Influence of a functional gradation on crack propagation in real structures

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Abstract. In technical practice functional graded structures gain increasingly in importance. Given that the gradation of the material has a remarkable influence on the crack propagation behavior, experiments and simulations are necessary. Within this paper the influence of a functional fracture mechanical gradation is examined. Investigations show that this kind of a functional gradation has an effect on the crack velocity as well as on the area of stable fatigue crack growth. In addition it is to query in which form the gradation has an impact on the crack propagation direction. These considerations are presented and discussed within this contribution. A new crack propagation concept is presented describing crack growth in fracture mechanical graded materials and is confirmed by first experimental investigations.

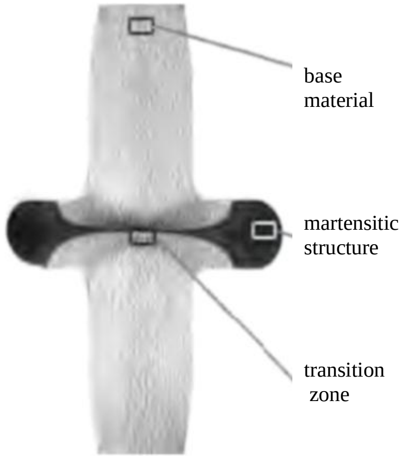
Introduction

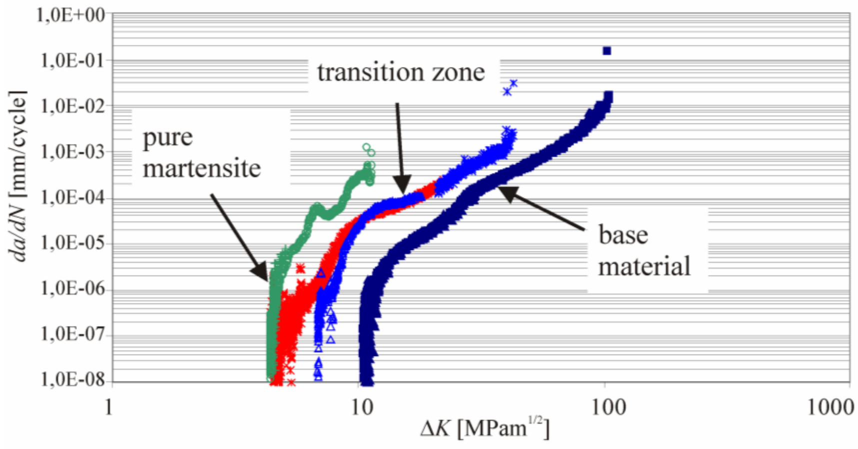
Functional graded structures become more and more important because they possess a large potential considering lightweight construction and a large application spectrum. Furthermore they meet demands with regard to absorption, abrasion and fatigue of structures. The advantage of these materials is among other things the adaptation of the material properties to the local stress situation, reduction of further treatment and hence the lowering of cost and time.

The collaborative research centre Transregio 30, a cooperation of the German Universities of Kassel, Paderborn and Dortmund, investigates "*functionally graded materials in industrial mass production*" [1]. Using a thermo-mechanically production process defined properties are generated for the demonstrator flanged shaft (Fig.1a). A ferritic-perlitic base material of the heat treatable steel 51CrV4 can be found in the unformed region of the shaft, whereas the formed flanged consists of martensitic structure. The transition zone between both structures is neglected in this contribution. The different microstructures are characterized by different fracture mechanical properties (Threshold value ΔK_{th}, fracture toughness K_{IC}, crack velocity da/dN) as can be seen in Fig.1b.

Under consideration of the material gradation the following questions are of importance:

- Under which condition is crack growth possible?
- In which direction does the crack grow in case of stable crack growth?
- How fast does the crack grow?
- When does the unstable crack growth start?





a) b)

Fig.1. a) Metallographic micrograph of the flanged shaft [1],
b) Crack velocity curves of different microstructures [2].

Influence of a fracture mechanical gradation on the limits of fatigue crack growth

Fig.2 shows the connection between the cyclic applied load $\Delta\sigma$ and the crack depths a for a crack with the geometry factor $Y = 1$ and displays the Threshold value curves and the fracture value curves of the base material and the martensite of the treatable steel 51CrV4. Below the Threshold value curves a crack is not able to grow, whereas the region above the fracture value curves is characterized by unstable crack growth. Stable crack growth is between these two curves. In addition a fracture mechanical gradation is considered by assuming that the crack starts within the base material and reaches the martensitic region at a crack depth of a = 6 mm. The area of stable crack growth of the base material is more distinct than the area of the martensite. Furthermore the transition at the crack depth a = 6 mm is very marginal leading to unstable crack growth and hence to the failure of the structure immediately after reaching martensite.

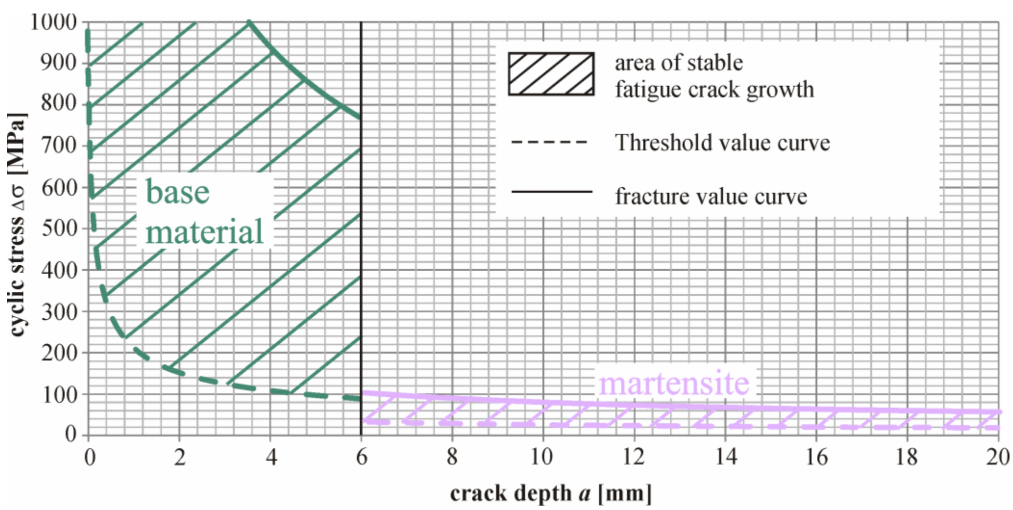


Fig.2. $\Delta\sigma$-a-diagram for a fracture mechanical material gradation: base material - martensite [2].

In Fig.3 besides the ferritic-perlitic base material and the martensite a bainitic transition zone is considered meaning the sharp transition between base material and martensite is replaced by a smoother transition zone. The respective changes happen at the crack depths a = 5mm and a = 7mm. It’s evident that bainite possesses better fracture mechanical properties than the martensitic structure. The fracture toughness of the bainite is considerably larger than the fracture toughness of the martensite. Accordingly due to the utilization of bainitic structure a distinct increase of the life time of structure can be obtained.

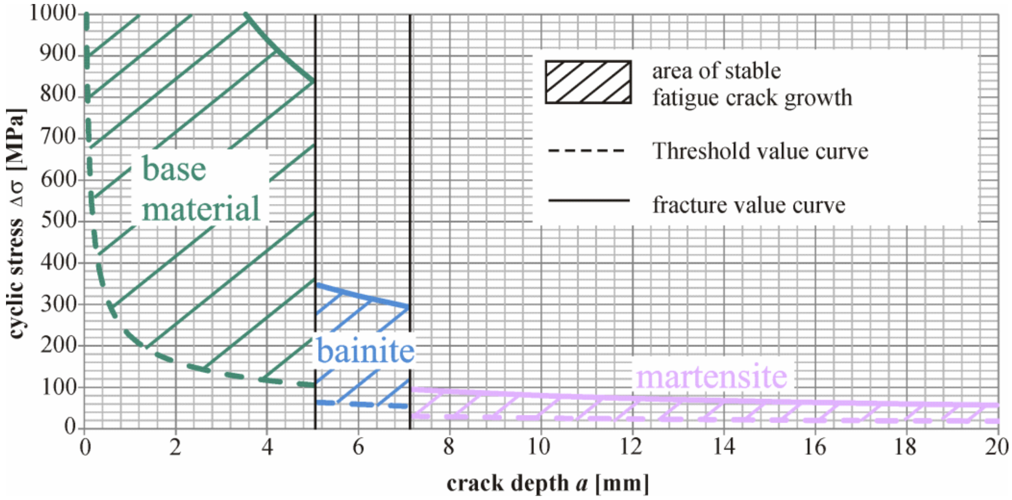
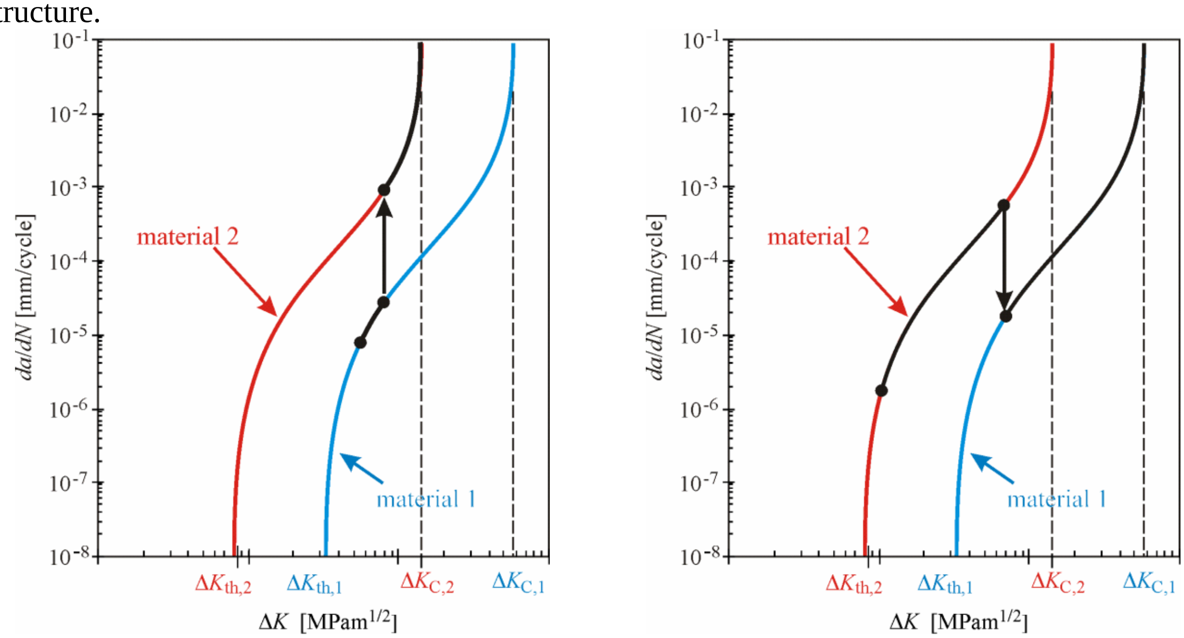


Fig.3. $\Delta\sigma$-a-diagram for a fracture mechanical material gradation: base material - bainite - martensite [2].

Influence of a fracture mechanical gradation on crack growth velocity

Fig.4 shows two schematic crack velocity curves for two different materials [2]. At the beginning the crack grows within material 1. The change to material 2 leads to a change between both curves to the crack velocity curve of material 2. The consequence is that the crack becomes unstable faster as if the crack grows further within material 1. Besides the different fracture mechanical limits a difference in the crack velocity can be identified. The velocity changes leading to a faster crack growth. The worst imaginable case at this gradation constellation is, that at the change to the crack velocity curve of material 2 the cyclic stress intensity factor ΔK is larger than the fracture value $\Delta K_{C,2}$ of material 2 with the consequence of unstable crack growth and the immediate failure of the structure.



a) b)

Fig.4. Schematic crack velocity curves with transition from material 1 to material 2 (a) and from material 2 to material 1 (b).

In Fig.4b the gradation is oriented opposite. The crack starts in material 2 and reaches material 1 afterwards. This constellation has a positive impact on the prospective life time. If the crack grows further in material 2 the instability is achieved very fast. Due to the change to the other crack velocity curve with the better fracture mechanical properties the crack slows down und more load cycles are possible until the failure of the structure. The best imaginable case is possible if the cyclic stress intensity factor ΔK is smaller than the Threshold value $\Delta K_{th,1}$ of material 1.

$\sigma_{\varphi TSSR}$-concept for the description of crack propagation in fracture mechanical graded materials

For determination of the crack propagation direction and the start of stable and unstable crack growth in fracture mechanical graded structures, the development of a new crack propagation concept is necessary. For this reason the $\sigma_{\varphi TSSR}$-concept was developed. Fig.5 shows a crack-afflicted structure which consists of two materials with different fracture mechanical properties. The position of the material transition in relation to the crack tip is identified by the gradation angle φ_M.

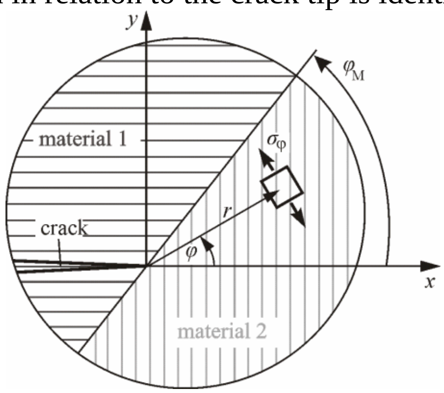


Fig.5. Fracture mechanical graded structure with the gradation angle φ_M [2].

This $\sigma_{\varphi TSSR}$-concept uses the tangential stress (Eq.1) and is based on the assumption that unstable crack propagation occurs when the cyclic tangential stress $\Delta\sigma_\varphi$ reaches a material limit value $\Delta\sigma_{\varphi C}$ or rather when a cyclic stress intensity factor ΔK, determined by the means of the cyclic tangential stress $\Delta\sigma_\varphi$, reaches the cyclic fracture value ΔK_{IC} of one of the materials.

$$\Delta\sigma_\varphi = \frac{\Delta K_I}{\sqrt{2\pi r}}\cos^3\frac{\varphi}{2} - \frac{\Delta K_{II}}{\sqrt{2\pi r}}\frac{3}{2}\sin\varphi\cos\frac{\varphi}{2} \tag{1}$$

The first intersection of the stress function $\Delta\sigma_\varphi\sqrt{(2\pi r)}$ (Eq.2) with one of the two cyclic fracture value curves identifies the occurrence of final failure of the structure, whereas the starting of crack growth is identified by the intersection of the stress function $\Delta\sigma_\varphi\sqrt{(2\pi r)}$ with one of the Treshold value curves.

$$\Delta\sigma_\varphi\sqrt{2\pi r} = \Delta K_I\cos^3\frac{\varphi}{2} - \Delta K_{II}\frac{3}{2}\sin\varphi\cos\frac{\varphi}{2} \tag{2}$$

Determination of the kinking angle φ_{TSSR}. The kinking angle φ_{TSSR} describing the kinking in a fracture mechanical graded structure is determined by comparing the stress function $\Delta\sigma_\varphi\sqrt{(2\pi r)}$ with both Threshold value curves. The first intersection leads to the kinking angle. In theory two kinking angles are possible: the gradation angle φ_M (Fig.5) and the kinking angle φ_0 according to the load situation. A case-by-case analysis (Fig.6) is necessary to identify the right kinking angle φ_{TSSR}. The gradation angle φ_M can be easily determined by looking at the material distribution around the crack tip.

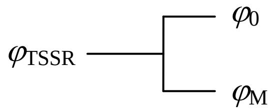


Fig.6. Case-by-case analysis to determine the kinking angle φ_{TSSR}.

For the determination of the kinking angle φ_0 crack propagation concepts for isotropic homogenous materials [3,4] have to be used. The $\sigma_{\varphi TSSR}$-concept uses the concept of the maximum tangential stress (MTS-concept) of ERDOGAN and SIH [5] to calculate the kinking angle $\varphi_{0,MTS}$. This MTS-concept is based on the assumption that the crack propagates perpendicular to the maximum tangential stress $\sigma_{\varphi,\max}$ and unstable crack growth occurs if the maximum comparative stress intensity factor $K_{V\max}$ reaches the fracture toughness K_{IC}.

The kinking angle φ_0 depends on the stress situation. A single Mode I situation (ΔK_{II}=0) due to tensile loading or bending leads to a symmetrical opening of the crack surfaces and a crack propagation perendicularly to the normal stress, hence the kinking angle $\varphi_{0,MTS}$ is 0°. The stress function is presented in Eq.3. Mode II (ΔK_I=0, Eq.4) evokes an opposite sliding of the crack surfaces leading to a kinking angle of $\varphi_{0,MTS}$ = -70°. A plane Mixed Mode situation considers an interference of Mode I and Mode II. The cyclic stress function is presented in Eq. 5.

Mode I: $$\Delta\sigma_\varphi\sqrt{2\pi r} = \Delta K_I \cos^3\frac{\varphi}{2} \qquad (3)$$

Mode II: $$\Delta\sigma_\varphi\sqrt{2\pi r} = -\Delta K_{II}\frac{3}{2}\sin\varphi\cos\frac{\varphi}{2} \qquad (4)$$

Mixed Mode: $$\Delta\sigma_\varphi\sqrt{2\pi r} = \Delta K_I \cos^3\frac{\varphi}{2} - \Delta K_{II}\frac{3}{2}\sin\varphi\cos\frac{\varphi}{2} \qquad (5)$$

Different $\Delta K_I/\Delta K_{II}$-ratios can be examined using the ratio V (Eq.6) to describe the Mixed Mode ratio. In the following a V-ratio of 0,33 is considered leading to the kinking angle of $\varphi_{0,MTS}$ =-40.3° due to the considered Mixed Mode situation.

$$V = \frac{\Delta K_{II}}{\Delta K_I + \Delta K_{II}} \qquad (6).$$

The schematic stress functions $\Delta\sigma_\varphi\sqrt{(2\pi r)}$ depending on the loading situation are presented in Fig.7.

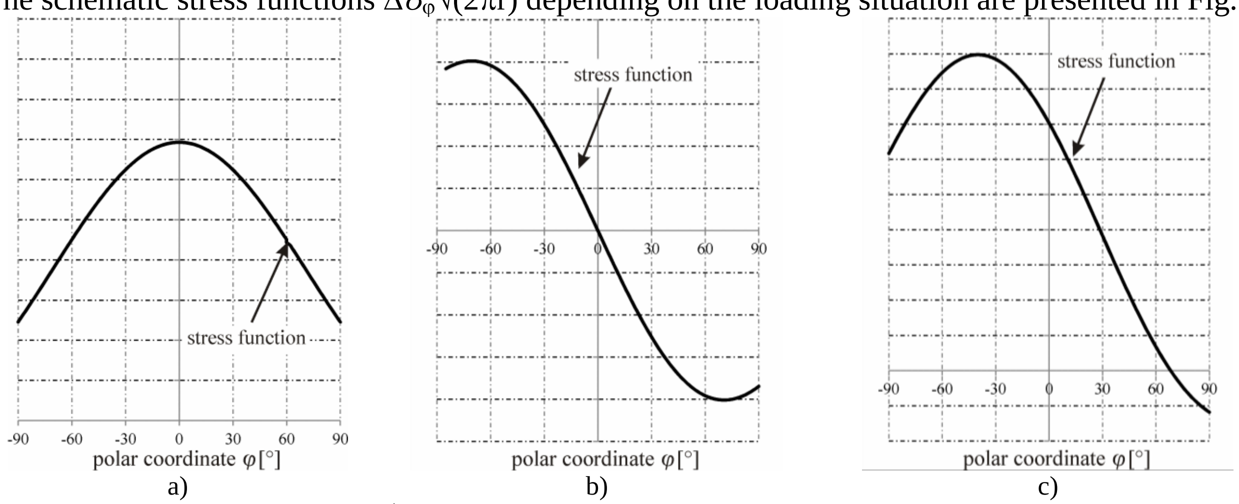


Fig.7. Stress functions $\Delta\sigma_\varphi\sqrt{(2\pi r)}$ in dependency of the load situation: Mode I (a), Mode II (b), Mixed Mode (c).

Application of the $\sigma_{\varphi TSSR}$-concept

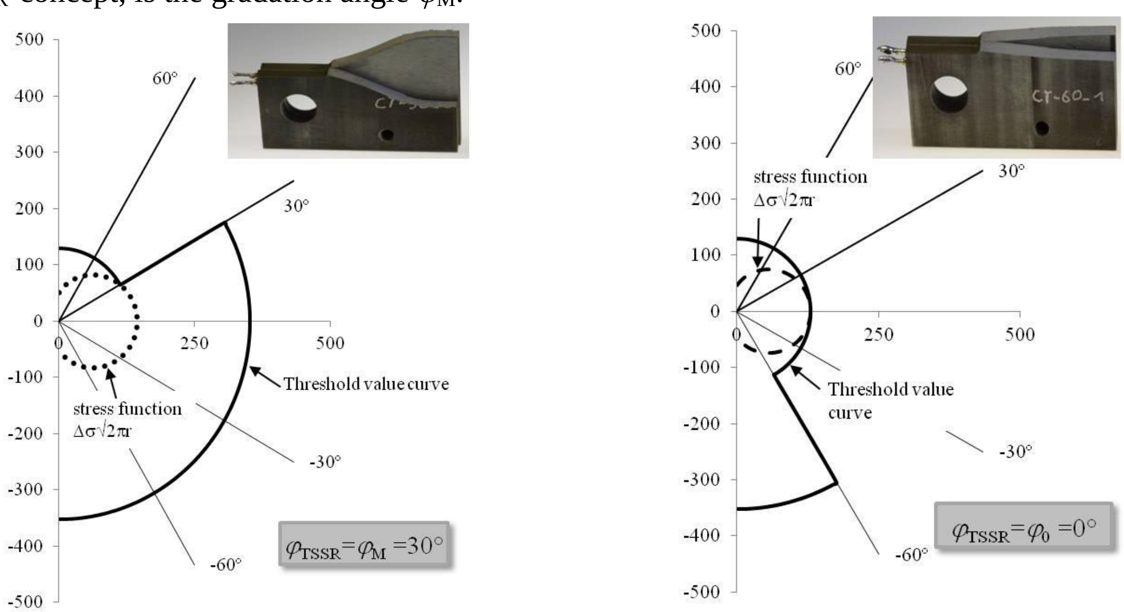
In the following the application of the φ_{TSSR}-concept is presented exemplary for different loading situations and chosen gradation angles φ_M.

Mode I. In Fig.8a the stress function $\Delta\sigma_\varphi\sqrt{(2\pi r)}$ due to Mode I and the Threshold value curves of the base material and the martensite can be seen under consideration of a gradation angle of φ_M=30°. Using the $\sigma_{\varphi TSSR}$-concept the intersection of the stress function $\Delta\sigma_\varphi\sqrt{(2\pi r)}$ with one of the two material curves has to be identified. In this case the intersection can be found at φ=30°, the gradation angle φ_M. Experimental investigations confirm that the kinking angle φ_{TSSR} due to the gradation with a gradation angle φ_M=30° is defined by this angle.

In Fig.8b a gradation angle of φ_M=-60° is considered. The intersection of the curves is found at φ=0° which is in accordance with the stress-depending kinking angle $\varphi_{0,MTS}$. Experimental investigations confirm the theoretically determined kinking angle φ_{TSSR}.

Mode II. Fig.9 presents the cyclic stress function $\Delta\sigma_\varphi\sqrt{(2\pi r)}$ of a pure Mode II loading situation and the Threshold value curves of the base material and martensite considering a gradation angle of φ_M=60° (a) and φ_M=-30° (b). For the structure with the gradation angle φ_M=60° the first intersection between the presented curves can be found at φ=-70° which is the stress controlled kinking angle $\varphi_{0,MTS}$. In the other case (b) the gradation angle φ_M defines the kinking angle φ_{TSSR} leading to a kinking of -30° in this fracture mechanically graded structure.

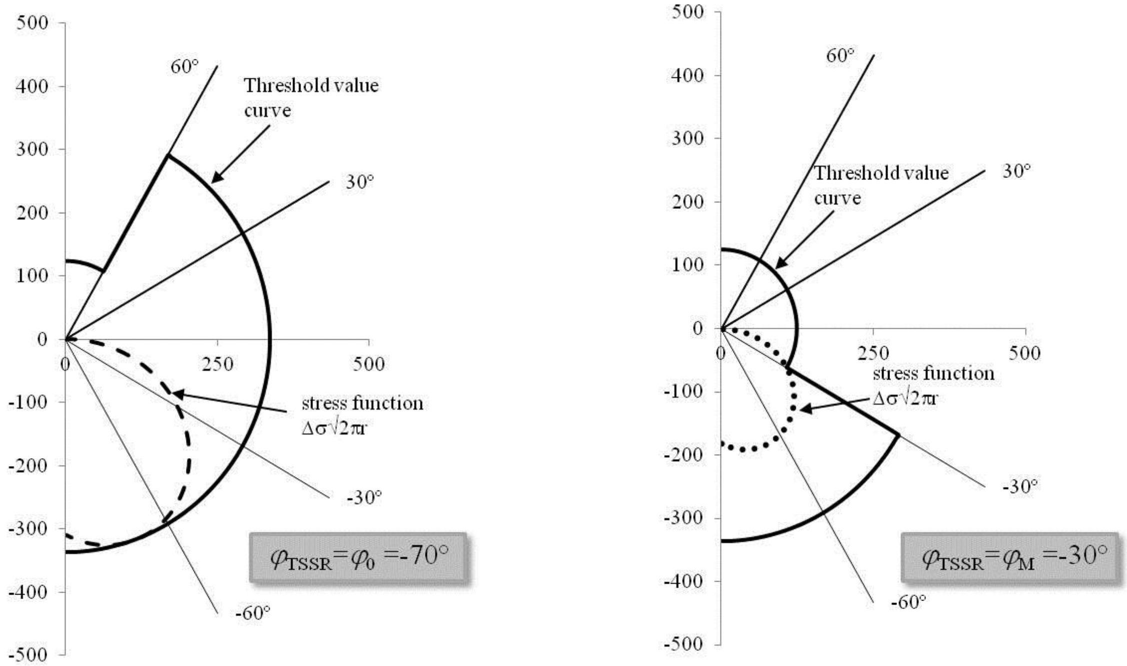
Mixed Mode. A gradation angle of φ_M=60° is presented in Fig.10a considering a Mixed Mode loading situation with a *V*-ratio of 0,33. As can be seen the kinking angle φ_{TSSR} is defined by the stress-controlled kinking angle $\varphi_{0,MTS}$ for this loading situation at -40,3°. A gradation angle of φ_M=0° is shown in Fig. 10b. In this case the kinking angle φ_{TSSR}, theoretically determined using the $\sigma_{\varphi TSSR}$-concept, is the gradation angle φ_M.



90°<φ< φ_M: martensite; φ_M <φ< -90°: base material

a) b)

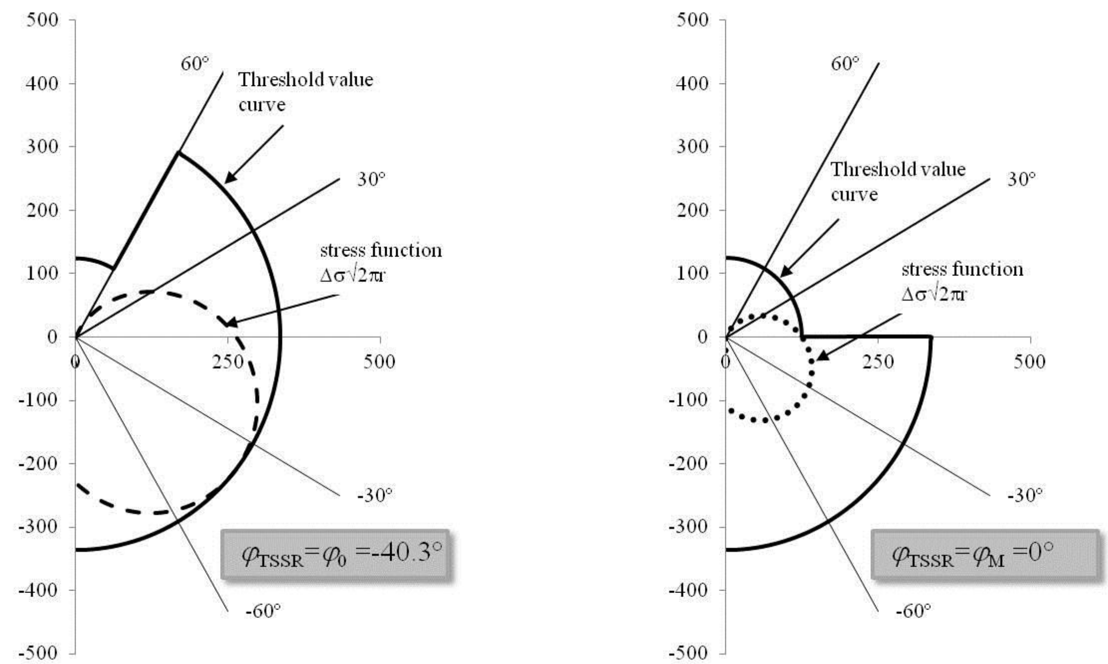
Fig.8. Mode I stress functions $\Delta\sigma_\varphi\sqrt{(2\pi r)}$ and Threshold value curves with a gradation angle of φ_M=30° (a) and φ_M=-60° (b) in comparison with experimental investigations.



90°<φ< φ_M: martensite; φ_M <φ< -90°: base material

a) b)

Fig.9. Mode II stress functions $\Delta\sigma_\varphi\sqrt{(2\pi r)}$ and Threshold value curves with a gradation angle of φ_M=60° (a) and φ_M=-30° (b).



90°<φ< φ_M: martensite; φ_M <φ< -90°: base material

a) b)

Fig.10. Mixed Mode stress functions $\Delta\sigma_\varphi\sqrt{(2\pi r)}$ and Threshold value curves with a gradation angle of φ_M=60° (a) and φ_M=0° (b).

Summary

The presented considerations show that a fracture mechanical material gradation has a significant influence on the region of stable fatigue crack growth as well as on the crack velocity. According to the gradation constellation either a positive or a negative impact on the expected lifetime of the structure is possible. In addition a hypothesis was presented to determine the start of unstable crack growth as well as the direction of crack propagation in fracture mechanically graded materials. First experimental investigations were carried out and confirm the presented $\sigma_{\varphi\mathrm{TSSR}}$ – concept.

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