

Incubation Time Based Fracture Mechanics

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Abstract. Structural-temporal theory for the analysis of the multi-scale nature of dynamic fracture based on the notion of a spatial-temporal fracture cell for different scale levels is presented. It was shown that incubation time based approach can be an effective instrument for both modeling principal dynamic fracture effects and establishing interconnections between dynamic strength properties on different scales.

Introduction

The dynamic fracture of quasi-brittle heterogeneous materials is governed by processes at several different scale levels. Each of these processes is either independent or dependent on the others. In order to model the dynamic fracture of such materials, it is necessary to account for all the rupture processes that contribute to the overall fracture process.

The most common difficulty in predicting the dynamic fracture of typical heterogeneous quasi-brittle materials (concrete, rocks) is the lack of adequate experimental data on the material properties at the relevant scale level. The tests on these materials are performed on laboratory scale samples (ranging in size from several centimeters to a couple of meters), while the size of most structures made of them ranges from tens of meters (concrete structures) to kilometers (geological objects). It is not easy or even possible to obtain reliable experimental data on samples of such large dimensions. It is thus necessary to predict the behavior of the material on the large size level using the properties measured on a smaller scale level. In order to do so certain assumptions have to be made consistent with the knowledge about the dependency of the fracture properties on the process scale level. Should these assumptions be confirmed by experiment at the laboratory scale, then it will be possible to predict the fracture properties on the “next” scale level to the laboratory scale. It is even conceivable that for some quasi-brittle materials it will be possible to predict fracture on any scale level from the properties measured on the laboratory scale level.

Unfortunately, no satisfactory approach to the problem exists even today. Several attempts have been made to compare the strength properties of the same material on different scale levels [1], but these have not provided a systematic explanation of the observed dependencies.

In this paper we shall propose a model for the dynamic fracture of quasi-brittle heterogeneous materials on different scale levels based on our experience in experimental, theoretical and numerical investigations of multi-scale fracture. The idea of interconnected fracture scale levels originates from the concept of a fracture cell implicit in the quasi-static fracture criterion introduced by Neuber [2] and later, but independently, by Novozhilov [3,4] and in its later generalization to dynamic fracture by Petrov and Morozov [5-7]. The generalization to dynamic fracture involves the

notion of incubation time, thus introducing a spatial-temporal discretization and “quantizing” the fracture process [7].

Incubation Time Approach

Experiments on the dynamic loading of solids reveal a number of effects indicating a fundamental difference between the fast dynamic rupture (breakdown) of materials and a similar process under slow quasistatic loads. For example, one of the basic problems in testing of dynamic strength properties of materials is associated with the dependence of the limiting rupture characteristics on the duration, amplitude, and growth rate of an external load, as well as on a number of other factors. While a critical value for strength parameter is a constant for a material in the static case, experimentally determined critical characteristics in dynamics are found to be strongly unstable, having a behavior that is unpredictable. The indicated (and some other) features of the behavior of materials subjected to pulsed loads are common for a number of seemingly quite different physical processes, such as dynamic fracture (crack initiation, propagation, arrest and spalling), cavitation in liquids, electrical breakdown in insulators, initiation of detonation in gaseous media, etc. Unified interpretation for fracture of solids, yielding and phase transforms is possible, constituting structural-time approach [5-7], based on the concept of the incubation time of a transient dynamic process.

The main difficulties in modeling the aforementioned effects of mechanical strength, yielding and phase transitions is the absence of an adequate limiting condition that determines the possibility of rupture, yield or phase transform. The problem can be solved by using both the structural fracture macromechanics and the concept of the incubation time of the corresponding process, representing nature of kinetic processes underlying formation of macroscopic breaks, yield flow or phase transformation. The above effects become essential for impacts with periods comparable to the scale determined by the fracture incubation time that is associated with preparatory relaxation processes accompanying development of micro defects in the material structure.

The criterion of fracture based of a concept of incubation time proposed in [5-8] makes it possible to predict unstable behavior of dynamic-strength characteristics. These effects are observed in experiments on the dynamic fracture of solids. The fracture criterion can be generalized:

$$\frac{1}{\tau} \cdot \int_{t-\tau}^t \left(\frac{F(t')}{F_c} \right)^\alpha dt' \leq 1. \quad (1)$$

Here, $F(t)$ is the intensity of a local force field causing the fracture (or structural transformation) of the medium, F_c is the static limit of the local force field, and τ is the incubation time associated with the dynamics of a relaxation process preparing the break. It actually characterizes *the strain (stress) rate sensitivity* of a material. The fracture time t_* is defined as the time at which condition Eq. (1) becomes equality. The parameter α characterizes *the sensitivity of a material to the intensity (amplitude)* of the force field causing fracture (or structural transformation).

Using an example of mechanical break of a material, one of the possible methods of interpreting and determining the parameter τ is proposed. It is assumed that a standard sample made of a material in question is subjected to tension and is broken into two parts under a stress P arising at a certain time $t = 0$: $F(t) = PH(t)$, where $H(t)$ is the Heaviside step function. In the case of quasi-brittle fracture, the material should unload, and the local stress at the break point should decreases rapidly (but not instantaneously) from P to 0. In this case, the corresponding unloading wave is generated,

propagates over the sample, and can be detected by well-known (e.g., interferometric) methods. The stress variation at the break point can be conditionally represented by the dependence $\sigma(t) = P - Pf(t)$, where $f(t)$ varies from 0 to 1 within a certain time interval T . The case $f(t) = H(t)$ corresponds to the classical theory of strength. In other words, according to the classical approach, break occurs instantaneously ($T = 0$). In practice, the break of a material (sample) is a process in time, and the function $f(t)$ describes the *micro-scale level* kinetics of the transition from a conditionally defect-free state ($f(0) = 0$) to the completely broken state at the given point ($f(t_*) = 1$) that can be associated with the macro-fracture event (Fig. 1). On the other hand, applying fracture criterion (1) to *macro-scale level* situation ($F(t) = PH(t)$), the relation for time to fracture $t_* = T = \tau$ for $P = F_c$ is received.

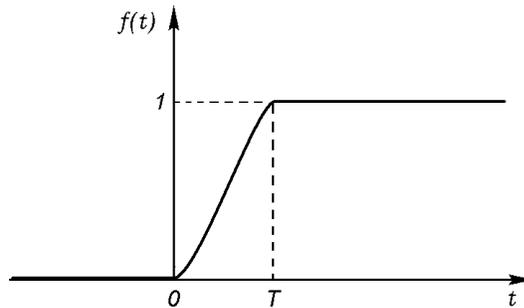


Fig. 1. Schematic representation of *micro-scale level* kinetics of fracture of a sample at the break point.

In other words, the incubation time introduced above is equal to the duration of the fracture process after the stress in the material has reached the static breaking strength *on the given scale level*. This duration can be measured experimentally statically fracturing samples and controlling the rupture process by different possible methods, e.g., measuring the time of the increase pressure at the unloading wave front, which can be determined by the interferometric (visar-based, or photoelasticity-based) method using the velocity profile of points of the sample boundary. Below, we analyze examples of the actual application of general criterion Eq. (1) to various physical and mechanical problems.

Dynamic Fracture Toughness Prediction

Principal effects in the behavior of the dynamic fracture toughness can be predicted and explained on the basis of incubation time notion. Rate dependences $K_{I,d}$ of the dynamic fracture toughness, which were observed in experiments, are characterized by a strong instability and can noticeably change when varying the duration of the load rise stage, the shape of the time profile of a loading pulse, sample geometry, and the method of load application (Ravi-Chandar and Knauss [11], Kalthoff [12], Dally and Barker [13], Smith [14]). The calculations based on the concept of the incubation time corresponding to the conditions of a number of experiments were carried out by Petrov and Morozov [1,2]. The total rate dependence of fracture toughness can be obtained on the basis of incubation time criterion Eq. (1). For the crack initiation problem under symmetrical loading conditions (K_I -mode) this criterion takes the form:

$$\frac{1}{\tau} \int_{t-\tau}^t K_I(t') dt' \leq K_{Ic} \quad (2)$$

where $K_I(t)$ is the time dependence of stress intensity at the crack tip, K_{Ic} is the mode I fracture toughness, measured under quasi-static experimental conditions. The scheme for the application of criterion Eq. (2) to crack growth initiation problems is given in [5-8].

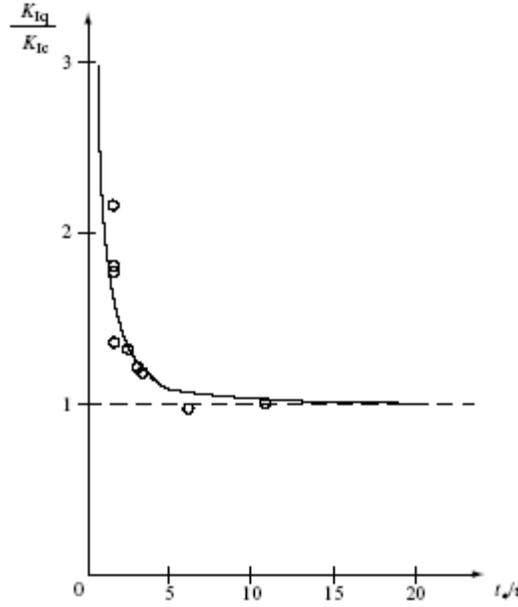


Fig. 2. Critical stress intensity factor K_{Iq} (dynamic fracture toughness K_{I_d}) versus time to fracture for Homalite-100: theoretical dependence calculated by the incubation time criterion [5] and experimental points by Ravi-Chandar and Knauss (1984) [11].

An example of our *analytical* calculation using criterion Eq. (2) for the rate dependence of fracture toughness for Homalite-100 ($\tau = 9\mu s$, $K_{Ic} = 0.48MPa\sqrt{m}$) for endless trapezoidal pulses of loading pressure applied to the crack faces is represented in Fig. 2 by the solid curve. Experimental points in Fig 2 were reported by Ravi-Chandar and Knauss [11].

On the other hand a decreasing and non monotonic behavior of dynamic fracture toughness with the decrease of time to fracture is also observed in a number of experiments (e.g. Kalthoff [12]). This phenomenon also can be predicted and explained by means of criterion Eq. (2) (Petrov and Morozov [5]). All manners of dynamic fracture toughness behavior – decrease, increase, and non monotonic change can be received for any particular material with the same fixed material constants τ and K_{Ic} .

Thus, our results show that the dynamic fracture toughness is not an intrinsic characteristic of a material. Moreover, the employment of both the criterion of the critical intensity factor $K_I(t) \leq K_{I_d}$ and the characteristic K_{I_d} as a *material parameter* defining the dynamic strength (in analogy to the static parameter K_{Ic}) are incorrect.

Spatial – Temporal Discreteness of the Fracture Process

The incubation time fracture criterion, originally proposed in [5-7] for predicting crack initiation

under dynamic loading conditions, states that fracture will initiate at a point x at time t when

$$\frac{1}{\tau} \int_{t-\tau}^t \frac{1}{d} \int_{x-d}^x \sigma(x',t') dx' dt' \geq \sigma_c . \quad (3)$$

Here, τ is the incubation time of the dynamic fracture process (or the fracture micro-structural time). It characterizes the response of the material to the applied dynamic loads; it is constant for a given material in the sense that it does not depend on the geometry of the test specimen, the way the load is applied, or the shape or amplitude of the load pulse. d is a characteristic size of the fracture process cell (zone) and is a constant for the given material and the chosen spatial scale. σ is the normal stress at the point which varies with time and σ_c is its critical value (i.e. the ultimate tensile strength evaluated under quasi-static conditions).

Assuming, as in the Irwin's small scale yielding approximation, that $d = \frac{2}{\pi} \frac{K_{Ic}^2}{\sigma_c^2}$. It can be shown

that within the framework of linear elastic fracture mechanics (LEFM), the dynamic crack initiation criterion (3) for an existing mode I loaded crack is equivalent to Eq. (2). This follows from the requirement that (3) is equivalent to Irwin's criterion, $K_I(t) \geq K_{Ic}$, under quasi-static conditions ($t_*/\tau \rightarrow \infty$). This means that a certain size characterizing the fractured material appears in the dynamic fracture initiation criterion. This size is associated with the size of the failure cell on the current spatial scale – all ruptured cells sized less than d cannot be regarded as failure cells on the current scale level.

Thus, by the introduction of τ and d the temporal-spatial domain is discretized. Once the current working scale for a given material has been chosen, τ gives the time in which the energy accumulated in the cell of size d is enough to rupture it. We believe that a correct description of high loading rate effects requires the introduction of this temporal-spatial discreteness. The advantage of the incubation time approach is that one can remain within the framework of continuum linear elasticity and allow for the discreteness of the dynamic fracture process only inside the critical fracture condition.

As has been demonstrated previously ([7-10]), the dynamic fracture criterion (3) successfully predicts fracture initiation in brittle solids. For slow loading rates when the times to fracture are much longer than τ , the criterion (3) is equivalent to classic Irwin's criterion. For high loading rates when the times to fracture are comparable with τ , a variety of effects observed in dynamical experiments ([11-14]) has been explained qualitatively and quantitatively using (3) [15]. The application of (3) for the description of real experiments or in the finite element analysis of dynamic fracture allows us to gain a better understanding of the nature of dynamic fracture [16] and even to predict new effects typical for dynamical processes ([9,17]).

Interconnection of Rupture Processes on Different Scale Levels

Consider a dynamic fracture process that is described by the incubation time criterion. Assume that the set of material parameters in this criterion σ_c, d, τ (or σ_c, K_{Ic}, τ) has been determined at a given scale level. We now assume that the characteristic length at this scale level is bounded from above and below. Thus, material volumes smaller in size than the lower bound

$$d \cong \frac{2}{\pi} \frac{K_{Ic}^2}{\sigma_c^2} \quad (4)$$

cannot display fracture at this scale level. Therefore, test samples smaller in size than this bound cannot be used to determine experimentally the dynamic fracture parameters at this scale level.

The upper bound of the characteristic length is

$$D \cong c \tau \quad (5)$$

where c is the speed of the energy transport in the material. This sets an upper limit on the volume of material in which the total energy accumulated during the incubation period is able to produce fracture. Test samples larger in size than this bound cannot be used to determine experimentally the dynamic fracture parameters at this scale level.

The admissible characteristic size L of the test specimen at a given scale level (at which fracture properties, σ_c, K_{Ic}, τ , are measured) should be in the range:

$$d \leq L \leq D$$

It follows that the i -th scale level is characterized by a pair of linear sizes $\{d_i, D_i\}$ and the admissible range of test specimen size is

$$d_i \leq L \leq D_i. \quad (6)$$

We now choose

$$d_{i+1} = D_i \quad (7)$$

so that

$$\tau_i = \frac{d_{i+1}}{c} = \frac{D_i}{c} \quad (8)$$

Thus, knowing the strength properties and the fracture incubation time on some scale level, it is possible to estimate the upper bound for the smaller scale level, the lower bound for the larger scale level and the incubation time for the smaller scale level. In this manner, the incubation time dynamic fracture criterion allows us to establish interconnections between the fracture properties on different scales. It also makes it possible to assign the fracture scale levels for different experiments on the same material.

“Non Local” Spatial - Temporal Modeling

In many practical situations for which nonlinear fracture mechanics is more appropriate another approach based on a generalization of the spatial-temporal approach can be followed to account for substantial plastic deformations (e.g. in highly plastic steels) or large “fracture process zones” (e.g. in quasi-brittle materials like concretes or rocks). A large “process zone” observed at the laboratory scale level can be regarded as a material parameter that remains constant in absolute size as the sample size increases. Thus, the “process zone” for large samples will play the same role as a “small scale yielding zone” of fracture (allowing the application of LEFM) for much larger scale fracture processes. This (sudden) transition from large scale “yielding” to small scale “yielding” can result

in catastrophic fracture as, for example, in the case of dynamic fracture of gas pipelines [19,20] or the fracture of large concrete beams [26,27]. The analysis of fracture in these situations can be effectively performed using the “nonlocal” alternative of the spatial-temporal approach. In this alternative approach the incubation time criterion can be written as:

$$\int_{t-\tau}^t \int_{x-l_p}^x \sigma(x',t') f(x') dx' dt' \geq G_F \cdot \tau \quad (9)$$

where $f(x) = -\frac{dw}{dx}$ is a weight function describing the spatial rate of the crack opening $w = w(x)$, l_p is the ‘process zone’ size and G_F is the specific fracture energy, i.e. the energy dissipation per unit area in the ‘process zone’. In [26, 27] several different variants of the weight function have been proposed for quasi-brittle materials. Mathematically, the only difference between the criteria (1) and (9) is in the choice of the weight function $f(x)$. In the simplest special case of a linear crack opening in the process zone $f(x)=1$ so that $G_F = \sigma_c \cdot l_p$, where σ_c is the ultimate strength in quasi-static conditions and τ is the incubation time of fracture measured in laboratory scale tests of the dynamic strength of ‘defect-free’ standard samples. The dynamic fracture criterion (3) that is a generalization of the classic static LEFM criterion can be recovered as a limiting case of (9) for the linear crack opening function and a small ‘process zone’.

The linear crack opening approximation is the easiest for analysis and convenient for practical qualitative estimations, although it is not suitable for analyzing the precise behaviour of the fracture zone. It gives results that are qualitatively similar to those of the more sophisticated nonlinear models of large “process zones”. Should further research however suggest the need for a detailed nonlinear model, the present analysis will not require major modifications. Assume that tests have been performed on samples which are “sufficiently large” in size on a given scale level in the sense that the size of the “process zone” approaches its upper limit [26]: $l_p \rightarrow l_{p\infty} = D = c \cdot \tau$ and $G_F \rightarrow G_D = \sigma_c \cdot D$. Then, for the prediction of fracture of sufficiently large structures made of nonlinear (“plastic”) materials or quasi-brittle materials with extensive “process zones” it is sufficient to use the following fracture condition:

$$\int_{t-\tau}^t \int_{x-D}^x \sigma(x',t') dx' dt' \geq G_D \cdot \tau \quad (10)$$

Now let us consider two examples demonstrating that interconnections of scales introduced in previous section permits the prediction of fracture parameters on a higher (real) scale level based on the test data obtained on a lower (laboratory) scale. This predictive capability is of vital importance in many applications in which it is not possible to evaluate the dynamic material properties on the real structural scale level (e.g. geological formations, large concrete structures, trunk pipelines, etc.).

Modeling Propagation of Cracks in Trunk Gas Pipelines

The above multi-scale approach was applied to the modeling of dynamic crack propagation in a trunk gas pipeline [18, 19, 20]. The data on the strength properties of the pipeline steel were obtained on laboratory scale level (on test samples several centimeters in size), while the actual pipe had a diameter of 1.22 m and was (practically) infinitely long. Laboratory tests on relatively small notched specimens of the pipe steel exhibited large plastic zones and ductile tearing, while large

pipe sections failed in tests after long crack propagation without significant plastic deformation akin to quasi-brittle fracture. The multi-scale approach generally is based on the scaling of the material properties from the laboratory scale to the large scale of the tested pipe sections.

The “non-local” spatial-temporal criterion (10) was used to conduct a qualitative analysis of the above problem of crack propagation in a trunk gas pipeline. The large scale fracture of a pipe section (length 9 m, diameter 1.22 m) was modeled. The size of an element on the crack path was chosen equal to $D = c_1 \tau = 90\text{mm}$, where c_1 is the speed of the longitudinal wave in the pipe steel and τ is the fracture incubation time measured on laboratory scale specimens of this steel in which the fracture process was accompanied by large plastic deformations.

The pipe was subjected to internal pressure close to the operational pressure in the gas pipeline. The drop in pressure in the pipeline as a result of crack extension was modeled by the motion of two wave fronts: the front wave of the pressure drop (velocity of this front is equal to the velocity of the acoustic wave in the gas – approximately 400 m/s for natural gas) and the back front of the pressure drop (travelling at a lower speed). After the passage of the back front, the pressure inside the pipeline is equal to the external atmospheric pressure. Between the two fronts, the pressure was assumed to vary linearly with the distance along the pipe. The fracture was initiated from a small artificial precursor crack, mimicking the appearance of a real crack in the pipeline.

The incubation time fracture criterion (10) was used to predict the conditions for release of the nodes along the crack path (i.e. for the initiation of crack growth) [18]. The modeling was performed on three different grades of steel (X80, X90 and X100), differing only in the ultimate tensile strength (625 MPa for X80, 711 MPa for X90, and 748 MPa for X100). Other material properties of all three grades were the same: density $\rho = 7800 \text{ kg/m}^3$, Young’s modulus = 210 GPa, Poisson’s ratio = 0.3, and the fracture incubation time $\tau = 15 \mu\text{s}$. Figure 3 shows the crack extension histories of the pipeline made from three different steels.

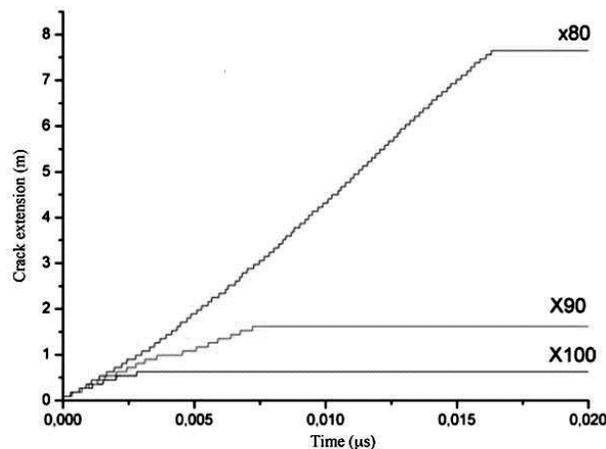


Fig.3. FEM crack extension histories of a pipeline made of different steels [18]

It was found that the speed of the crack is close to the speed of the acoustic wave in the gas (i.e. the speed of the wave front of the pressure drop). The small difference in the ultimate tensile strength of the pipeline steel leads to a qualitative change in the crack propagation regime; if the speed of the crack exceeds the speed of acoustic wave in the gas, the crack will never arrest.

The instability of crack propagation regimes revealed by the finite element analysis is in good agreement with experimental observations on dynamic cracking in gas pipelines [19]. In the experiments a section of the pipe (length 9 m) was subjected to an internal pressure close to the

operational pressure inside the gas pipeline. A furrow was made in a part of the pipe surface along its length. This furrow was filled with an explosive. When the explosive was detonated, a crack started to propagate from the base of the furrow. Pipe sections made of several different pipe steel grades were tested. It was found that length of the crack before it arrested depended strongly on the ultimate tensile strength of the pipe steel and varied over two orders of magnitude from 3 to 300 m, despite the fact that the ultimate static tensile strength of the three steel grades differed by less than 20%.

Dynamic Fracture of Reinforced Concrete

The high-performance fiber reinforced concrete, CARDIFRC, developed and produced at Cardiff University [21-23] was studied in [24] using the Hopkinson Split Bar (HSB) equipment (with steel bar diameter 20 mm). Standard Brazil tests were conducted on circular discs of CARDIFRC (diameter 15 mm and thickness 10 mm). The standard Kolsky method was used.

These experiments also were analyzed using the incubation time theory [24]. The incubation time criterion for dynamic fracture (3) was applied in order to calculate the variation of the critical failure stress as a function of the stress or strain rate. The dynamic split tensile strength can be calculated on the basis of the following critical condition flowing from (3) in the special case that the material is initially ‘intact’:

$$\frac{1}{\tau} \int_{t-\tau}^t \sigma(t') dt' \leq \sigma_c \quad (11)$$

Here, $\sigma(t)$ is time dependence of the local tensile stress at the fracture point, σ_c is the quasi-static split tensile strength (for CARDIFRC = 23 MPa), and τ is the incubation time of fracture.

The test results and the predictions based on (11) are presented in Figure 4. The best agreement between the test results and predictions is achieved when the incubation time τ equals $15 \mu\text{s}$.

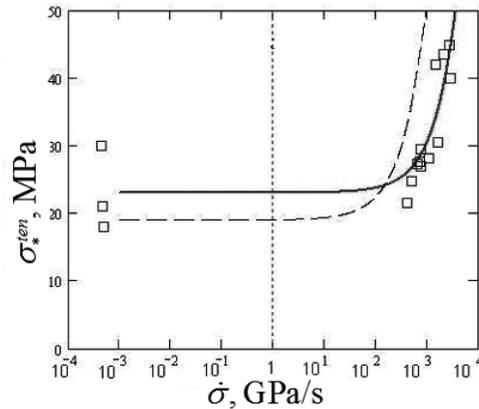


Fig.4. Dynamic tensile split strength of CARDIFRC measured experimentally and calculated with help of criterion (11) in [24] (solid line $\sigma_c = 23 \text{ MPa}$ and $\tau = 15 \mu\text{s}$); dashed line corresponds to granite ($\sigma_c = 19 \text{ MPa}$ and $\tau = 70 \mu\text{s}$ - data from [25])

In order to demonstrate the possible effect of loading rate on the strength of a material a comparison is made between CARDIFRC and granite. The dynamic split strength (stress at fracture) of the latter obtained under the same experimental conditions is also shown on Fig. 4 as a function of stress rate. The quasi-static split tensile strength of granite is 19 MPa and its incubation time is $70 \mu\text{s}$ [25]. As seen on Fig. 4, although CARDIFRC has a higher quasi-static split strength than that of granite, its dynamic split strength is lower at high stress rates ($>10^{2.5}$).

The incubation time for CARDIFRC determined above can be used to estimate the upper bound for the experimental scale level $D = c\tau = 67.5 \text{ mm}$, as $c = 4500 \text{ m/s}$ for CARDIFRC. Following the previous discussion about the interconnection of rapture processes on different scale levels, we can expect that LEFM can be used on samples much larger in size than 67.5 mm to study the fracture of CARDIFRC. In fact, this is confirmed by the theoretical and experimental investigations reported in [27, 28], albeit on concrete without fibers.

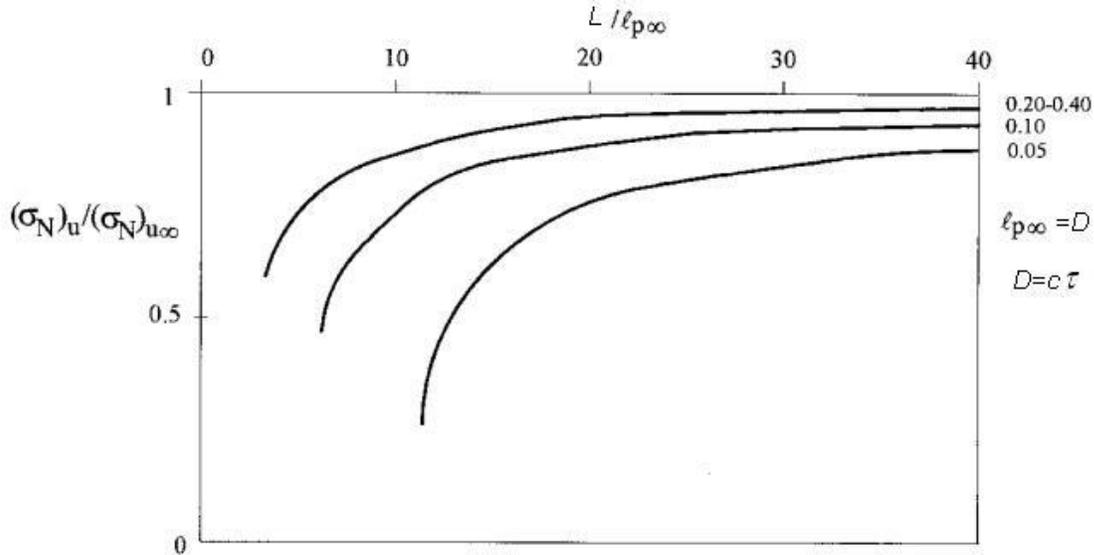


Fig.5. Variation of the nominal failure strength with depth for several values of the pre-crack size [26]. In the range $0.2 < \alpha < 0.4$ the variation of the nominal failure strength is nearly the same ($\alpha =$ notch to depth ratio).

If the size of the process zone in an “infinitely” large specimen is identified with D , i.e. $l_{p\infty} = D = c\tau$ then from Fig. 5 (and in agreement with the observations reported in [27, 28]) it is clear that LEFM can be applied to concrete structures with characteristic sizes $L \approx 1.5 - 2 \text{ m}$ (i.e. 20-30 times larger than the estimated $D = 67.5 \text{ mm}$).

Summary

Examples illustrating typical dynamic effects inherent in dynamic fracture process are analyzed. A unified interpretation for fracture of solids utilizing structural-temporal approach based on the concept of the fracture incubation time is discussed. Corresponding generalized model accounting for fracture scale level is also presented. Notion of spatial-temporal cell for different scale levels is introduced. Problem of experimental determination of a fixed scale level is considered. Two characteristic linear sizes specifying lower and upper boundaries of the scale level should be put in consideration in order to determinate correct experiments and the admissible characteristic size of the test specimen at a given scale level. Knowledge of both static and dynamic properties of the fracture process is principally important for this specification. The problem of the experimental determination of fracture parameters at a given scale level and their possible interconnections with higher and lower scale levels is stated. It is shown that these interconnections can make possible the prediction of fracture parameters on a higher (real) scale level based on the test data obtained on a lower (laboratory) scale. This possibility is of extreme importance for many applications where the possibility to evaluate material strength properties on real structure scale level does not exist (ex. geological objects, big concrete structures, trunk pipelines, etc.).

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