Improvement of fatigue performance by residual stresses: a fracture mechanics approach

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Abstract. The fatigue crack growth behaviour of the quenched and tempered steel 25CrMo4 is investigated at various load ratios and notch depths. The growth behaviour in the notch stress field is monitored in detail, whereby also information about the buildup of crack closure effects and the transition from short to long crack behaviour is gained. A crack growth model which is able to describe the growth behaviour for arbitrary crack length and stress ratio, is developed and adapted to the experimental results. Finally, the application of the model to real components is discussed, taking account of locally variable loads and residual stresses.

Introduction

The high number of cycles and the resulting low allowable flaw size pose a challenge in assessing the damage tolerance of drivetrain components. To decide whether a flaw of a certain length is still safe (at least until the next inspection of the component), it is important to be able to model the crack growth behaviour as precisely as possible. So far, the growth behaviour of short cracks has been described only inadequately or not at all in analytical models. In this paper a simple analytical model for describing the crack growth behaviour for any crack length and stress ratios will be introduced.

Analytical description of the crack growth behaviour

The following considerations on crack growth behaviour are based on the simplified model in Fig. 1a. Starting from a sharp notch with parallel flanks of length a_0, a crack of length Δa grows. The sharp notch can be regarded as a crack of length a_0 which is not subject to any crack closure. Therefore the stress intensity factor is calculated via $K \sim \sigma(\pi a)^{1/2}$ using the whole crack length $a = a_0 + \Delta a$, whereas for the build-up of crack closure only the crack extension Δa takes effect. The analytical description of crack growth under cyclic loading, with the maximum stress $\sigma_{\max}$, the minimum stress $\sigma_{\min}$ and the stress range $\Delta\sigma = \sigma_{\max} - \sigma_{\min}$ as well as the respective stress intensity factors $K_{\max}$, $K_{\min}$, ΔK for any stress ratio $R = \sigma_{\min}/\sigma_{\max} = K_{\min}/K_{\max}$ and crack extension Δa, is based on the crack growth equations according to Erdogan/Ratwani [1] and Forman/Mettu [2] including the Newman's crack closure function [3]. These equations were modified to take account of the short crack behaviour (i.e. for small Δa). A summary and short discussion of these equations can be found in [4]. Already the crack growth equation according to Erdogan/Ratwani [1]

$$\frac{da}{dN} = C' \frac{\left(\Delta K - \Delta K_{\mathrm{th}}\right)^m}{(1-R)K_{\mathrm{c}} - \Delta K} \quad (1)$$

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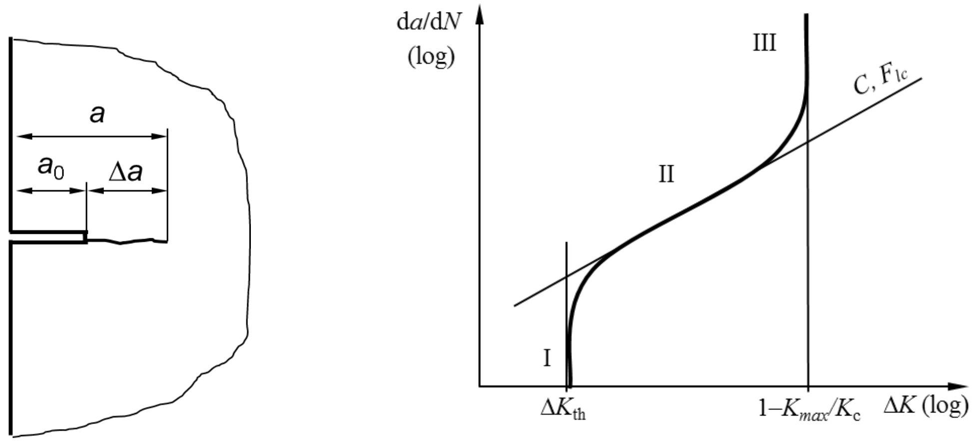
represent the three regions of the crack growth curve (Fig. 1b). Here the stress ratio R is only used to determine the asymptote for unstable crack growth, $\Delta K=(1–R)K_c$ (region III), from the fracture toughness K_c. The position of the asymptote for the crack growth threshold (region I) is determined by ΔK_{th}, and the position of the curve for stable crack growth (region II) is determined by C' and m. Therefore it is necessary to find an analytical description for the dependence of the parameters ΔK_{th} and C' on the stress ratio R and the crack extension Δa.



(a) **(b)**

Fig.1. (a) Sharp notch with growing crack; (b) fatigue crack growth curve

The Forman/Mettu equation [2]

$$\frac{da}{dN}=C\left[\left(\frac{1-f}{1-R}\right)\Delta K\right]^m\frac{\left(1-\frac{\Delta K_{\mathrm{th}}}{\Delta K}\right)^p}{\left(1-\frac{K_{\max}}{K_{\mathrm{c}}}\right)^q}=C\left(\frac{1-f}{1-R}\right)^m\Delta K^{m-p}K_c{}^q\,\frac{(\Delta K-\Delta K_{\mathrm{th}})^p}{(K_{\mathrm{c}}-K_{\max})^q} \quad (2)$$

also represents the three regions of the crack growth curve. Additionally, the curvature at the transition between the different areas can be adjusted with p and q. Setting $p=m$ and $q=1$ leads to

$$\frac{da}{dN}=C\cdot F_{\mathrm{lc}}\cdot\frac{(\Delta K-\Delta K_{\mathrm{th}})^m}{1-\frac{K_{\max}}{K_C}}\ \text{with}\ F_{\mathrm{lc}}=\left(\frac{1-f}{1-R}\right)^m \quad (3, 4)$$

F_{lc} describes the position of region II depending on the stress ratio R for long cracks (large crack extensions Δa, index “lc”). The cause of this dependence is seen in crack closure effects, which are described by the crack opening function

$$f=\frac{K_{\mathrm{op}}}{K_{\max}} \quad (5)$$

Newman [3] achieved, based on finite element simulations of plasticity-induced crack closure for long cracks, the following analytical approximation for the crack opening function:

$$f = \begin{cases} \max(R; A_0 + A_1 R + A_2 R^2 + A_3 R^3, & R \geq 0 \\ A_0 + A_1 R, & -2 \leq R < 0 \\ A_0 - 2A_1, & R < -2 \end{cases} \tag{6}$$

$$A_0 = \left(0{,}825 - 0{,}34\alpha + 0{,}05\alpha^2\right)\left[\cos\left(\frac{\pi\sigma_{\max}}{2\sigma_F}\right)\right]^{1/\alpha} \qquad A_2 = 1 - A_0 - A_1 - A_3$$
$$A_1 = \left(0{,}415 - 0{,}071\alpha\right)\frac{\sigma_{\max}}{\sigma_F} \qquad A_3 = 2A_0 + A_1 - 1 \tag{7}$$

In what follows, the values $\sigma_{\max}/\sigma_F$=0.3 and α=3 are assigned, as it applies for approximate plane strain condition and largely elastic crack behaviour; for a more detailed discussion of these parameters cf. [3,4]. The curves in Fig. 5 (left) for the crack opening function f and Fig. 5 (right) for the crack velocity factor F for Δa = 10 mm ($\approx F_{lc}$) show the behaviour valid for long cracks in graphical form. For cracks with arbitrary crack extension Δa an empirical approach for the crack velocity factor F is developed further below.

The R-dependence of the threshold for long crack growth propagation is approximated by [2,4]

$$\Delta K_{\text{th,lc}} = \Delta K_0 \sqrt{\frac{a}{a + a_{0,\text{fikt}}}}\left[\frac{1-f}{(1-A_0)(1-R)}\right]^{-(1+C_{\text{th}}R)} \tag{8}$$

where ΔK_0 is the threshold for long crack growth at R=0, and C_{th} is an adjustment parameter. $a_{0,\text{fikt}}$ is a fictional intrinsic length scale based on the concept of El Haddad [5] for the approximate consideration of short crack effects. It is assumed that a crack of the total length a=0 shows a threshold value of 0 and crack closure builds up until about the total length a=$a_{0,\text{fikt}}$. At least in the presence of an initial notch a_0, see Fig. 1a, this concept proves to be unsustainable because only the crack extension Δa and not the total crack length a is relevant to the build-up of crack closure effects. Furthermore, also in the absence of crack closure the threshold value is not 0 but equal to the intrinsic threshold for crack propagation $\Delta K_{\text{th,eff}}$. For these reasons the application of the El Haddad correction is not considered (i.e., $a_{0,\text{fikt}}$=0) and Eq. 8 is used exclusively to describe the R-dependence of the long crack threshold.

Instead, for the description of the threshold build-up starting from the intrinsic value of $\Delta K_{\text{th,eff}}$ at a crack extension of Δa=0 to the long crack growth threshold $\Delta K_{\text{th,lc}}$ (for large Δa) the empirical approach

$$\Delta K_{\text{th}} = \Delta K_{\text{th,eff}} + (\Delta K_{\text{th,lc}} - \Delta K_{\text{th,eff}}) \cdot \left[1 - \sum_{i=1}^{n} v_i \cdot \exp\left(-\frac{\Delta a}{l_i}\right)\right] \text{ with } \sum_{i=1}^{n} v_i = 1 \tag{9, 10}$$

is proposed. The l_i can be interpreted as fictitious length scales for the formation of crack closure effects (similar to $a_{0,\text{fikt}}$) and determined in conjunction with the v_i by fitting of the experimentally obtained crack growth resistance curve (ΔK_{th} plotted against Δa), see Fig. 4.

Since the crack velocity factor F also describes the effects of the same crack closure mechanisms, in analogy to Eq. 9 the approximation

$$F = 1 - (1 - F_{lc}) \cdot \left[1 - \sum_{i=1}^{n} v_i \cdot \exp\left(- \frac{\Delta a}{l_i} \right) \right] \quad (11)$$

with the same values for the l_i and v_i is chosen. F_{lc} is calculated according to equations (4) and (6). In the crack growth equation (3) then F_{lc} is to be replaced by F. Fig. 5 shows this behaviour graphically.

Experimental investigations

As material for the experimental investigations the QT steel 25CrMo4 widely used for drivetrain components was chosen. The material has a bainitic microstructure and a hardness of ~245 HV10. In the tensile test, a 0.2% yield strength of 512 MPa and a tensile strength of 674 MPa with an elongation at fracture of 18.9% is obtained. For determining the cyclic crack resistance curves, SENB (Single Edge Notched Bending) specimens measuring 100x6x20 mm with different notch depths a_0 (0.35mm, 1 mm, 5.3 mm) were machined. The notches were sharpened by means of a razor blade coated with diamond paste (1 µm). The samples were then compression pre-cracked at a stress ratio of R=10 to obtain an incipient fatigue crack, which is fully open due to the residual tensile stresses from compression pre-cracking so that crack closure effects can be excluded. The samples were then subjected to cyclic loading under eight-point bending in a resonance testing machine at a test frequency of 108 Hz. The crack growth was measured by DC potential drop method.

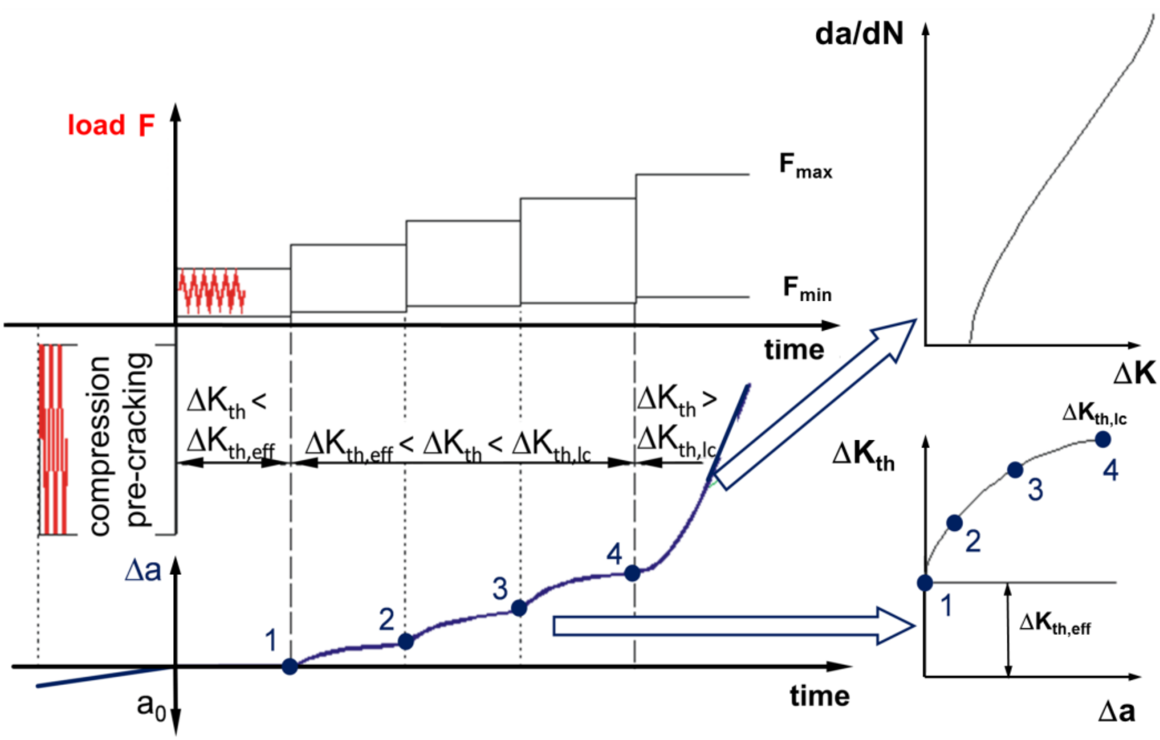


Fig.2. Experimental procedure [6]

The experiments are conducted with step-wise increasing loads, see also [6]. The crack grows initially, but after a certain crack extension Δa crack arrest occurs due to the build-up of crack closure. Subsequently, the load is increased so that the crack can grow further. In this way, the crack resistance curve is obtained point by point (Fig. 2). As soon as the crack starts to grow through, one gets the crack growth curve. This crack growth curve is the one of long cracks, because at that time the crack closure has already built up completely.

Parameter determination

For the investigated material, in place of Eq. 3 the slightly modified crack growth equation

$$\frac{da}{dN} = C \cdot F \cdot \frac{\Delta K^{m} - \Delta K_{\text{th}}^{m}}{1 - \dfrac{K_{\max}}{K_C}} \qquad (12)$$

is used, which gives a significantly better fit of the transitions between the three regions of the crack growth curve.

F is calculated according to Eq. 11 with F_{lc} according to Eq. 4, and ΔK_{th} is calculated according to Eq. 9 with $\Delta K_{\text{th,lc}}$ according to Eq. 8. The model parameters determined from the experiment are summarized in Table 1.

The parameters were obtained as follows: first the parameters for the growth of long cracks were determined from the crack growth curves, Fig. 3. Subsequently, the crack resistance curves were adapted, Fig. 4.

Fig. 5 right shows the curves of the crack velocity factor F according to Eq. 11 for different crack extensions Δa as a function of the stress ratio R. At all stress ratios R, a clear difference between short and long crack behaviour is recognized. In addition, the threshold ΔK_{th} is much lower for short cracks, as is evident from the crack resistance curves in Fig. 4. All this leads to a marked deviation of the growth behaviour of short cracks compared to that of long cracks, see Fig. 3 right.

Table 1. Parameters of the fatigue crack growth model for 25CrMo4

Parameter	**Value**	**Unit**
C	8	[nm / (MPa m$^{1/2}$)]
K_c	90	[MPa m$^{1/2}$]
m	2,88	-
α	3	-
σ_{max}/σ_F	0,3	-
$\Delta K_{\text{th,eff}}$	2,2	[MPa m$^{1/2}$]
ΔK_0	8,5	[MPa m$^{1/2}$]
C_{th}	0,115	-
l_1	0,08	[mm]
l_2	1,55	[mm]
v_1	0,45	-
v_2	0,55	-

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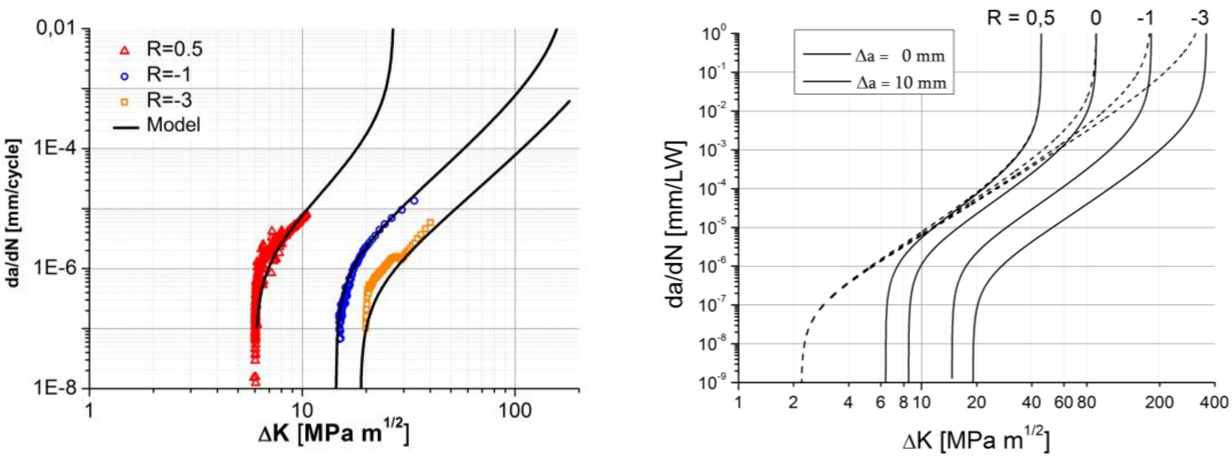

Fig.3. Fatigue crack growth curves at different crack extensions Δ*a* and stress ratios *R* – experiment and analytical model

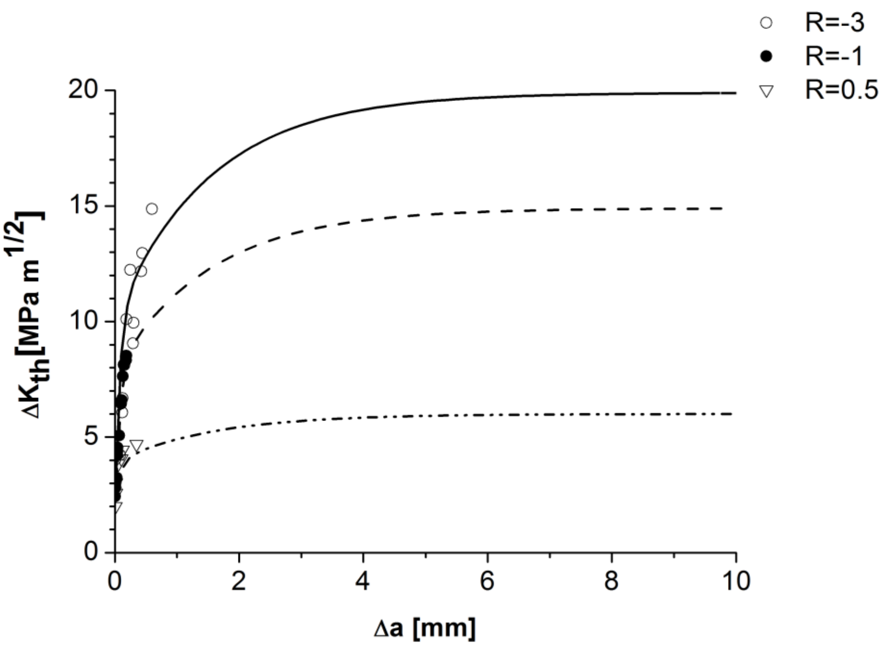

Fig.4. Crack resistance curves: crack growth threshold ΔK_{th} vs. crack extension Δ*a* from Eq. 9 for different stress ratios *R* – experiment and analytical model

Model verification

In order to verify the model, the growth of a short crack starting from a notch of depth a_0 = 0,812 mm (*R*=-1) is calculated and compared with the measured data. Good agreement between measurement and calculation is observed, see Fig. 6. The left limiting curve corresponds to a crack extension Δ*a*=0 (short crack behaviour), the right to a very large Δ*a* (long crack behaviour). In the first two load steps the crack slows down and stops (curves 1, 2). Only after a further increase of the load the crack grows, after an initial slight deceleration, finally through, in the course of which it approaches the behaviour of long cracks (curve 3).

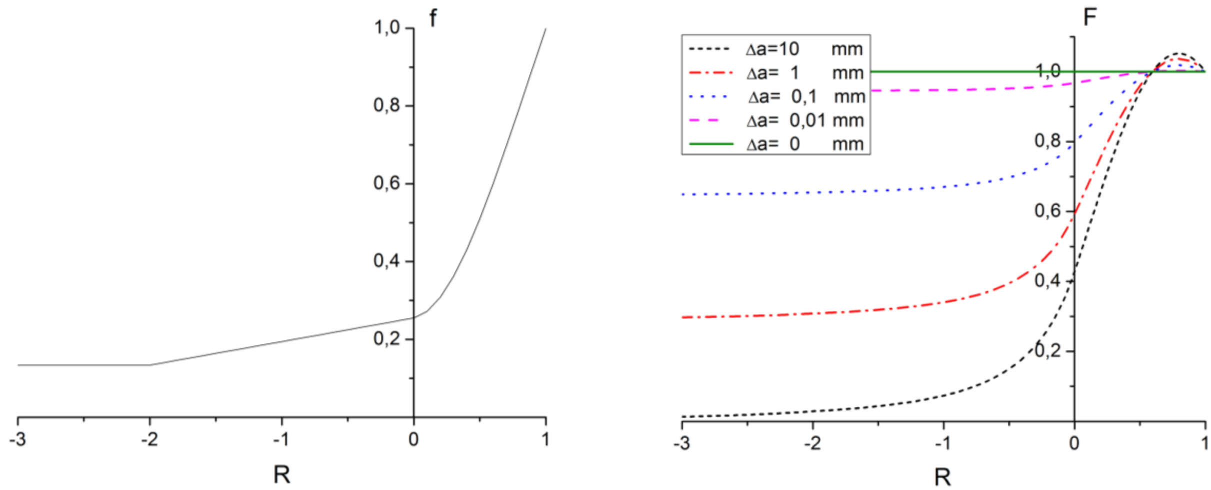

Fig.5. Crack opening stress function f from Eq. (6) and crack velocity factor F from Eq. (11) for different crack extensions Δa depending on the stress ratio R

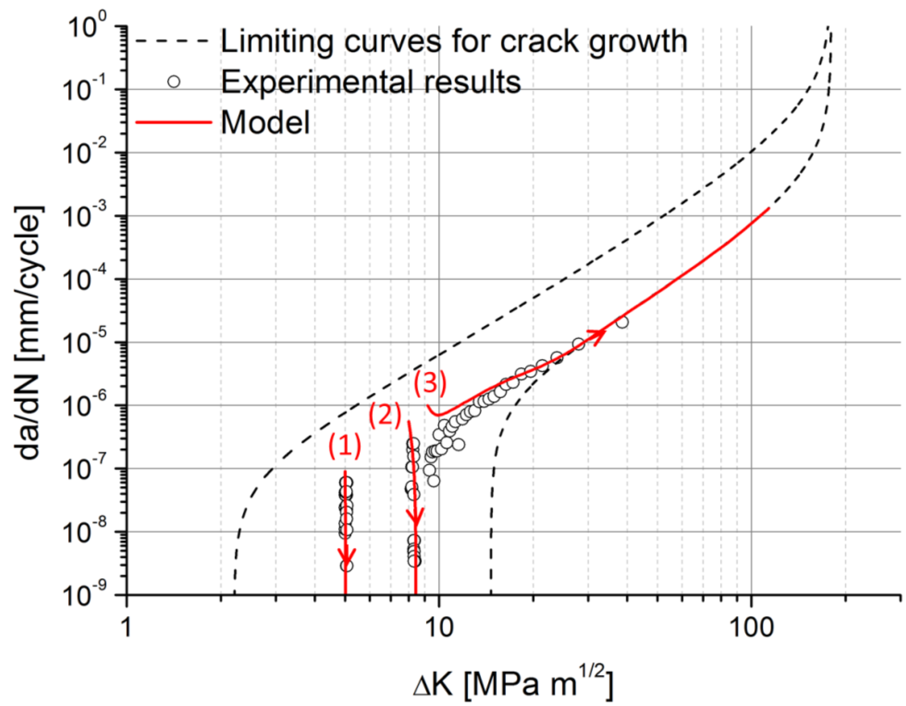

Fig.6. Growth of a short crack – experiment and computation

Damage tolerance assessment procedure

For assessing the damage tolerance of structural component, especially the influence of the locally varying stress fields due to the external loading as well as due to residual stresses must be considered. This is most easily done by using a Green's function [7]

$$\Delta K = \frac{2 \cdot 1{,}12}{\sqrt{\pi a}} \int_0^a \frac{\Delta\sigma(x)}{\sqrt{1-(x/a)^2}} dx \tag{13}$$

where at each point of the crack flank x the cyclic load range $\Delta\sigma(x)$ is given by the difference between maximum and minimum load stress $\sigma_{\max,L}(x)$ and $\sigma_{\min,L}(x)$, respectively. Both for determining the threshold value (for endurance evaluation) from Eq. 9 and for determining the crack growth rate (for lifetime assessment) from Eq. 12 the stress ratio R must be known. Since $\sigma_{\max,L}(x)$, $\sigma_{\min,L}(x)$ and $\sigma_{\mathrm{res}}(x)$ vary locally, the local stress ratio $R(x)$ is obtained from

$$R(x) = \frac{\sigma_{\min,L}(x) + \sigma_{\mathrm{res}}(x)}{\sigma_{\max,L}(x) + \sigma_{\mathrm{res}}(x)} \tag{14}$$

A conservative approach is to determine the maximum of R along the crack flanks; in a somewhat less conservative manner, also be the mean value of R can be used.

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