Experimental and expected fatigue fracture plane according to variance and damage accumulation methods under multiaxial loading

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Abstract. The paper presents comparison of fracture plane position gained from experiment tests of specimens under multiaxial loading and theoretical ones from calculation according to variance and damage accumulation methods. In the variance method it is assumed that the plane in which the maximum variance of the equivalent stress appears is critical for a material and the fatigue fracture should be expected in this plane. In the damage accumulation method as fracture plane, the plane which suffered the greatest damage during service loading is adopted. For both methods the equivalent stress was calculated according to the fatigue failure criterion of maximum normal and shear stresses in the critical plane.

Introduction

To establish the critical plane position two variance and damage accumulation methods were used. The method of variance [1] relies on search of the maximum variance of equivalent stress (or another parameter) according to the selected fatigue failure criterion. The plane where the variance reaches its maximum is assumed as the critical plane. The method of damage accumulation [2, 3] seems to be the most interesting because of its close relation to the idea of critical plane. Here, the selected fatigue failure criterion is applied for search of the plane of the maximum damage, i.e. the plane of the minimum fatigue life. This is an iterative method, so search of the critical plane requires repetition of all the calculation algorithm many times.

The aim of the paper is comparison of the variance method and the method of damage accumulation for determination of the critical plane position with the results of experiments.

Material and test procedure

Specimens of round sections were tested (see Fig. 1a). Those were cut from the sheets 16 mm thick, according to the rolling direction. The specimen surface has been obtained by turning using conventional polishing with progressively finer emery papers. A final average roughness 0.16 μm has been measured. Diameter of the specimens was d = 8 mm for pseudo-random loading. Some mechanical properties under bending of the tested steel are given in Table 1. The tests were performed, in the high cycle fatigue regime (HCF) under pseudo-random bending with torsion loading, at Opole University of Technology [4, 5]. The fatigue stand was used to carry out fatigue tests. The stand MZGS-200PL (pseudo-random loading), with the dominating frequency 28.8 Hz for bending and 30 Hz for torsion, is applied for fatigue tests of specimens made of structural materials subjected to non-proportional combinations of the bending moment M_B and the torsional moment M_T.

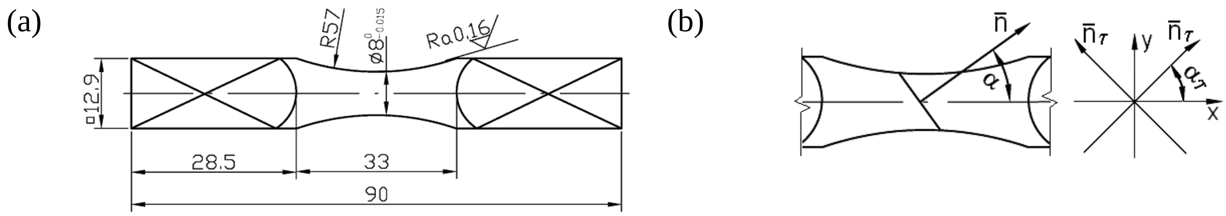

Fig.1. Shape and dimensions of a specimen a) and definition of the critical plane position with estimated shear fracture plane α_τ b).

Histories of these moments are independent (separate drives and control systems) and they are polyharmonic (pseudo-random). The histories are sums of four harmonic components with different amplitudes, frequencies and phases. The stand enables testing of the influence of cross-correlation between normal and shear stresses, their frequencies and amplitudes on the fatigue life of the tested material.

Table 1. Mechanical properties of the 10HNAP steel

Yield stress σ_{YS}, MPa	Ultimate stress σ_U, MPa	Elastic modulus E, GPa	Poisson`s ratio ν	Fatigue limit σ_{af}, MPa	Number of cycles N_0 for σ_{af}
418	566	215	0.29	300	$3.135\cdot10^6$

Experimental results and discussion

The critical plane position for a given material depends on values of loadings, the cross-correlation coefficient of stresses, and the ratio of maximum stresses. Using mathematical relationships, while calculations one or more positions of the critical plane are obtained. Moreover, material is never perfect and a damage can occur in the point where its structure is heterogeneous. Fig. 1b shows definition of the critical plane position and how the angle $\alpha_\tau = \alpha_{FP} \pm 45°$ (α_{FP} – average angle of fracture plane position) were determined.

In the present paper, on base of earliest analysis experimental tests [5], the failure criterion of maximum normal and shear stress in the critical plane was assumed for fatigue life calculation. According to this criterion, the equivalent stress σ_{eq} (t) in the plane of maximum shear stress τ_1 (t) takes the following form

$$\sigma_{eq}(t) = (2-B)\left[\sigma_x(t)\cdot\cos^2(\alpha) + \tau_{xy}(t)\sin(2\alpha)\right] + B\left[-0.5\sigma_x(t)\sin(2\alpha) + \tau_{xy}(t)\cos(2\alpha)\right] \qquad (1)$$

where: $B = \sigma_{af}/\tau_{af} = 1.73$ (for $N_f = 3.135\cdot10^6$ cycles) – fatigue limits ratio under bending and torsion, respectively, σ_x (t) - normal stress along the specimen axis, τ_{xy} (t) - shear stress in the specimen cross section, α - angle determining the critical plane position (Fig. 1b).

From Eq. (1) it appears that the equivalent stress σ_{eq} (t) is linearly dependent on the stress state components σ_x (t) and τ_{xy} (t), so it can be expressed as

$$\sigma_{eq} = \sum_{j=1}^{n} a_j x_j = a_1 x_1 + a_2 x_2, \qquad (2)$$

where: $a_1 = (2-B)\cos^2(\alpha) - 0.5B\sin(2\alpha)$, $a_2 = (2-B)\sin(2\alpha) + B\cos(2\alpha)$, $x_1 = \sigma_x$, $x_2 = \tau_{xy}$.

From theory of probability [6] it results that the variance of random variable being a linear function of some random variables is expressed by the following formula

$$\mu_{\sigma eq} = \sum_{j=1}^{n} a_j^2 \mu_{xj} + 2\sum_{j\langle k} a_j a_k \mu_{xjk} = a_1^2 \mu_{x_1} + a_2^2 \mu_{x_2} + 2a_1 a_2 \mu_{x_1 x_2}, \qquad (3)$$

where: $\mu_{\sigma eq}$, μ_{x1}, μ_{x2} - variance of equivalent stress σ_{eq}, normal stress σ_x and shear stress τ_{xy}, respectively, μ_{x1x2} - covariance of normal σ_x and shear τ_{xy} stresses.

Under biaxial random stationary and ergodic stress state, the variances μ_{x1}, μ_{x2} and the covariance μ_{x1x2} in Eq. (3) are constant.

In the method of variance for determination of the critical plane position the maximum function of Eq. (3) is searched in relation of the angle α occurring in coefficients a_1 and a_2. After reduction, the variance of equivalent stress $\mu_{\sigma eq}$ versus the angle α can be written as

$$\begin{aligned}\mu_{\sigma eq} &= \left[(2-B)\cos^2(\alpha) - 0.5B\sin(2\alpha)\right]^2 \mu_{x_1} + \left[(2-B)\sin(2\alpha) + B\cos(2\alpha)\right]^2 \mu_{x_2} \\ &+ 2\left\{(2-B)\cos^2(\alpha)\left[B\cos(2\alpha) + (2-B)\sin(2\alpha)\right] - 0.5B\left[(2-B)\sin^2(2\alpha) + 0.5B\sin(4\alpha)\right]\right\}\mu_{x_1 x_2}\end{aligned}. \qquad (4)$$

In the method of damage accumulation the damage degree $S_{PM}(T_O)$ during observation time T_O was calculated with use of the rain flow algorithm and Palmgren-Miner hypothesis

$$S_{PM}(T_O) = \begin{cases} \sum_{i=1}^{k} \dfrac{n_i}{N_o\left(\dfrac{\sigma_{af}}{\sigma_{eq,ai}}\right)^m}; & \text{for} \quad \sigma_{eq,ai} \geq a \cdot \sigma_{af} \\ 0; & \text{for} \quad \sigma_{eq,ai} < a \cdot \sigma_{af} \end{cases}, \qquad (5)$$

where: $\sigma_{eq,ai}$ – amplitude of the equivalent stress, m – coefficient of Wöhler's curve slope, σ_{af} – fatigue limit under bending, N_o – number of cycles corresponding to the fatigue limit σ_{af}, n_i – number of cycles with amplitude $\sigma_{eq,ai}$, a – coefficient allowing to include amplitudes below the fatigue limit in the process of fatigue damage accumulation (a = 0.5 was assumed in this paper), k – number of intervals of the amplitude histogram.

Fatigue life T_{cal} was calculated according to the following equation

$$T_{cal} = \frac{T_O}{S_{PM}(T_O)}. \qquad (6)$$

According to the method of damage accumulation, the critical plane position is identified by the maximum damage.

Graphs in Figs. 2a – 14a present relation between the normalized value of equivalent stress variance (see Eq. (4)), while graphs in Figs. 2b – 14b show relation between the normalized value of the damage degree (see Eq. (5)) and the critical plane angle α, respectively. The pseudo-random loadings generated the same cross-correlation coefficient between normal and shear stresses $r_{\sigma\tau}$ = 0.16 and different ratios of maximum stresses $\lambda_\sigma = \tau_{xy\,max} / \sigma_{x\,max}$.

From Figs. 2–14 it appears that the variance of equivalent stress $\mu_{\sigma eq}$ and the fatigue damage degree $S_{PM}(T_O)$ are continuous function of the critical plane angle α with some maximum values. Taking into account a certain randomness of the material structure and assuming that the critical plane position can occur at 5% deviation from the maximum variance and the maximum damage degree, ranges of variation of the angle α for particular loading cases can be distinguished (Table 2).

Table 2. Range of the critical plane angle variation (5%) according to the criterion of maximum normal and shear stresses and experimental fracture plane position under pseudo-random loadings

Type of loadings $r_{\sigma\tau} \approx 0.16$	Ratio of maximum stresses $\lambda_\sigma = \tau_{xy}/\sigma_x$	Experimental(*) fracture plane angle α_{FP} (deg) (stage II) with scatter (sc.)	Estimated(*) shear fracture plane angle α_τ (stage I) with scatter (sc.)	Range of critical plane angle $\Delta\alpha$ (deg) according to the method of	
				variance	damage accumulation
K01	0.189	4 (±3)	49 (±3)	-47 ÷ -33	-43 ÷ -37
K05	0.214	2 (±1)	47 (±1)	-47 ÷ -39	-30 ÷ -23
K02	0.274	$4\binom{+5}{-2}$	$49\binom{+5}{-2}$	-47 ÷ -32	-42 ÷ -34
K06	0.309	$9\binom{+15}{-9}$	$54\binom{+15}{-9}$	-47 ÷ -31	-42 ÷ -36
K03	0.358	3 (±2)	48 (±2)	-48 ÷ -31	-38 ÷ -29
K09	0.394	2 (±1)	47 (±1)	-24 ÷ -12	-33 ÷ -26
K07	0.405	$7\binom{+8}{-7}$	$52\binom{+8}{-7}$	-50 ÷ -31	-57÷-50 and -32÷-25
K04	0.442	$3\binom{+1}{-2}$	$48\binom{+1}{-2}$	-48 ÷ -28	-30 ÷ -23
K08	0.500	$9\binom{+16}{-7}$	$54\binom{+16}{-7}$	-50 ÷ -19	-23 ÷ -17
K10	0.515	$23\binom{+12}{-8}$	$68\binom{+12}{-8}$	-46 ÷ -23	-26 ÷ -20
K11	0.636	$2\binom{+1}{-2}$	$47\binom{+1}{-2}$	-21 ÷ 3	-17 ÷ -11
K12	0.680	37 (±11)	82 (±11)	-16 ÷ 3	-14 ÷ -9
K13	0.840	$13\binom{+15}{-11}$	$58\binom{+15}{-11}$	-89 ÷ -81 and -6 ÷ 8	-7 ÷ -1

(*) Angles α_{FP} and α_τ may be positive or negative – it depends on point of view

(a)

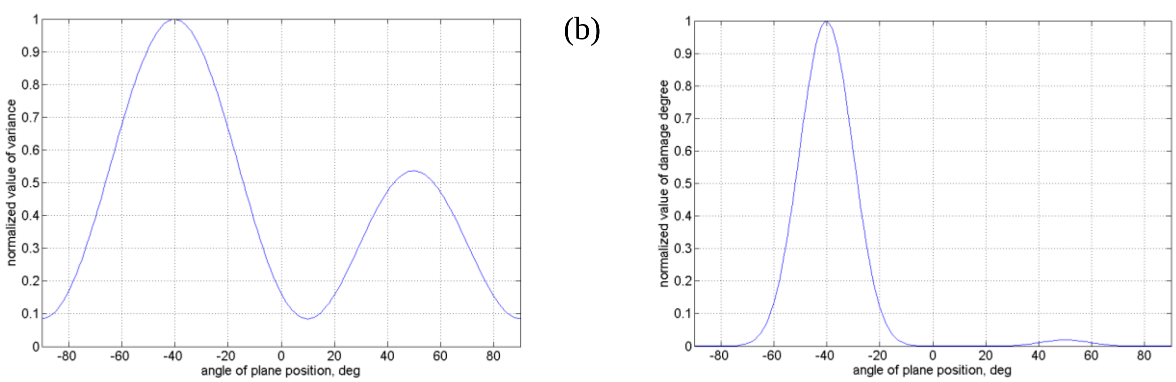

Fig.2. Dependence of the normalized value of: a) variance, b) damage accumulation on the angle α of critical plane position for loading combination K01 ($\lambda_\sigma = 0.189$).

While experiments, two kinds of crack planes could be seen. One of them occurred at the stage I (crack initiation), the other one occurred at the stage II (crack propagation, fracture plane). The initiation plane is inclined to the propagation plane at the angle of 45°.

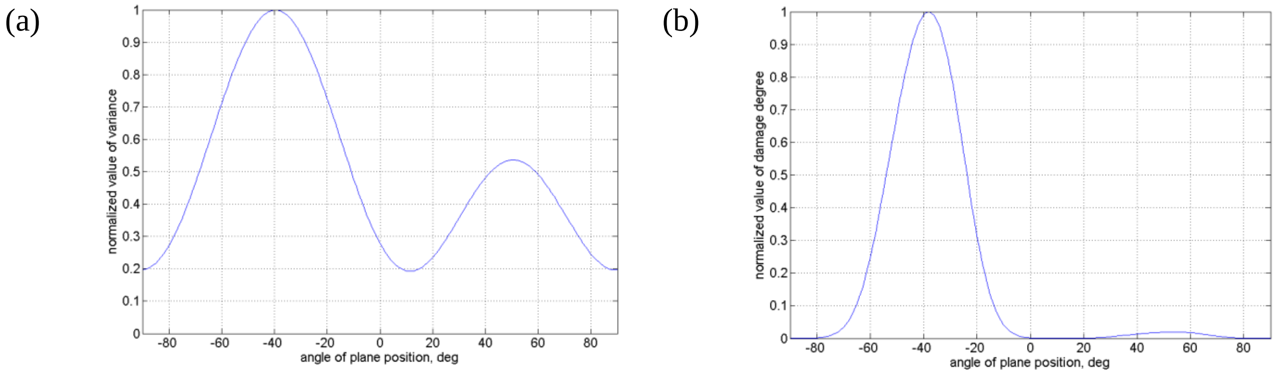

Fig.3. Dependence of the normalized value of: a) variance, b) damage accumulation on the angle α of critical plane position for loading combination K02 ($\lambda_{\sigma} = 0.274$).

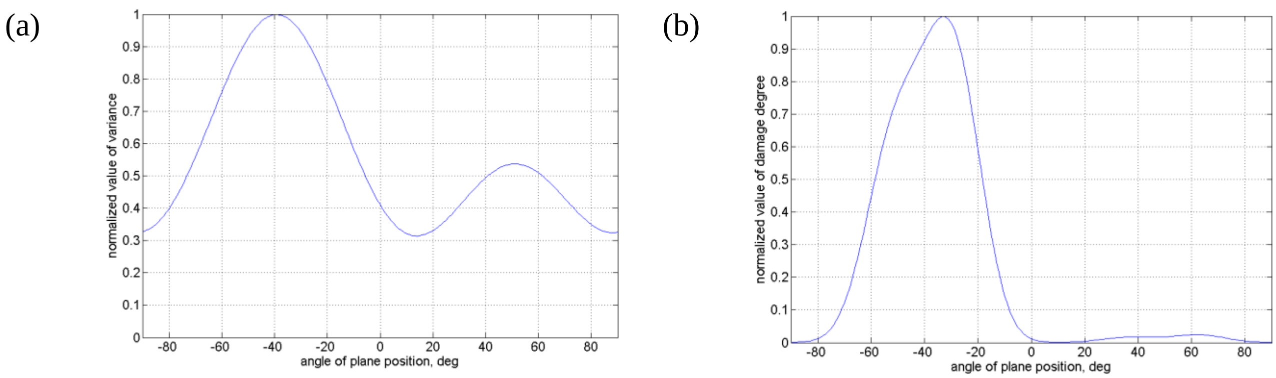

Fig.4. Dependence of the normalized value of: a) variance, b) damage accumulation on the angle α of critical plane position for loading combination K03 ($\lambda_{\sigma} = 0.358$).

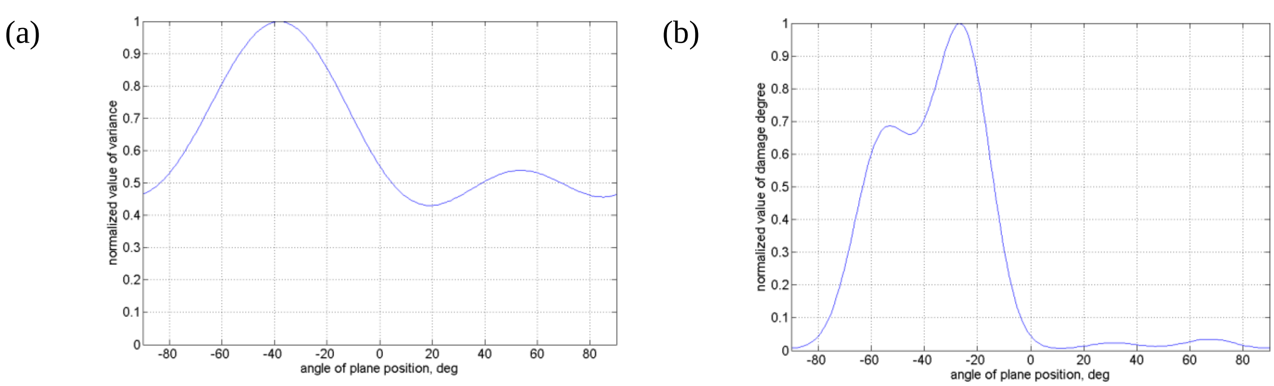

Fig.5. Dependence of the normalized value of: a) variance, b) damage accumulation on the angle α of critical plane position for loading combination K04 ($\lambda_{\sigma} = 0.442$).

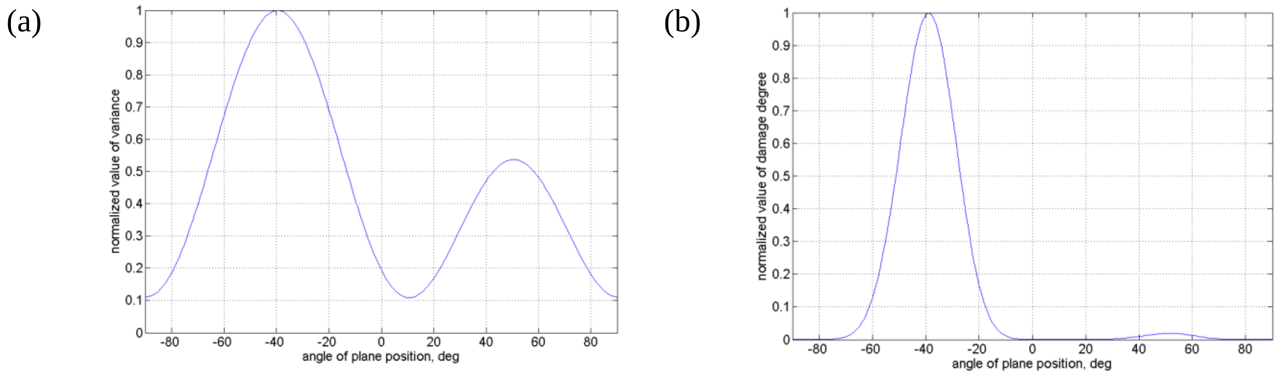

Fig.6. Dependence of the normalized value of: a) variance, b) damage accumulation on the angle α of critical plane position for loading combination K05 ($\lambda_\sigma = 0.214$).

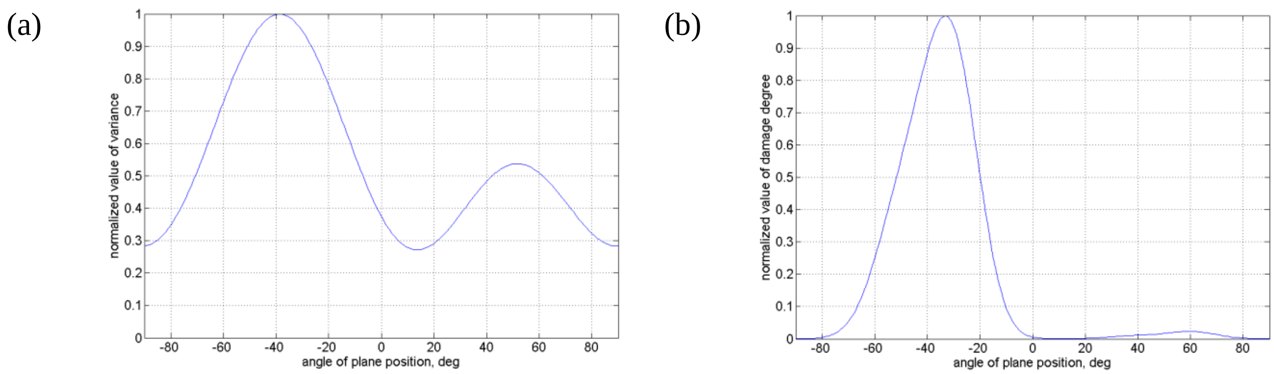

Fig.7. Dependence of the normalized value of: a) variance, b) damage accumulation on the angle α of critical plane position for loading combination K06 ($\lambda_\sigma = 0.309$).

For the cases K01-K08 in Table 2, the crack initiation plane (stage I) obtained from experiments coincides with the results of calculations with the variance method. A poor conformity of the experimental and calculated results was obtained by means of the method of damage accumulation. There is one case K07, which confirms calculation and experimental results. However, there were the cases K09-K13 where the crack initiation plane obtained from experiments did not coincide with that defined from the calculations.

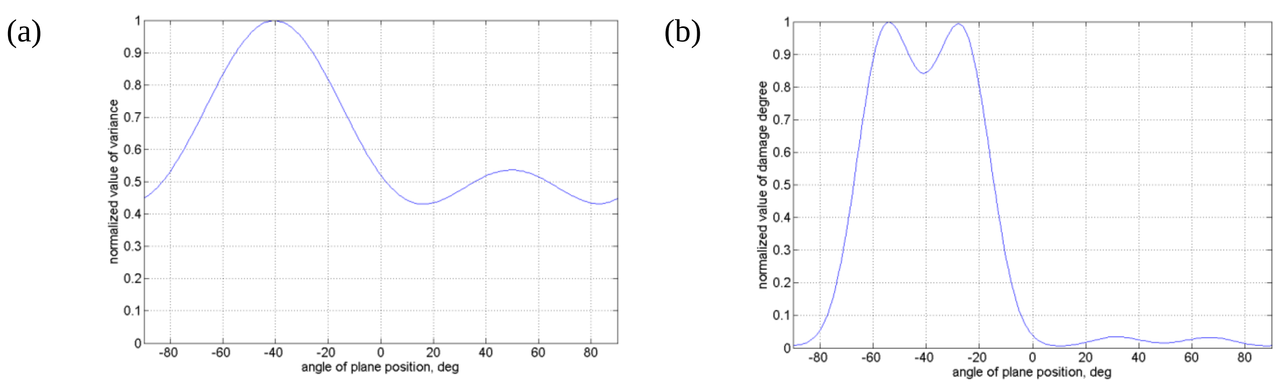

Fig.8. Dependence of the normalized value of: a) variance, b) damage accumulation on the angle α of critical plane position for loading combination K07 ($\lambda_\sigma = 0.405$).

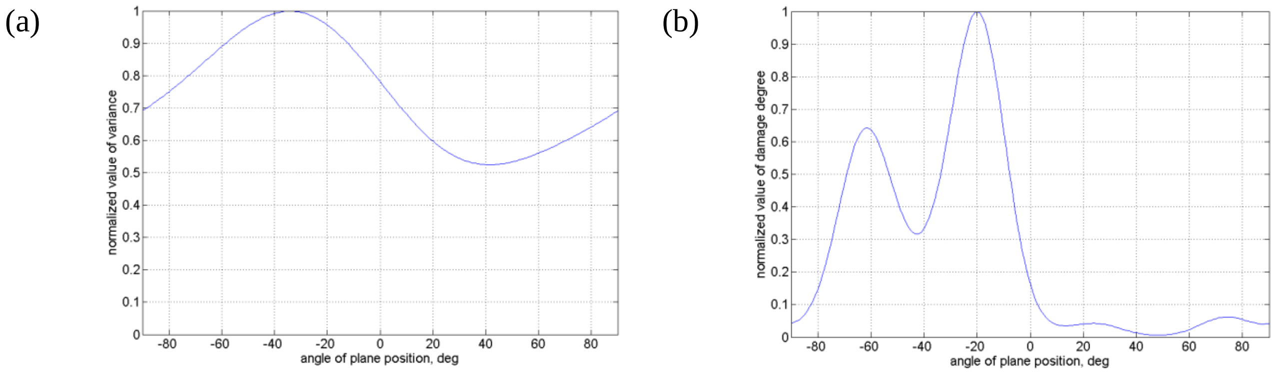

Fig.9. Dependence of the normalized value of: a) variance, b) damage accumulation on the angle α of critical plane position for loading combination K08 ($\lambda_\sigma = 0.5$).

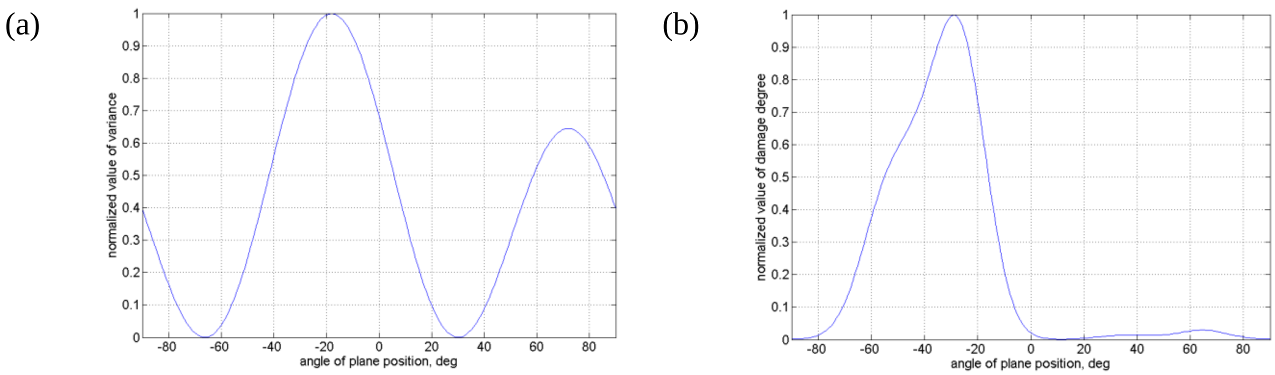

Fig.10. Dependence of the normalized value of: a) variance, b) damage accumulation on the angle α of critical plane position for loading combination K09 ($\lambda_\sigma = 0.394$).

For the cases K10 and K13 the crack propagation plane (stage II) obtained while experiments coincided with the calculation results obtained by means of the variance method and the method of damage accumulation. The critical plane angle ranges $\Delta\alpha$ (with 5% variation of α) according to the variance and damage accumulation methods overlap in almost all cases except two K05 and K09. Test measurements of the fracture plane angle α_{FP} (stage II) were performed on the specimen surface with the optical method (magnification 10x) to an accuracy of 1°, and the scatter band of the

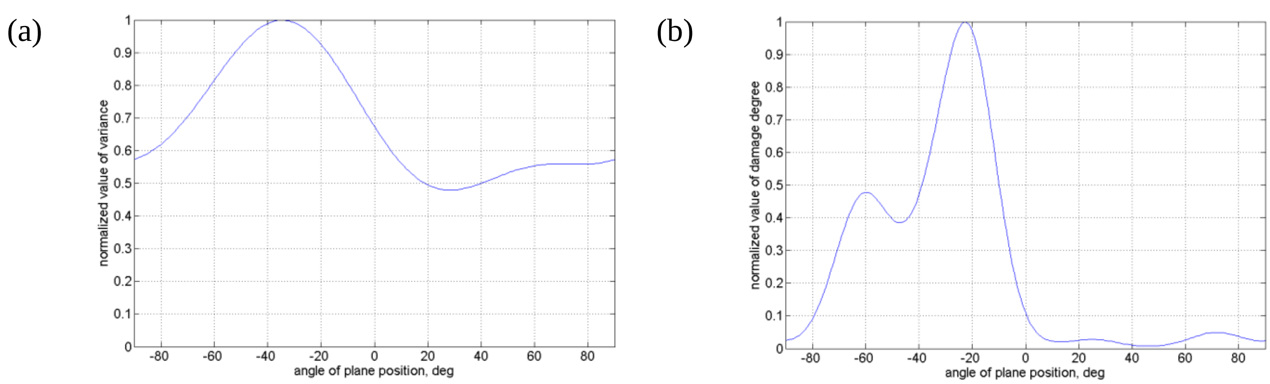

Fig.11. Dependence of the normalized value of: a) variance, b) damage accumulation on the angle α of critical plane position for loading combination K10 ($\lambda_\sigma = 0.515$).

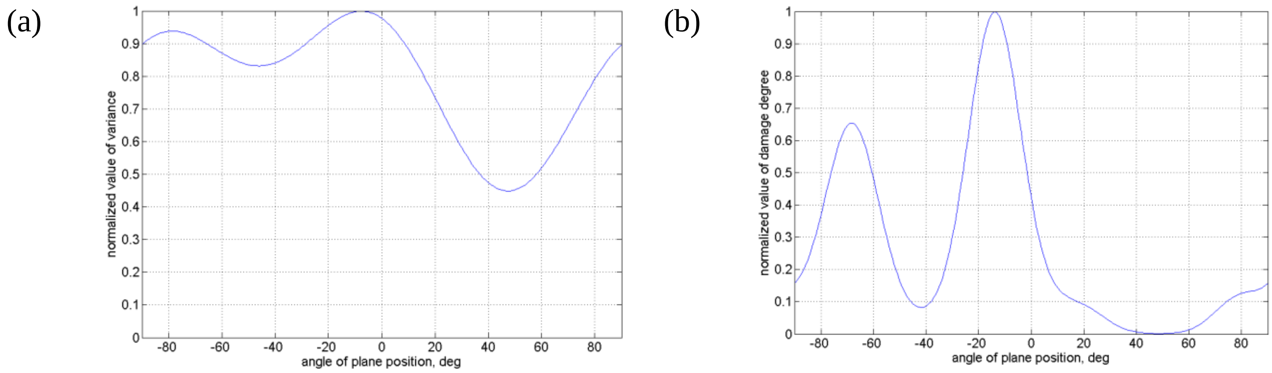

Fig.12. Dependence of the normalized value of: a) variance, b) damage accumulation on the angle α of critical plane position for loading combination K11 ($\lambda_\sigma = 0.636$).

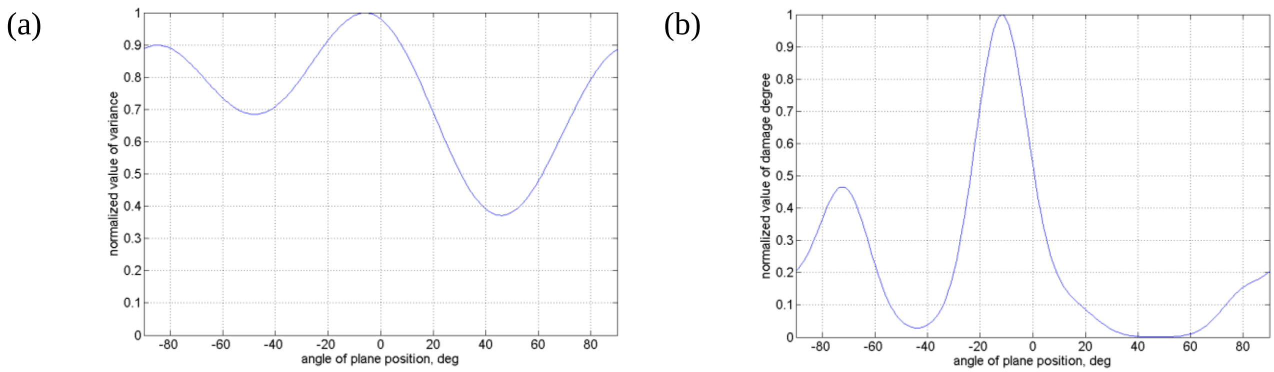

Fig.13. Dependence of the normalized value of: a) variance, b) damage accumulation on the angle α of critical plane position for loading combination K12 ($\lambda_\sigma = 0.68$).

experimental results was shown in Table 2. The angle of the plane position was determined from linear approximation with the least square method. Measurement of the angle α_{FP} was done for the stage II (propagation), and the angle α_τ for the stage I (initiation) was obtained by the plane rotations by the angle of 45°.

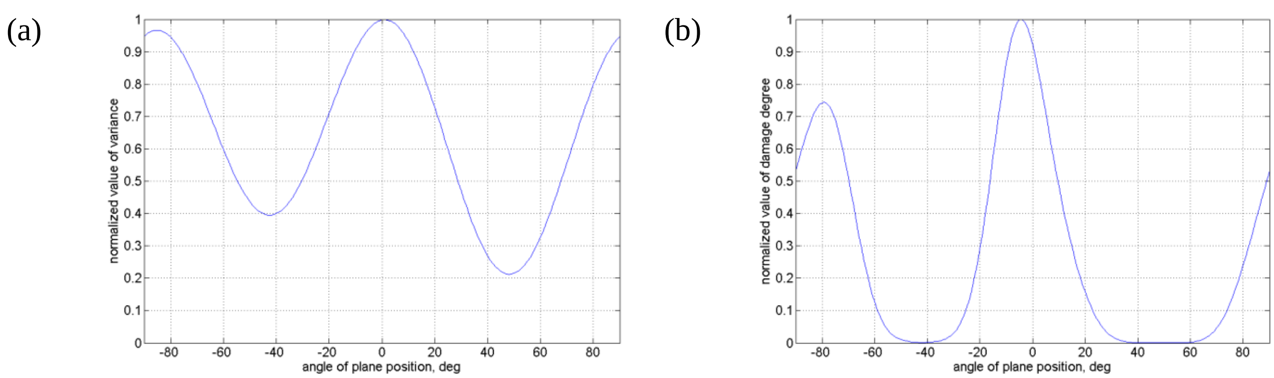

Fig.14. Dependence of the normalized value of: a) variance, b) damage accumulation on the angle α of critical plane position for loading combination K13 ($\lambda_\sigma = 0.84$).

Figure 15 shows differences in experimental and calculated fatigue lives with critical planes determined according to variance and damage accumulation methods. In spite of differences in angle of critical plane position fatigue lives are almost the some for both methods. Determination of the critical plane with the method of variance is connected only with the stress values, while the method of damage accumulation includes also characteristics of the fatigue strength of the given material.

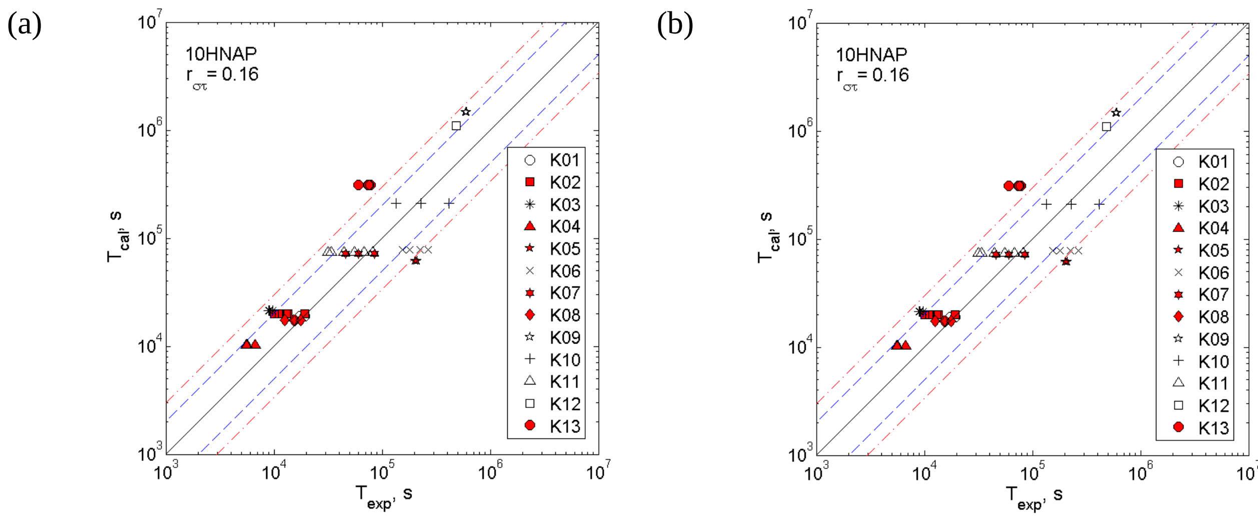

Fig.15. Comparison of experimental and calculated fatigue lives according to the failure criterion of maximum normal and shear stresses in the critical planes determined with: a) variance and b) damage accumulation methods.

Summary

The normalized values of the equivalent stress variance and damage accumulation depending on the angle of the critical plane position α are different for different combinations of loading. The method of variance gives the angles more similar to the experimental data for the initiation plane position (stage I). The variance and damage accumulation methods indicate the critical plane angle ranges (with 5% variation of angle) which overlap in almost all 13 analysed cases of pseudo-random loadings except two. A conformity of the experimental fatigue life with the calculated results was obtained by means of the failure criterion of maximum normal and shear stress in the critical plane according to the both variance and damage accumulation methods.

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References

[1] W. Będkowski, E. Macha: Mat.-wiss. U. Werkstofftech., Vol. 23 (1992), pp. 82-94.
[2] T. Łagoda, E. Macha, W. Będkowski: Int. J. Fatigue, Vol. 21 (1999), pp. 431–443.
[3] A. Karolczuk, E. Macha: Fortschritt-Berichte VDI, Mechanik/Bruchmechanik, reihe 18, no. 298. Düsseldorf: VDI Verlag (2005), p. 204.
[4] D. Rozumek, Z. Marciniak, C. T. Lachowicz: Int. J. of Fatigue, Vol. 32 (2010), pp. 1343-1350.
[5] Z., Marciniak, D. Rozumek, E. Macha: Int. J. of Fatigue, Vol. 30 (2008), pp. 800-813.
[6] G.A. Korn, T.M. Korn : Mathematical Handbook, Sec. Ed., Mc Graw-Hill Book Company, New York (1968).