# Intergrated Evaluation Method for Fatigue Limit of Notched Steel

# with $\Delta K_{th}$ Value

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#### Abstract

It is the big problem to evaluate the fatigue limit  $\sigma_w$  in order to safely design structures and machines. Under the condition of the fatigue crack initiation limit  $\sigma_{w1}$ , the present author defined the initial crack size  $\sqrt{\text{area}}$  using the characteristic elastic field of the notch, the half length of the fatigue crack *a* and a notch depth *t*. Using this $\sqrt{\text{area}}$ , the methods of calculating  $\Delta K_{th}$  and  $\sigma_{w1}$  were proposed. In addition to it, in this condition it is made sure that  $\Delta K_{th}$  exists on the band from  $\Delta K_{th}$  prediction line of the micro fatigue crack within 18% error. Using  $\Delta K_{th}$  calculated by the prediction line and the value of  $\sqrt{\text{area}}$ , this evaluating method of  $\sigma_{w1}$  was confirmed with annealed 0.35% carbon steel. As a result, it was confirmed that this method to evaluate  $\sigma_{w1}$  is within 18% error. Moreover it was also confirmed that the  $\sigma_{w1}$  value by this prediction method is within 10% error when the predicted  $\sigma_{w1}$  value is greater than  $\sigma_{w2}$ .

### Introduction

The fatigue limit of materials and these structures is controlled by defects. The fatigue crack nonpropagating limit  $\sigma_{w2}$  of notched the specimen can be easily evaluated using fatigue crack properties [1]. However there is no method for evaluating the fatigue crack initiation limit  $\sigma_{w1}$  with the threshold stress intensity factor range  $\Delta K_{th}$  including effect of the characteristic elastic filed of notch, because there is no fatigue crack under the condition of  $\sigma_{w1}$ . It is important to evaluate  $\sigma_{w1}$ , because the fatigue limit of a blunt notch specimen depends on  $\sigma_{w1}$ . Therefore, the method of evaluating  $\sigma_{w1}$ using a fatigue crack property is proposed and examined with 0.35% carbon steel in this study.

### The method of evaluating fatigue limit

The fatigue properties of notched specimen. The fatigue limits of a notched specimen consist of  $\sigma_{w1}$  and  $\sigma_{w2}$ . It is well-known that a notch root radius  $\rho$  and a notch depth *t* affect these two fatigue limits. The material broken is controlled by the highest limit of the two fatigue limits. Fig.1 shows it. By the way, in carbon steel, the initial crack of  $\sigma_{w1}$  is the micro fatigue crack nucleated at the notch root. On the other hand, the initial crack of  $\sigma_{w2}$  is the notch regarded as the crack. However, both  $\sigma_{w1}$  and  $\sigma_{w2}$  are caused by the plastic-induced crack closure.

The fatigue crack is nucleated at the grain having biggest fatigue damage. Therefore, in notch specimen, the fatigue crack is nucleated at the biggest grain at notch root. In this study, it is

supposed that the fatigue crack is nucleated at the grain of average size at the notch root. Figs.2 and3 show the model of the initial crack and the non-propagating crack at the notch root.



Fig.1 Relation between  $\sigma_{w0}$ ,  $\sigma_{w1}$ ,  $\sigma_{w2}$  and  $1/\rho$ 



The method of defining the initial crack. In the initial crack, the stress intensity factor  $K_{\rm I}$  on the crack front is same value and the fatigue crack stops propagating only by the plastic-induced crack closure. These two conditions define the initial crack size. We supposed that when the crack tip is affected by the stress field of a defect, the initial crack length is decided by the fatigue crack length. When the crack tip is not included in the stress field of a defect, the initial crack length is decided by the fatigue crack length and defect length. Fig.4 shows the relationship between the stress field and

the fatigue crack length.



The stress field from notch root. The stress field from the notch root is calculated by Eq.1 [2].  $K_t$  is a stress concentration factor and x is a distance from the notch root.

$$\sigma_{y} = K_{t} \sigma_{\infty} / \sqrt{1 + 4.5 x / \rho} \tag{1}$$

The criterion of the characteristic elastic field from the notch root  $R^*$  is evaluated by Smith's parameter [2]. When we know a notch depth *t* and a notch root radius  $\rho$ , we can calculate  $R^*$  using Eq. 2.

$$R^* = 0.75 \sqrt{\rho t} \tag{2}$$

When  $R^*$  is longer than the half length of the fatigue crack *a*, we can use Eq.2. By contrast, when  $R^* < a$ , the stress at the fatigue crack tip is not decided using Eq.1 because Eq.1 is not precise in this region.

Method of calculating  $\Delta K_{th}$  at notch root. The square-root of the average grain area projected on the plane perpendicular to the axial direction is the initial crack size  $\sqrt{\text{area}}$ . When  $R^* > a$ , the characteristic elastic stress filed of the notch affects the fatigue crack tip. Therefore the initial crack size  $\sqrt{\text{area}}$  is calculated using Eq.3.

$$\sqrt{\text{area}} = \sqrt{\pi/2} a \tag{3}$$

When  $R^* < a$ , the characteristic elastic field of the notch does not affect the crack tip. Thus the initial crack size is calculated using Eq. 4

$$\sqrt{\text{area}} = \sqrt{\pi/2} \left( a + t \right) \tag{4}$$

The  $\Delta K_{th}$  value of the fatigue crack at the notch root is calculated from a,  $\sqrt{\text{area}}$  and the stress ratio R using Eqs. 5 and 6. On the other hand, when the  $\Delta K_{th}$  value of the notch specimen is obtained,  $\sigma_{w1}$  is calculated using Eqs. 5 and 6. Figs. 6 and 7 show the model of the initial crack at the notch root.

$$\Delta K_{th} = (1-R)(0.44 / \sqrt{1 + 4.5a/\rho} + 0.15) K_t \sigma_{w1} \sqrt{\pi^4} \sqrt{\text{area}} \qquad (R^* > a)$$
(5)  
$$\Delta K_{th} = 1.12(1-R)\sigma_{w1} \sqrt[4]{2\pi} \sqrt[4]{area} / \sqrt{2.464} \qquad (R^* < a)$$
(6)



Fig.6 An initial crack of a specimen with notch  $(R^*>a)$ 



Fig.7 An initial crack of a specimen with notch ( $R^* < a$ )

The examination of this method using the average grain size of annealed 0.35% carbon steel Method of calculating the initial crack length of annealed carbon steels. In annealed carbon steels, the fatigue crack is nucleated at the ferrite grain at the notch root. Thus the half length of the fatigue crack *a* is calculated from the average ferrite grain size  $d_f$  using Eq. 7

$$a = d_f / 2 \tag{7}$$

Fig.8 shows the relationship between  $\sqrt{\text{area}}$  and  $\Delta K_{th}$ . The experimental values are quoted from [3], [4]. The  $\Delta K_{th}$  values are on the same band which is from the  $\Delta K_{th}$  prediction line of the micro fatigue crack [1]. The maximum error from the prediction line of micro fatigue crack is 18%. In other word, We can predict  $\Delta K_{th}$  within 18% using Eq.8[2].



Fig. 8 Relation ship between  $\sqrt{\text{area}}$  and  $\Delta K_{th}$  of annealed 0.35% carbon steels

Fig.9 show the predicted and experimental values of the fatigue limits of annealed 0.35% carbon steel specimens with various types of notch. The fatigue limit is decided by Eq.9. In addition to it,

the Non-propagating fatigue limit is decided by Eq.10[2] using the Vickers hardness HV and the initial crack size  $\sqrt{area}$ .

$$\sigma_{w} = \max(\sigma_{w1}, \sigma_{w2})$$
(9)  
$$\sigma_{w2} = 1.43 (HV + 120)^{/12} \sqrt{\text{area}}$$
(10)

When using this method, we can predict the fatigue limit of notch specimens within 12% error.



Fig.9 Fatigue limits of annnealed S35Cwith various notch shape

#### Investigation of using the average ferrite grain size regarded as the initial crack size

Method of calculating the surface ferrite grain size. Grain sizes distribution is said that it follow the normal distribution. In addition to it, surface grain size is not always the diameter of the ferrite grain because of cut by material surface. In this study, ferrite grains are regarded as spheres. This spheres diameter is d. Therefore, the suface ferrite grain size  $d_a$  is calculated from d and distance from the center of sphere to the suface h using Eq.10. Fig.10, 11 show the model of the relationship between d and  $d_a$ .

$$d_{a}/2 = \sqrt{(d/_{2})^{2} - h^{2}}$$
(10)  
A specimen surface  
$$h \oint A \text{ ferrite}_{\text{grain}}$$

Fig.10 Relationship between the diameter of grain and surface grain size



Fig.11 Relationship among d,  $d_a$  and h

Simulation of using the average grain size regarded as the initial crack size. According to Eqs. 5~8,  $\sigma_{w1}$  has proportional connection with  $\sqrt[12]{area}$ . Therefore, Eq. 11 is valid.

$$(\sigma_{w_1} \text{using } d_{a,max}) / (\sigma_{w_1} \text{using } d_{ave}) = (d_{a,max} / d_{a,max})^{1/6}$$
 (11)

Fig.12 shows the result of the simulation to calculate ratios of  $\sigma_{w1}$  using average grain size  $d_{ave}$ , random  $d_a$ . In this simulation, it is assumed that d follows the normal distribution and the standerd deviation  $\sigma$  is 10µm,  $d_f$  is 30µm and number of samples are 5, 10, 25, 100, 500, 1000. The upper limit of random  $d_a$  is setted  $2d_{ave}$  and The lower limit is setted 0. Number of samples are regarded as notch radius  $\rho$  because the bigger  $\rho$  is, the more number of grains at notch root increase. Therefore, this simulation is regarded as the investigation of  $\sigma_{w1}$  of notched specimen which has various  $\rho$ . When we calculate the fatigue crack initiation limit  $\sigma_{w1}$  using  $d_{ave}$ , the maximum error is near by 10%. Therefore, it seems to be valid to use the average grain size regarded as the initial crack size.



Fig.12 Ratios between  $\sigma_{w1}$  using  $d_{a,max}$  and  $\sigma_{w1}$  using  $d_{ave}$ 

## Conclusion

We proposed the method of evaluating  $\sigma_{w1}$  using  $\Delta K_{th}$  value, moreover examined this method with 0.35% carbon steel. The following results were obtained. (1) This method to evaluate  $\Delta K_{th}$  and  $\sigma_{w1}$  is within 18% error. (2) When the predicted  $\sigma_{w1}$  value is greater than  $\sigma_{w2}$ , the prediction value using this method is within 10% error. (3) From result of simulation, it seems to be valid to use the average grain size as the initial crack size.

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