Failure Criteria for Wood Under Multiaxial Loading

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Abstract. The first part of the paper reviews the different failure criteria proposed for wood under condition multiaxial loading. A statistical analysis is performed about the fitting of these criteria to the experimental data, from literature, for clear spruce wood under different loading conditions. Eventually a new failure criterion is proposed. It is a non-continuous failure criterion that, without using a larger number of fitting parameters than the other criteria, provides a substantial better fitting to the observed experimental results. This non-continuous criterion distinguishes fiber failure (as previously proposed by Norris) from the other failure mechanisms. It is claimed that the same criteria may be used for other wood species.

Introduction

Wood is a sustainable building material that captures CO_2 and it is easily recyclable. In this sense, its use as a structural element contributes to protect the environment, but its use requires a good knowledge about its load carrying ability. Wood is a natural biological material with a large scatter in its behavior. Its behavior depends on the wood type, age, humidity, and of course, orientation with respect to the stresses. Wood structure is anisotropic and in most cases its behavior is simplified to orthotropic.

There are a large number of criteria proposed for wood and composite materials. To what extent criteria used for artificial, fiber reinforced materials accurately represent failure in wood will be treated with some extension. These criteria have a different number of parameters to fit the experimentally observed behavior. The larger the number of parameter the better the fitting to the experiments, but on the other hand also the characterization of a new type of wood will require a larger campaign of tests to estimated all the parameters.

Many different phenomenological failure criteria have been proposed; most of them are based on quadric surfaces, in which certain constraints are taken strictly from geometrical considerations to achieve a closed failure envelope. Some of the criteria include dependent or independent interaction coefficients for bi-axial stress conditions. The models do not explain the mechanism of failure itself. They merely identify failure (yes or no), and are usually regarded in practice as a simple and reliable tool for design. Most of the criteria were developed for composite materials, but are extensively applied for wood.

Most failure criteria are continuous in the stress space, but a few are non continuous accounting for different failure mechanisms. Different failure mechanisms are present in the failure of wood: fibers might break, delamination might take place among the fibers, buckling in compression of fibers is observed with kinks, etc. Because of the large scatter of wood (even dealing with clear blocks) hinders these micromechanisms of failure.

A large amount of experiments were conducted by Eberhardsteiner [1] on clear spruce under multiaxial loading. These raw data are analyzed for the different proposed criteria and their ability to predict the experiments is discussed, including the effect of the number of parameters.

Experimental data

In the open literature there is an extensive amount of data about the (uniaxial) tensile behavior of different kinds of woods. But it is not the case under multiaxial loading conditions. Eberhardsteiner [1] has published a numerous amount of experimental results carried out on cruciform test-pieces of clear spruce wood, tested in multiaxial conditions under plane stress conditions. This work uses this data to analyze and compare the different failure criteria published in the literature.

Description of the tests. The test-pieces used by Eberhardsteiner [1] had 140 × 140 mm, and a variable thickness depending on the kind of test: 4.5 mm were used for the biaxial loading tests and between 7.5 and 9.5 mm for the compression tests. The angle φ between the loading direction and the fiber direction is variable. Fig. 1 shows a picture of the experimental set-up and the biaxial testing rig.

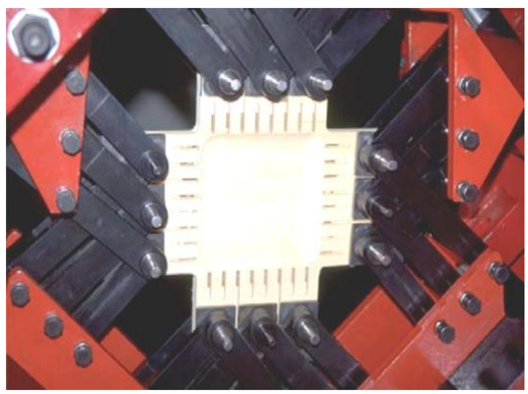


Fig. 1. Experimental set-up for multiaxial testing of wood panels (from Eberhardsteiner [1]).

Two coordinates systems are considered: the system (x_1, x_2) in the loading direction, and the system (x, y) in the fibre direction (see Fig. 2). The angle φ is determined by the fibre orientation.

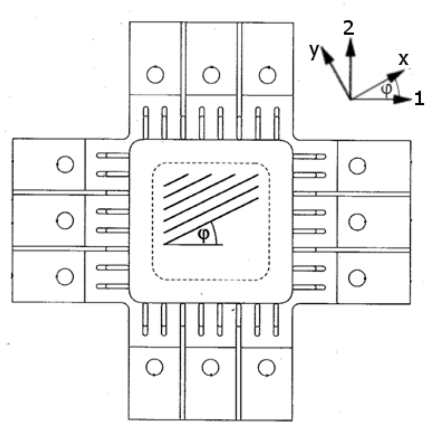


Fig. 2. Test-piece geometry and coordinates used by Eberhardsteiner (from [1]).

The relation between the stresses and the coordinate systems are given by the matrix transformation equation (Boding and Jayne [2])

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} m^2 & n^2 & 2mn \\ m^2 & m^2 & -2mn \\ -mn & mn & m^2 - n^2 \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ t_6 \end{bmatrix} \qquad (1)$$

where $m = \cos \varphi$ and $n = \sin \varphi$. Under plane stress conditions $t_6 = 0$.

Data analysis. A total of 414 tests are considered. These tests are the results of testing under different fiber and load orientations. Table 1 summarizes the load ranges tested versus the fiber orientations.

Table 1. Range of tested load vs. fiber orientations

φ (º)	σ_1 (MPa)		σ_2 (MPa)	
	min	max	min	max
0.0	- 49.0	80.0	- 9.9	6.3
7.5	- 38.0	66.0	- 9.5	6.0
15.0	- 37.0	39.0	- 8.4	6.0
30.0	- 19.0	18.0	- 7.7	7.2
45.0	- 10.0	10.0	- 12.9	10.7

The following figures represent the 414 experiments in the material coordinate system.

This representation of the data shows a much larger material strength in the fiber directions (Fig 3) than in the transverse directions, and an elliptical distribution in the projection σ_y τ_{xy} (Fig. 4).

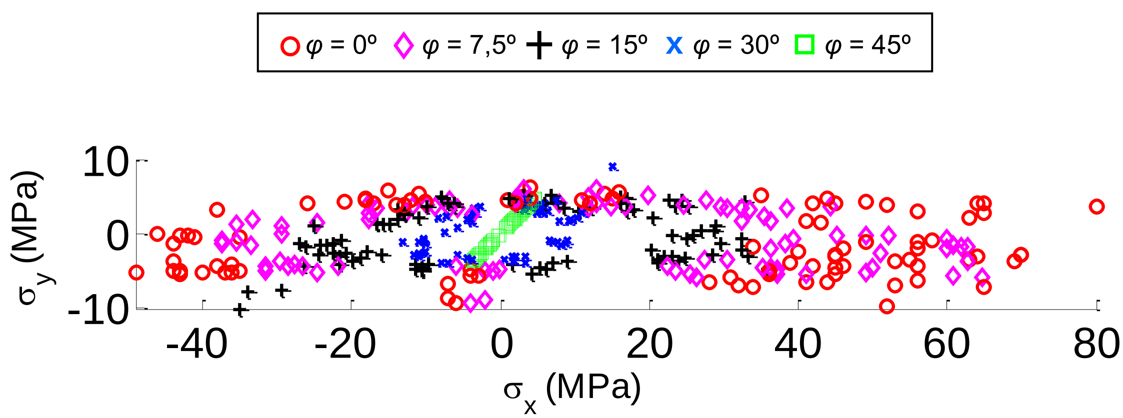


Fig. 3. Experimental failure locations projected on σ_x σ_y plane.

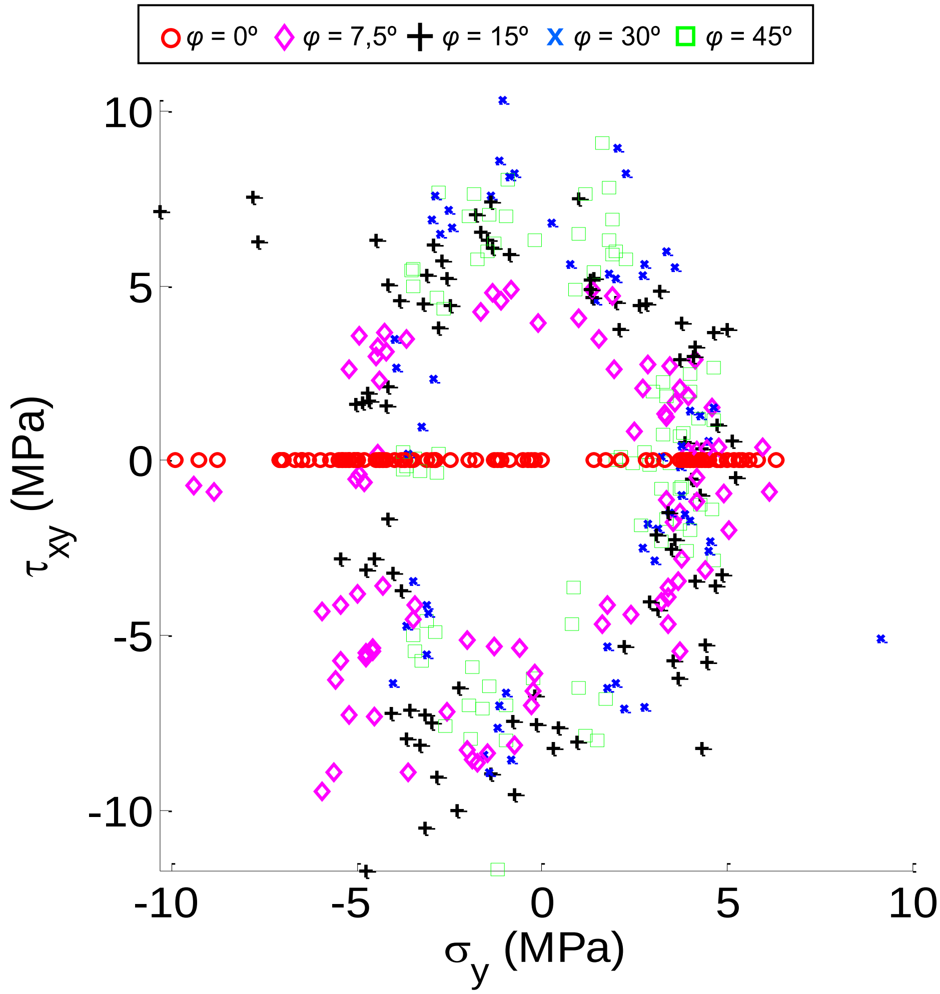


Fig. 4. Experimental failure locations projected on σ_y τ_{xy} plane.

Also, the failure locations, with a common fiber orientation (φ), share a common plane when observed from σ_x = σ_y axis (see Fig. 5).

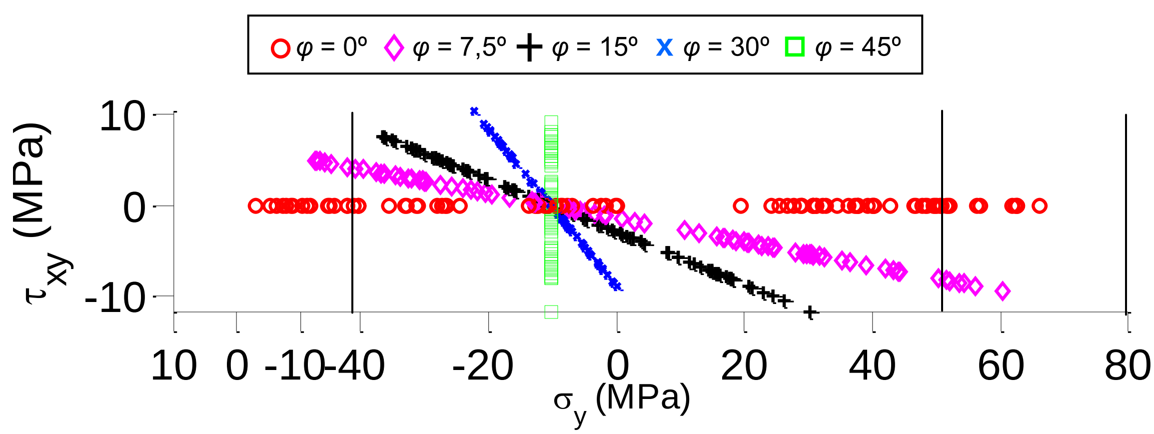


Fig. 5. Failure locations when observed from the direction σ_x = σ_y. Experiments with a common φ (fiber vs. main loading direction angle) are observed on different planes.

Failure criteria. Along the last decades a number of different criteria have being proposed for wood failure assessment under multiaxial loading conditions. Most frequently these criteria are closed surfaces in the stress space, using polynomial expressions, with coefficients that depend on the material strength for the different directions x and y (fiber or longitudinal and the two transverse directions to the fibers). X_t and Y_t are used for tension; X_c and Y_c for their strengths in compression and S for their strength under shear. All these values should be previously measured by dedicated experiments.

- *Linear models.* The simplest model is the lineal one [3] expressed by the equation of a plane

$$\frac{\sigma_x}{X} + \frac{\sigma_y}{Y} + \frac{\tau_{xy}}{S} = 1 \tag{2}$$

This model has been proved to fit the experimental evidence very poorly and will not be analysed any further here.

- *Quadratic models.* Most failure criteria proposed for wood until now, use ellipsoidal surfaces. A few of them are described in the following paragraphs.

Aicher and Klök´s [3] model consists in an ellipsoid centered at the origin and with their axis lying along the coordinate directions, i.e.

$$\left(\frac{\sigma_x}{X}\right)^2 + \left(\frac{\sigma_y}{Y}\right)^2 + \left(\frac{\tau_{xy}}{S}\right)^2 = 1 \tag{3}$$

So, the strengths under tension and compression are considered to be identical, what is not always the case for wood samples.

Tsai-Hill [4] model proposes an ellipsoidal failure surface centered at the origin, but the difference with the previous criterion, the elliptical axis on the x and y plane form and angle with the reference coordinate axis, so it considers an interaction between σ_x and σ_y stresses. The failure surface is given by

$$\left(\frac{\sigma_x}{X}\right)^2 - \frac{\sigma_x \sigma_y}{X^2} + \left(\frac{\sigma_y}{Y}\right)^2 + \left(\frac{\tau_{xy}}{S}\right)^2 = 1 \tag{4}$$

Norris's model [5] is non-continuous. i.e. it is not a smooth surface. It postulates that failure will take place whenever one of the three following equations are met

$$\left(\frac{\sigma_x}{X}\right)^2 - \frac{\sigma_x \sigma_y}{XY} + \left(\frac{\sigma_y}{Y}\right)^2 + \left(\frac{\tau_{xy}}{S}\right)^2 = 1 \tag{5a}$$

$$\left(\frac{\sigma_x}{X}\right)^2 = 1 \tag{5b}$$

$$\left(\frac{\sigma_y}{Y}\right)^2 = 1 \tag{5c}$$

The quadratic Eq. 5a is similar to Eq. 4. The introduction of the other two equations means that in the case of tension-tension and compression-compression (2nd and 4th stress quadrants) the failure will be produced whenever one the stresses σ_x or σ_y first reaches their mechanical strength limits.
Von Mises [6] criterion is similar to the quadratic equation proposed by Norris [5] in Eq. 5a, with the only difference that the term corresponding to shear is being multiplied by a factor of 3

$$\left(\frac{\sigma_x}{X}\right)^2 - \frac{\sigma_x \sigma_y}{XY} + \left(\frac{\sigma_y}{Y}\right)^2 + 3\left(\frac{\tau_{xy}}{S}\right)^2 = 1 \qquad (6)$$

Van der Put proposed [7] a failure surface for wood as an ellipsoid with their axis oriented along the coordinate directions, but its centre is not located at the origin; its centre being on the $x\ y$ plane.

$$\left(\frac{1}{X_t} - \frac{1}{X_c}\right)\sigma_x + \left(\frac{1}{Y_t} - \frac{1}{Y_c}\right)\sigma_y + \frac{1}{X_t X_c}\sigma_x{}^2 + \frac{1}{Y_t Y_c}\sigma_y{}^2 + \frac{1}{S^2}\tau_{xy}{}^2 = 1 \qquad (7)$$

Tsai-Wu's model [8] is the most general one. Under plane stress conditions, it is expressed as

$$\left(\frac{1}{X_t} - \frac{1}{X_c}\right)\sigma_x + \left(\frac{1}{Y_t} - \frac{1}{Y_c}\right)\sigma_y + \frac{1}{X_t X_c}\sigma^2{}_x + \frac{1}{Y_t Y_c}\sigma^2{}_y + 2a_{xy}\sqrt{\frac{1}{X_t X_c}\frac{1}{Y_t Y_c}}\sigma_x\sigma_y + \frac{1}{S^2}\tau^2{}_{xy} = 1 \qquad (8)$$

where $|a_{xy}| \leq 1$

Ellipsoid center being shifted and their axis form and angle with the coordinate axis, which depends on the interaction coefficient a_{xy}. This coefficient should be previously measured. Van der Put's model is a particular case of this criterion ($a_{xy} = 0$).
Van der Put's and Tsai-Wu's models consider, in a single equation, different behaviours in tension and compression. All the others criteria, presented until here, should be used -at best- with different equations/parameters for the different loading modes/quadrants.

Criteria comparison

In the following the quadratic criteria will be statistically compared versus the experimental results provided by Eberhardsteiner for clear spruce wood [1].
For this purpose a generic ellipsoid equation is used with six degrees of freedom: two correspond to its centre location (x_0, y_0), the angle β between the ellipsoid axis and the coordinated axis, and three more corresponding to the ellipsoid semi-axis (a, b, c). The mean quadratic error will be used to optimize those parameter versus the 414 experimental data.
A very large difference is observed in the fibre direction in comparison with the strength in the transverse directions; so, individual differences will be weighted/normalized in relation with their vector distances. I.e. the function to be minimized is given by

$$SS = \sum_{i=1}^{N} \frac{(\sigma_x - \hat{\sigma}_x)^2 + (\sigma_y - \hat{\sigma}_y)^2 + (\tau_{xy} - \hat{\tau}_{xy})^2}{\sigma_x{}^2 + \sigma_y{}^2 + \tau_{xy}{}^2} \qquad (9)$$

where the circumflex accents indicate the projection, with respect to the origin, of coordinates of the individual data being considered, on the ellipsoidal surface.

The most general case is that of Tsai-Wu, with 6 parameters. The same way is used with the other criteria, imposing the ellipsoids their corresponding restrictions, for example, $x_0 = y_0 = \beta = 0$ are imposed for the Aicher and Klök's [3] model, etc.
Table 2 shows the obtained parameter for the ellipsoids for the different criteria fitted. Semi-axes and centre location are expressed in MPa. *SS* represents the normalized sum of square error, so, it has no dimensions. The different models are indicated by its corresponding reference. In bold are represented their corresponding restrictions. Note that Tsai-Hill, Norris and Van del Put models do not have β as a free parameter because it is a function of the semiaxes of the ellipsoid.

Table 2. Ellipsoid parameters fitted to the different failure criteria

MOD	***a***	***b***	***c***	$\mathbf{x_0}$	$\mathbf{y_0}$	β (°)	***SS***
[3]	77.4	5.3	8.2	**0**	**0**	**0**	28.9
[4]	79.0	5.3	8.2	**0**	**0**	**0.1**	29.2
[5]	128.3	5.3	8	**0**	**0**	**1.4**	34.6
[6]	150.7	5.1	8	**0**	**0**	**1.1**	35.9
[7]	62.5	5.6	8.3	14.4	-1.0	**0**	20.2
[8]	62.5	5.6	8.3	14.4	-1.0	0.2	20.1

Tsai-Wu's model results in the least error, as it was expected because it also has the largest number of parameters (degrees of freedom). A shift of the centre of the ellipsoid is observed with respect to the origin of coordinates to the tensile side (x_0 = 14.4 MPa), thus the spruce strength in tension is larger than under compression. The β angle is nearly negligible, and in fact the advantage, with respect to Van de Put's model, which impose a $\beta = 0$, is barely appreciable.
Fig. 6 represents the best fitted ellipsoid and the experiments.

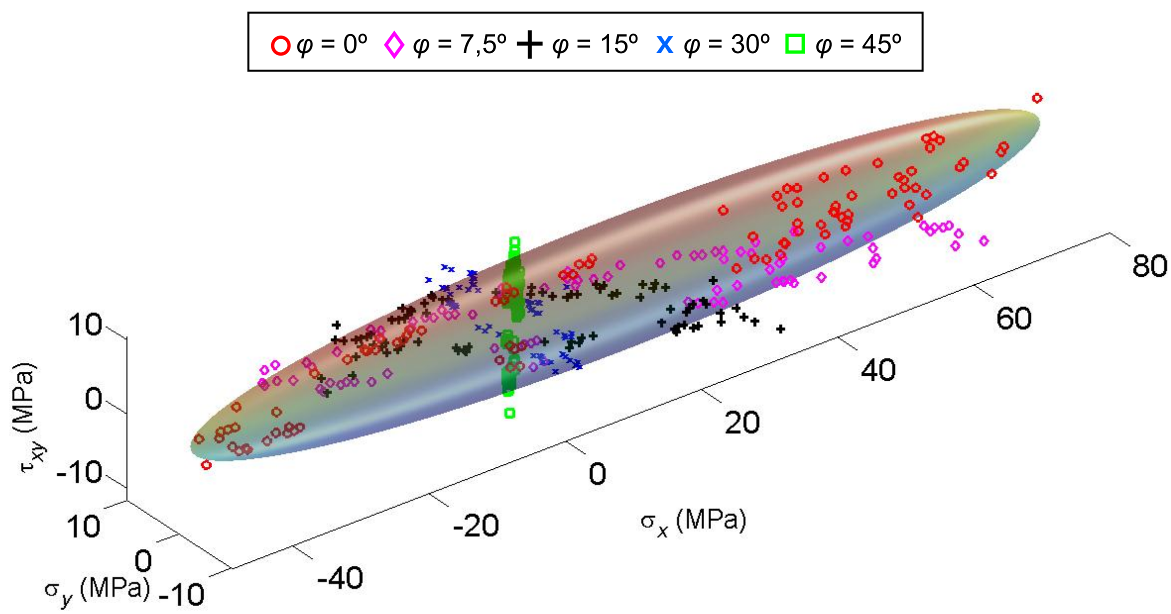


Fig. 6. Experimental failure locations and best fitted ellipsoid.

New failure criterion

It is quite obvious that woods fails by different micromechanisms when tested under different stress conditions: failure in tension parallel to the fibres is produced by the fracture of the fibres, which determines its strength; under compression the most frequent failure mechanisms is the buckling and kinking of the fibres. Under tension or compression in direction perpendicular to the fibres, the failure is produced by debonding of the fibre interfaces without fibre fractures. Eberhardsteiner and Mackenzie-Helnwein [9] clearly distinguishes four failure mechanisms after testing clear spruce wood panels under multiaxial conditions. Failure is the results of the competition among different failure mechanisms, so, the stress space is being limited by different surfaces, ideally representing the different failure mechanisms, intersecting each other. The first criteria to be met is which limits the acceptable load under this particular loading conditions (let us assume proportional loading conditions). Among all the criteria aforementioned, Norris' [5] is the only one represented by the minimum of a set of equations/conditions.

The introduction of a cut at the tensile tip of the ellipsoid (that represents the failure location in the stress space) was already suggested by Norris [5] add a large reduction in the residuals after fitting to the experiments. It is here proposed a new failure criterion with two equations defining a truncated ellipsoid. From the results to allow for a small angular deviation between the fibre and the coordinate axis does not improve the fitting in a significant way, so, $\beta = 0$ is imposed. Therefore, this model has not a larger number of parameters than the other criteria. Table 3 summarizes the value of its parameters and the residual sum of quadratic errors. The equation for the cutting plane under tension aligned with the fibres is

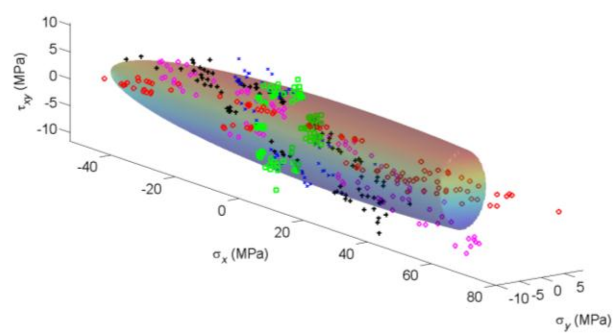
$$\sigma_x = X_t \,. \tag{10}$$

As for the previous Table 1, also the values are expressed in MPa.

Table 3. Parameter for the proposed truncated ellipsoid, failure criteria for clear spruce wood (and normalized error)

a	***b***	***c***	***x_0***	***y_0***	***X_t***	***SS***
76.4	5.7	8.2	23.7	-1.0	56.7	19.2

Fig. 7 shows the suggested failure surface in this work. The error is reduced by a 5% when compared to Tsai-Wu's model (the best model in Table 1).



○ φ = 0º ◇ φ = 7,5º + φ = 15º x φ = 30º □ φ = 45º

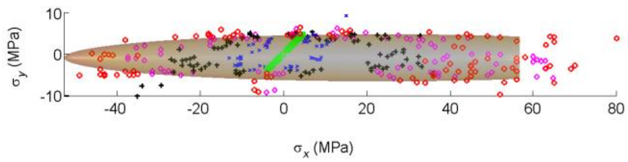


Fig. 7. Proposed failure criterion: tensile truncated ellipsoid.

A second (compression parallel to the fibres tip) cut does not improve the fitting in a statistically significant way. Taking into account that it will require the introduction of a new parameter and additional testing efforts to measure it, this ideas does not results attractive from a practical point of view (at least with the available data for the clear spruce wood).

Fig. 8 shows the projection of the 414 failure locations and the proposed failure criteria projected on the σ_y τ_{xy} plane. A shift of the ellipsoid to the left is observed (to compressive values of σ_y); it means that clear spruce is more resistant in compression than to tension, both perpendiculars to the fibres.

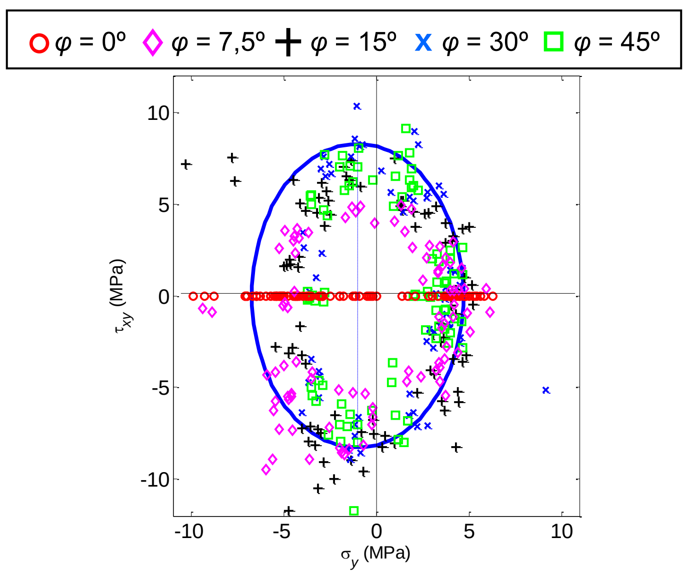


Fig. 8. Failure locations projected on σ_y τ_{xy} plane and proposed failure criterion.

Fig. 9 represents only the experiments for $\varphi = 0^{\circ}$ and the section of the proposed criterion in the σ_x σ_y plane. The failure location might seem not to be optimized to the data represented; note that it is optimized to the whole set of experiments (there is a number of them above and bellow the represented section) and not only for those experiments with orientation $\varphi = 0^{\circ}$. Red and black crosses represent the origin and the ellipsoid centres, respectively. A shift is clearly observed between both crosses on both directions: σ_x and σ_y.

If the errors are assumed to be normally distributed with a mean value of $\mu = 0$ and a variance $\sigma = \sqrt{\frac{SS}{N-1}}$, it is also possible to plot confidence bounds. Fig. 9 also shows (with dashed lines) 95% confidence bounds. So, the probability of failure in between both dashed lines accounts for the 95% of the cases. These bounds are useful for safe design purposes.

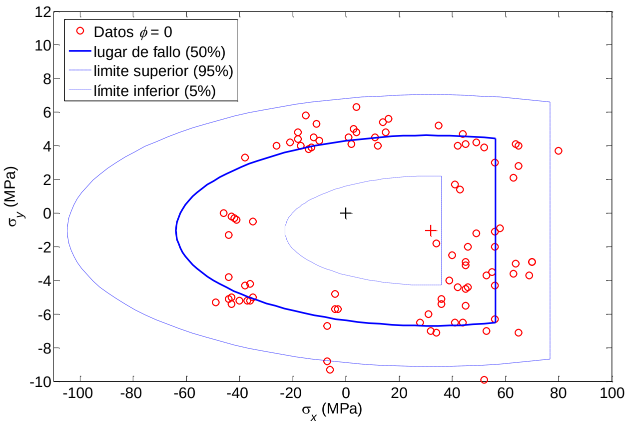


Fig. 9. Tests with a $\varphi = 0$º, proposed failure criterion and 95% confidence intervals.

Discussion

Eberhardsteiner [1], by using uniaxial tests, determines a fracture stress X_t = 62.33 MPa for spruce wood. In our model this value is reduced to X_t = 56.7 MPa, but it was fitted to all the set of 414 experiments (not to only uniaxial tensile experiments parallel to the fibers). Even considering only the experiments for φ = 0º (those in Fig. 9), our value seems in better agreement with the experiments than the value proposed by Eberhardsteiner.

Summary

Different failure criteria have been compared versus the experiments conducted by Eberhardsteiner [1] for clear spruce wood under multiaxial stress conditions. The best criterion, fitting the experimental results, is Tsai-Wu's [8] consisting in a free ellipsoid in the stress space σ_x, σ_y, τ_{xy}, with the only restriction of symmetry with respect to the plane $\tau_{xy} = 0$, so, with six degrees of freedom.

Most criteria consist in a single equation for defining the failure locus. Because the mode of failure varies with the loading conditions, ideally different set of equations should be used to account for the different failure mechanisms. Only Norris [5] proposed a set of (3) equations, so the failure surface has sharp edges and it is not a completely smooth surface.

In this work a new criterion is proposed consisting in an ellipsoid parallel to the coordinate axis, symmetric with respect to the plane $\tau_{xy} = 0$, and truncated by the plane $\sigma_x = X_t$ which accounts for the tensile fracture of fibers. This model improves the fitting to the experiments conducted by Eberhardsteiner [1] in clear spruce wood than the other model described, without increasing the number of parameters required by the other criteria.

Acknowledgment

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