Fracture initiation on the contact under shear

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Keywords: crack, shear, contact surface, slip, stresses, crack interaction with contact surface

Abstract

Within the contact region of elastic bodies at slipping one can separate the zones where stick of the surfaces being in contact takes place as well as the zones where slip occurs. It is convenient to represent the last zones as a transverse shear cracks such that on the crack surfaces the forces of resistance to slipping act. The stress concentration in the end zones of a transverse shear crack promotes faults initiation in contacting bodies. The results of experiments and an analytical model of the phenomenon are given for the situation which models an intersection of a contact plane inclined relative to acting loads by a fault.

Introduction

A problem on intersection of a plane boundary of two elastic media by a crack of normal tension seems important, in particular, because of possible technologic consequences. According to the observations [1-3] in many cases the intersection is accompanied by a shift of the fault axis along this boundary such that the fault takes a stepped form (Fig. 1). The shift of the fault axis under conditions of external compression is accompanied by local shear along the contact plane. In this place normal displacements of the fault surfaces are replaced by shear displacements of interacting surfaces. As a result a severe decrease of the fault section occurs. The effect can be essential, e.g., at a fluid motion in the crack of hydraulic fracturing in rock massive under compression.

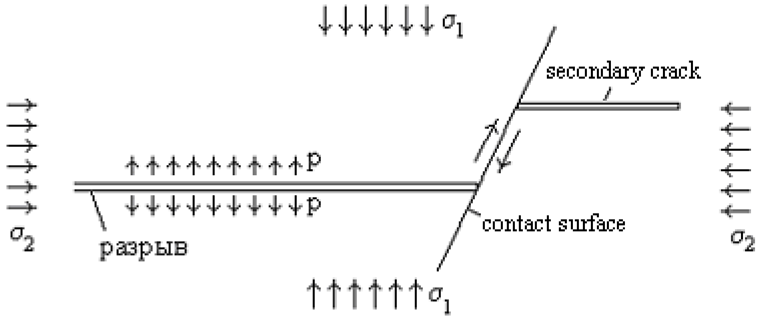

Fig. 1. Scheme of an interaction of the crack and the contact surface

Crack transition along the direction of a normal to the interface of two elastic brittle materials was considered, for instance, in [1]. Initiation of a secondary crack (fault) is related to the asymptotics of stresses in small vicinity of the initial crack tip. According to this asymptotics the maximal tension is observed at an angle to the crack direction instead of its plane. This is only one of the possible scenarios of a crack transition through an interface. In particular, another scenario is realized at the crack exit on an inclined interface. This scenario is realized and analyzed in the given paper.

1. Description of experiments

To model the process of an interface intersection by a tensile crack we performed a series of experiments on a model material (gypsum). The samples-cubes with the edge 50mm having an inclined contact plane (Fig. 2) were tested under uniaxial loading by using a special force band.

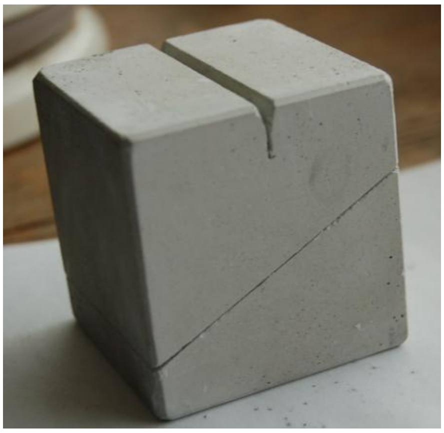

Fig. 2. Combined gypsum sample

Experiments required

- creation a uniform stresses in the sample by preliminary compression and fixation of these stresses by the force band;
- generation of the initial crack up to its exit on the contact plane;
- creation of displacements along the contact plane on its part adjoining to the initial crack and initiation of a crack on another side of the interface.

The scheme of the sample and the fixed parameters are given in Fig. 3. The force band aimed at fixation of the compressive tractions at uniaxial external loading is given in Fig. 4.

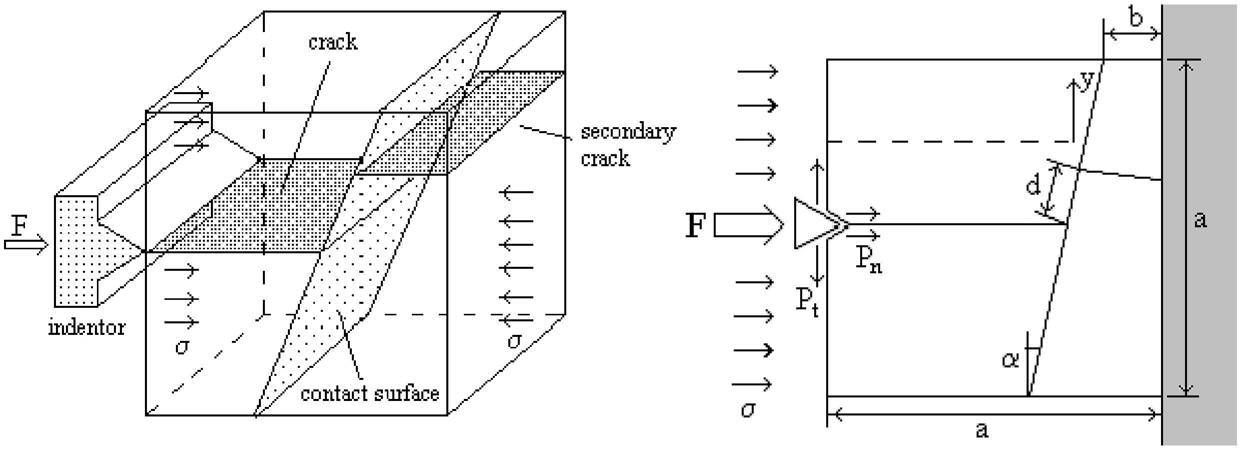

Fig. 3. Sample scheme (a – width and height of the sample, b – distance from the interface exit on the sample side to the foundation, d – distance between the cracks along the interface, α - angle of orientation of the interface)

The experiment consists of the following steps:

- sample compression by external load (testing system) along the vertical axis up to the required stress level;
- the load in the force band is fixed by the screws with spring washer;
- the band is taken out from the space between the testing system plates and a wedge is inserted into a slit on the upper plate of the band;
- the wedge inserted in the slit in the plate and the notch in the sample is pressed in the sample up to the moment of the crack initiation.

2. Experimental results

The experiments confirmed the scenario of the crack motion through the plane of contact of sample parts when the friction takes place on the contact plane. The main crack initiated as the crack of normal tension in its final state also includes an element of slipping along the contact surface and then the crack is continued on another part of the sample as a crack of normal tension.

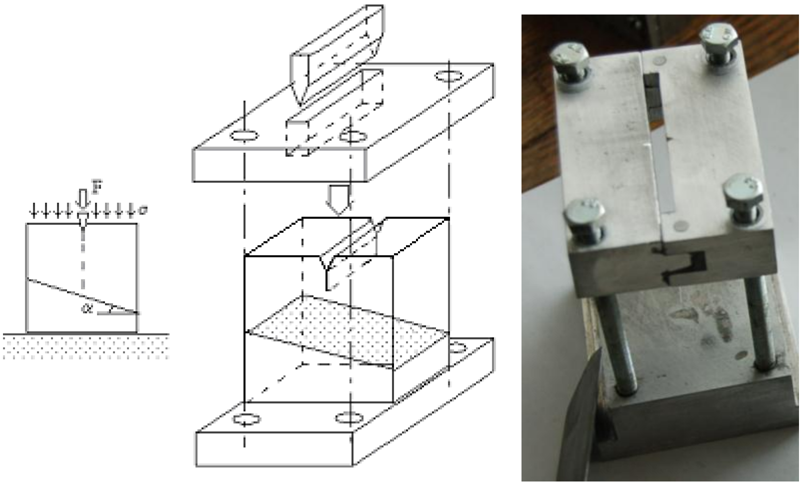

Fig. 4. Scheme of the force band for testing the combined samples. To the left – the loading scheme, to the right - the view of the band assemblage. The slit for the wedge is seen

Since the confined slip should be realized within the limits of a small laboratory sample, the range of the conditions for which the slip can occur is very narrow. This can be illustrated for the tested model material by an interrelation between the stress of the preliminary axial compression σ and distance between the cracks on the contact plane for the same contact plane orientation (Fig.5).

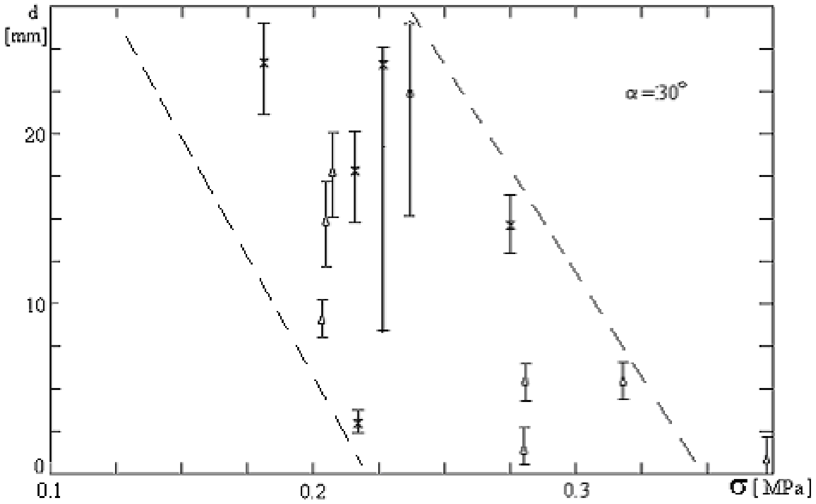

Fig. 5. Variation of the distance between the cracks at axial compression increase. The region of parameters where the mechanism of stepped fracture is realized is marked by dotted lines (x - plane samples, Δ - cubic samples)

At small compression (in the given experiments up to $\sigma < 0.15$ MPa) mutual slipping of the blocks occurs after the crack exit on the interface; the secondary crack is not formed. Such variant of fracture is given in Fig. 6. When the compressive stress exceeds a certain limit level (0.30-0.40 MPa for the interface angle $\alpha = 30^0$) no slipping occurs along the contact plane and the initial crack crosses the whole volume. The front of the main crack is separated after intersection with the interface; transverse cracks occur (Fig. 7).

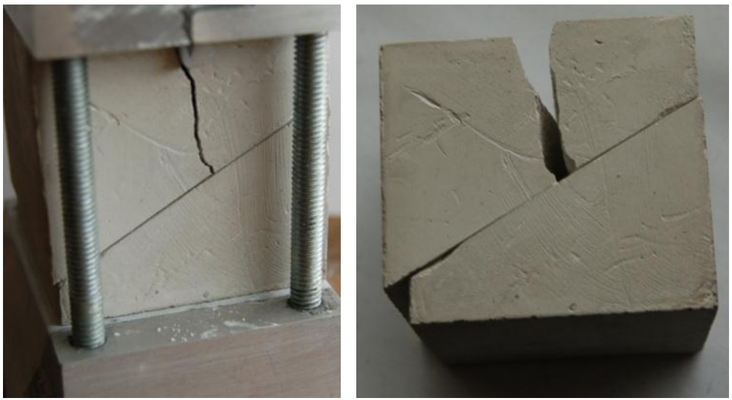

Fig. 6. Sample after testing ($\alpha = 30^0$, $\sigma = 0.15$ MPa)

The range of the loads is shifted at variation of the angle of the contact plane orientation. For instance, the upper stress level decreases down to 0.20-0.25 MPa while the lower level tends to 0.08 MPa at $\alpha = 10^0$. Note, that according to experimental data the limit value of the angle α for which the stepped crack can be formed do not exceed 35-40^0.

Fig. 7. Sample after testing. Secondary cracks - to the right ($\alpha = 30^0$, $\sigma = 0.45$ MPa)

Combined data on an interrelation between the distance from the point of the initial crack exit on the interface and the place of the secondary crack initiation, d, and the angle of the interface orientation are given in Figs 8-9. The distance d increases with the angle α increasing at other things being equal (Fig. 8). At the same time the limit load on the wedge is regularly decreased.

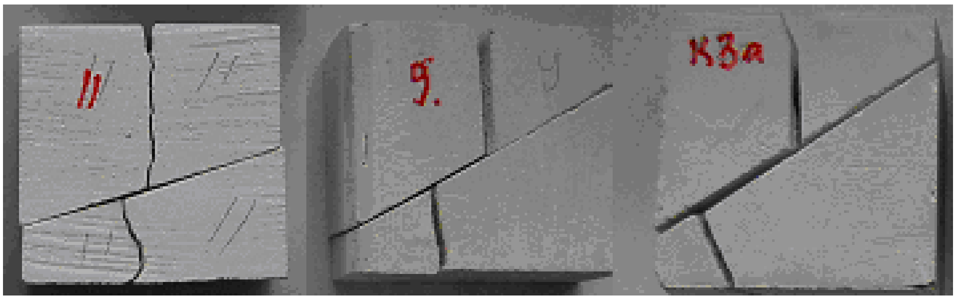

Fig. 8. Samples after testing. An increase of the size d of the slipping zone at increasing the angle α of the interface plane inclination is seen

For interpretation of the obtained data we performed special experiments aimed at determination of the necessary material properties: friction coefficient and tensile strength. Tensile strength (~ 4MPa) and static elasticity modulus (~ 4GPa) correspond to the properties of the materials of the given class [4]. In whole it was shown that the interrelation of normal and shear loads is linear in the state of the limit equilibrium and the Coulomb-Mohr concept is applicable within the loads range realized in the experiments on a crack interaction with an interface.

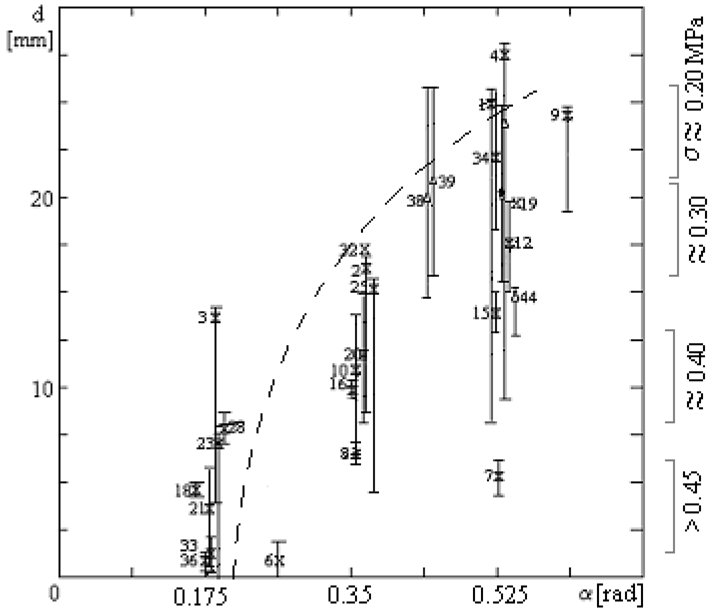

Fig. 9. Interrelation between d and α (x – plane samples, Δ - cubic samples).
Dotted line – results of calculations according to Eq. (12)
for the variants with the different level of compressive stresses

The coefficient of static friction calculated as a ratio of the maximal force F to the normal load P in all cases exceeds 1 and is varied within the limits 2-3 for the used range of normal loads. The effect of stepwise slip (stick-slip) is observed at mutual slipping of the blocks. Fracture of inner acting surfaces occurs in the contact zone. The difference between the friction coefficient for the conditions of motion and rest at the contact equals $\Delta f_{fr} \approx 0.2$-$0.3$.

3. Crack of transverse shear on the slipping plane

Let us make some estimates. The sample block separated by the crack is pressed to the contact plane by the applied forces. An overturn moment acts on this block. In these conditions one can separate the regions of stick and slip within the contact zone. The slip region can be represented as a crack of transverse shear. Let us apply this scheme for evaluation of the conditions of stepped crack growth up to the moment of a secondary tensile crack initiation. The normal stresses at the slipping surface can be represented in the beam approximation as follows

$$\sigma_n \approx \frac{N}{at} + \frac{12My}{t(a/2)^3} \tag{1}$$

where t is the sample thickness and coordinate y is referenced from the middle plane of the slipping block.

The overturn moment is created by the forces applied to the wedge

$$M \approx P_n \frac{a}{4} - P_t \left(a - b - \frac{a}{4} \mathrm{tg}\alpha \right) \tag{2}$$

where P_n, P_t are projections of the forces created by the wedge (see, Fig. 3).

The resulting linear load at the slipping surface represents a sum of the appropriate projections on the contact plane of all forces

$$-N = P_t \sin\alpha + P_n \cos\alpha + \sigma a t \cos\alpha , \tag{3}$$

The linear sliding down force at the slip plane equals

$$T = P_t \cos\alpha + P_n + \sigma a t \sin\alpha \tag{4}$$

Resolution of forces without accounting for small friction forces on the wedge sides leads to the expressions

$$P_t = \frac{P}{2}\operatorname{ctg}(\beta/2); \qquad P_n \approx \frac{P}{2} \qquad (5)$$

where β is the angle at the wedge tip (in the experiment - 15^0).

4. Limit equilibrium condition for the crack of transverse shear

Let us consider a limit equilibrium condition which needs to be fulfilled in the end zone of the transverse shear crack with surfaces being in contact and loaded by shear stresses caused by the friction resistance. Since the crack trajectory is given by the slipping plane – contact boundary it is convenient to represent this condition in the form usually used in the models of quasibrittle fracture. Namely, assume that the condition implies attaining a certain limit stress ahead of the shear tip in its vicinity [5]. The contact zone, as each interface, usually has mechanical properties and structure different from ones for the base material. Hence, one can separate in this zone the transverse size – the contact zone thickness. The end zone of the shear is located within this contact zone. Its size is indeterminate. In particular, this size can correlate with the contact zone thickness. Accounting for that the limit stresses in the shear tip are connected with the conditions of the limit friction and sticking in the contact (static friction), the value of the critical stress intensity factor for the transverse shear crack, K_{IIC}, can be written as follows

$$K_{IIC} \approx \tau_{max}\sqrt{2\pi R} \qquad (6)$$

where R is the characteristic distance (of order of the effective thickness of the contact region $R \approx 0.5h$),

$$\tau_{max} = -\Delta f_{fr}\sigma_n \qquad (7)$$

where Δf_{fr} is the difference between the static and dynamic friction coefficients, σ_n is the normal pressure at the contact plane.

Thus, the critical conditions of transverse shear do not represent the material constant or constant for the given system. These conditions depend on external loads providing the level of the contact pressure. Note, that the value σ_n is determined by Eq. (1) for the situation realized in the experiments. If the shear tip follows the y – coordinate then the value K_{IIC} linearly increases with the shear region increasing for the model under consideration.

Note, that the given variant of the process do not account for a possible local variation of volumes in the shear region at force contact realization. The surface structures can effectively decrease the material volume as this occurs at fracture of high porous bodies. The opposite effect can be realized because of dilatancy. In these cases the condition of the limit equilibrium should account for the normal component of stresses and displacements.

5. Condition for crack initiation in the vicinity of the transverse shear front

Within the framework of the suggested model of the transverse shear crack end zone one can wait for nucleation of a crack of normal tension near the boundary of the intermediate layer in the contact region. According to the observation data (Fig. 8) the angle of orientation of the new crack of normal tension corresponds to the angle 60-80^0 to the shear plane. Hence, fracture stresses in the crack nucleation zone acting transverse to this direction should attain the level of the local material tensile stress. The problem on searching for the location of the secondary fracture source is similar to the problem on choosing the direction of crack deviation in homogeneous material. For simplicity let us use the known asymptotic solution of the problem on the stress state in a vicinity of the transverse shear crack and the criterion of generalized rupture. According to this criterion the direction of rupture is determined by the maximum of circumferential stresses [5]

$$\sigma_\theta = \frac{3K_{II}}{4\sqrt{2\pi r}}\left(\sin\frac{\theta}{2} + \sin\frac{3\theta}{2}\right); \quad \sigma_{\theta max} \approx 0.3\frac{K_{II}}{\sqrt{2\pi r}}; \quad \theta \approx 71^0 \qquad (8)$$

If h is the thickness of the slipping zone then the distance from the shear tip to the rupture source will be equal to

$$r = R^* \approx \frac{h}{2\sin 71^0} = 0.53h \tag{9}$$

Hence, the condition of rupture initiation takes the following form

$$\sigma_{\theta\max} = \sigma_t \approx 0.29\frac{K_{II}}{\sqrt{\pi h}} \quad \text{or} \quad K_{11}^* \approx \frac{\sqrt{\pi h}}{0.29}\sigma_t^* \tag{10}$$

where σ_t^* is the limit stress of the material, K_{II}^* is the stress intensity factor of the transverse shear stresses at rupture initiation.

The sliding down forces create at the slip plane the stresses acting along this plane. The limit stresses are influenced by these forces

$$\sigma_t^* \approx \sigma_t - \sigma^*; \qquad \sigma^* \approx \frac{T}{at}\left(\frac{a}{2\cos\alpha} - \ell\right) / \left(b + \left(\frac{a}{2\cos\alpha} - \ell\right)\sin\alpha\right) \tag{11}$$

where σ_t is the tensile stress, σ^* are the stresses along the slip plane in the conditions of the experiment.

In the conditions of experiments, as mentioned above, the effective fracture toughness K_{IIC} increases with the increase of the shear zone. It means that the probability of secondary crack initiation increases with increasing the shear zone. Initiation of a secondary crack leads to a partial system unloading. As a result shear propagation can be stopped. Let us find an interrelation between the condition of the secondary crack initiation and the critical stress intensity factor for the transverse shear crack.

By incorporating Eqs (6), (7), (10), (11), we obtain the condition which determines the relation between the main properties of material and loading conditions

$$K_{IIC} \approx K_{II}^*; \qquad \Delta f_{fr}\sigma_n \approx 3.35\sigma_t^* \tag{12}$$

The interrelation between the parameters for the example of the experiments conditions is given in Figs 9, 10. One can see that, in accordance with the experimental data, the slip plane orientation is the dominant factor which determines the size of the slip zone between the initial and secondary cracks.

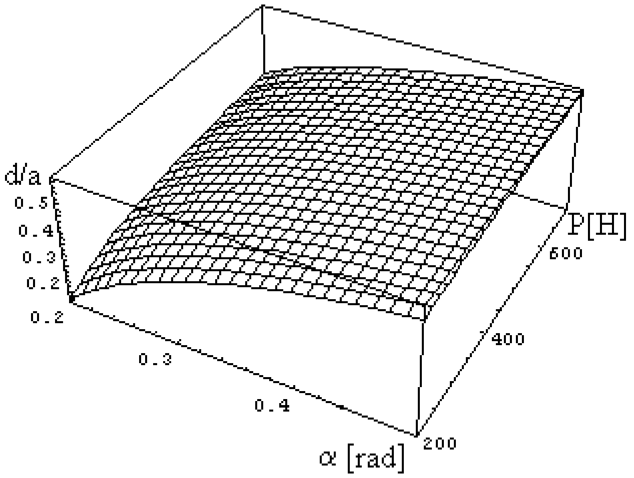

Fig. 10. Interrelation of the experimental parameters for the variant
$\sigma = 0.27\text{MPa}; \ \Delta f_{fr} = 0.2; \ \sigma_t = 4\text{MPa}$

Hence, it was shown that the effects of the sliding down forces at the crack intersection of an inclined slip plane can be more essential for the fracture scenario as compared to the stresses at the tip (front) of the initial crack as it is considered in case of normal orientation of the crack relative to the interface [1, 6].

Conclusion

In the paper we demonstrated a possibility for experimental modeling of the main tensile crack transition through a plane region of compressed block contact accompanied by formation of an intermediate region of transverse shear. A scenario of stepped fracture formation in the blocks of the same material was realized. The conditions of friction within the contact region were taken into account. A possibility was shown of searching for the system parameters which provide formation and growth of the initial tensile crack up to its exit on the contact plane, transverse shear along this plane and initiation of the secondary tensile crack on the opposite surface of contact of the blocks. This scenario was realized on the laboratory samples. The obtained data enable to specify the scenario of stepped crack formation.

Experiments on the laboratory gypsum samples showed that the range of the conditions (parameters) providing realization of the described fracture scenario is very narrow. For the used model material the stresses of pressing the sample parts (blocks) must be in the range 0.15MPa ÷ 0.40MPa at variation of the angle of the interface inclination to the normal to the pressing traction in the range $10^0 \div 35^0$. The blocks slipping along the interface occurs at smaller loads while the loads exceeding the upper bound lead to splitting of the sample as a whole at wedging. The tendency to increasing the distances along the interface between the initial and secondary cracks at increasing the angle of the interface inclination is observed. Shear zone propagation can be represented as the motion of a crack of transverse shear along the interface.

The study was supported by RFBR (Project 11-01-00358).

References

[1] C.E. Renshaw, D.D. Pollard. Int. J. Rock Mech. Min .Sci. Vol. 32(3) (1995), p.237.
[2] J. Zhou, M. Chen , Y. Jin, G. Zhang. Int. J. Rock Mech. Min .Sci. Vol. 45(3) (2008), p.1143.
[3] N.R. Warpinsky, L.W. Teufel. J. Petrol. Technol. 2 (1987), p.209.
[4] Handbook (cadaster) of physical properties of rocks. Moscow: “Nedra” Publ. (1975).
[5] K.L. Johnson. Contact mechanics. Cambridge: Cambridge Univ. Press (1987).
[6. K.Y. Lam, M.P. Cleary. Int. J. Numer. Analyt. Meth. Geomech. Vol. 8. (1984), p.589.