A. Meyers, Dynamic Behavior of Materials. New York: Wiley. Inc.; 1994.
[23] D. J. Steinberg, Report No. UCRL-MA-106439. Lawrence Livermore National Laboratory. Livermore. CA. USA. 1991.
[24] J. C. F Millett, N. K. Bourne, Y. J. E Meziere, R. Vignjevic, A. A. Lukyanov, Compos. Sci. Technol. 67 (2007), p. 3253.