Evaluation of Concentration of Cracks In Porous Material Using its Overall Deformability to Find Their Influence on its Strength

R.L. Salganik[1,a] , A.A. Fedotov[2,b]

[1] A. Ishlinsky Institute for Problems in Mechanics, Russian Academy of Sciences, prosp. Vernadskogo 101, block 1, Moscow 119526, Russian Federation

[2] «MATI» - Russian State University of Aviation Technology, Orshanskaya 3, Moscow 121552, Russian Federation

[a] salganik@ipmnet.ru, [b] afed83@gmail.com

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Abstract. Theoretical method of evaluation of concentration of cracks in an elastic porous material using its overall deformability found experimentally for ceramics is considered. The method is based on applying the theoretical dependences of effective elastic modules of an elastic solid with particulate spherical pores and disc-like cracks on their, generally non-dilute, concentrations obtained theoretically with help of the differential scheme. As a result, the parameter of crack concentration, which is the mean cube of crack radius times the crack number per unit volume, is obtained. This parameter is important when considering strength properties of the material under study. Special attention is paid to analysis with help of the aforementioned theoretical method of the recently published results on strength of SiC ceramics analyzed by their authors assuming that only porosity is responsible for the observed strength reduction (as compared to the intact material). Meantime, the analysis performed with help of the above differential scheme has shown that one may expect also the presence in the above ceramics of considerable concentration of cracks commensurable with experimentally investigated pores existing in the ceramics and therefore the role of these cracks should not be ignored, when considering the effect of the porosity on strength of SiC ceramics. The latter is anticipated to be true for other ceramics too. Some prospects of performing the mathematical modeling with application of the differential scheme to analyze the experimental results on strength of ceramics are discussed.

Introduction

The technology of production of ceramics necessarily entails appearance of great number of pores, forming porosity, and of great number of cracks in them essentially reducing their elastic modules and strength. Recently, research works have been published on investigation of mechanical properties of a class of ceramics widely applied in electronics and tribo-engineering, namely – SiC ceramics [1-3]. For SiC ceramics, used in nuclear industry, a concise handbook has been issued [4]. In the above research works, the effect of porosity on the mechanical properties of SiC ceramics has been investigated on the basis of rich experimental material. However, the effect of cracks, which very probably are present in ceramics, on the same properties has not been taken into account.

The aim of this work is to show by means of due analysis of known experimental data obtained on bulk samples that cracks under consideration are present in ceramics and should be taken into account when considering both overall deformability and strength of ceramics. As to the overall deformability, the aim is achieved by means of comparison of known experimental data with the results of calculations, performed, using theoretically derived, with help of the differential scheme, equations for effective deformation properties of an elastic solid with great number of disc-like cracks and spherical pores randomly distributed in it [5-7]. As to the strength, the aim is achieved by means of the well known facts, relating to the influence of cracks on strength. The

efficiency and the adequacy of the results obtained by means of the differential scheme to experimental data were confirmed in experiments performed on model samples and on samples of rocks close in their structure to ceramics [6-9]. This method of analysis of the experimental data under consideration was successfully applied to obtain clear evidence of the presence of multitude of cracks in a superconductive ceramics [10].
Real distribution of pores on their sizes in SiC ceramics is multi-scale (from tenths to hundreds of micrometers), the pores being interconnected [1-3]. Assuming that the diameters of relatively large-scale pores are large enough to neglect the contribution of the channels, interconnecting the large-scale pores, one may regard the large-scale pores as the non-connected ones, when considering the contribution of pores in the overall deformability. Next, let us: (i) consider all sufficiently oblate pores and cracks, as disc-like cracks (cracks – in the following), (ii) neglect any effect of too prolate pores, (iii) consider all the remaining pores as spherical ones (pores – in the following).
Taking into account that the typical diameters of the cracks and pores under study are much less than the length scales, which SiC ceramics manifests its overall deformability for, one may pass on from consideration of an elastic solid with great number of pores and cracks, as it was meant above, to regarding a homogeneous elastic solid of the same form, possessing the effective elastic properties (effective medium), which ensure the same deformability as the overall one in the above case of an elastic solid with great number of pores and cracks. For the case of isotropic distribution of pores and cracks that are present in a loaded elastic solid, ensure isotropy of the respective effective medium and markedly interact by virtue of the elastic field disturbances they produce, the above effective properties, namely – the Young's modulus E and Poisson's ratio υ, may be derived by applying the differential scheme [5-7]. The consideration below follows generally to that presented in [10].

Differential scheme for calculation of effective elastic properties of a solid with great number of pores and cracks

A) Effect of cracks

Assume at first that only cracks are present in the solid under consideration. In the case, when their concentration is so dilute that they may be considered non-interacting, the effective elastic properties determined by them may be obtained by simply summing up of the contribution of every crack in these properties, as if all the other cracks were absent. Assuming the crack orientation to be fully random, the cracks themselves to be disc-like and the solid by itself to be homogeneous and isotropic possessing the Young's modulus E_0 and Poisson's ratio ν_0, it is obtained [5]:

$$\begin{cases} E = E_0\left[1-\dfrac{16}{45}(10-3\nu_0)\dfrac{1-\nu_0^2}{2-\nu_0}\tilde{v}\right] \\ \nu = \nu_0\left[1-\dfrac{16}{15}(3-\nu_0)\dfrac{1-\nu_0^2}{2-\nu_0}\tilde{v}\right] \end{cases}, \tilde{v} = N\left\langle a^3\right\rangle \qquad (1)$$

Here N is the number of cracks per unit volume, $\left\langle a^3\right\rangle$ the mean value of the cubed crack radius, $\tilde{v}$ being the crack concentration, having the sense of quasi-porosity, that is the portion of volume occupied by quasi-pores: the unloaded regions (of the sizes of the order of magnitude of a) being adjacent to the surfaces of cracks (for more detail see [11]). The quasi-pore affects the overall deformability like a pore but does not affect the density. Eqs. 1 are valid at $\tilde{v} << 1$ (dilute concentration).
Let now the concentration of cracks be non-dilute but their distribution on their radii is wide enough, other conditions being unchanged. This means that all these cracks may be divided into

such groups, forming a sequence, that in every one of these groups the concentration of cracks is dilute and the radii of these cracks exceed sufficiently those of the cracks included in the preceding group in the sequence to such an extent that the cracks included in the group under consideration may be regarded as if they were situated in a homogeneous and isotropic elastic medium with the Young's modulus and Poisson's ratio being equal to the effective Young's modulus and Poisson's ratio determined by the cracks included in all the preceding groups in the sequence. Next, considering the situations, when the distribution of cracks on their radii is sufficiently wide in order to make the crack concentration increment occurring, when passing from one group of the cracks to the next one, so small that it becomes possible to consider the effective Young's modulus and Poisson's ratio as differentiable functions of the concentration of cracks, while the just mentioned increment of the concentration of cracks may be considered as differential $d\tilde{\nu}$. Then E and ν in Eq. 1 should be replaced with $E_0 + dE$ and $\nu_0 + d\nu$ respectively, where dE and $d\nu$ are differentials. After that E_0 and ν_0 should be replaced by E and ν respectively. Having done all that, the following system of two differential equations for finding two unknowns E and ν obtained at [5]:

$$\begin{cases} \dfrac{d\ln E}{d\tilde{\nu}} = -\dfrac{16}{45}(10-3\nu)\dfrac{1-\nu^2}{2-\nu} \\ \dfrac{d\ln \nu}{d\tilde{\nu}} = -\dfrac{16}{15}(3-\nu)\dfrac{1-\nu^2}{2-\nu} \end{cases} \tag{2}$$

The initial conditions are obviously as follows: $E\big|_{\tilde{\nu}=0} = E_0, \nu\big|_{\tilde{\nu}=0} = \nu_0$

B) Effect of pores

Using considerations analogous in their essence to those applied in subsection A) the following expressions are obtained for pores in the case of their dilute concentration [7]:

$$\begin{cases} E = E_0\left[1-2\pi\dfrac{(1-\nu_0)(9+5\nu_0)}{7-5\nu_0}\tilde{\nu}\right] \\ \nu = \nu_0\left[1+2\pi\dfrac{(1-\nu_0^2)(1-5\nu_0)}{7-5\nu_0}\tilde{\nu}\right] \end{cases}, \tilde{\nu} = N\left\langle a^3\right\rangle \tag{3}$$

where a is the pore radius, N the number of pores per unit volume, other notations being the same as in subsection A).

Applying the differential scheme generally like it has been done in subsection A), the following system of differential equations is obtained [7]:

$$\begin{cases} \dfrac{d\ln E}{d\tilde{\nu}} = -2\pi\dfrac{(1-\nu)(9+5\nu)}{7-5\nu} \\ \dfrac{d\ln \nu}{d\tilde{\nu}} = -2\pi\dfrac{(1-\nu^2)(1-5\nu)}{7-5\nu} \end{cases} \tag{4}$$

The initial conditions are: $E\big|_{\tilde{\nu}=0} = E_0, \nu\big|_{\tilde{\nu}=0} = \nu_0$.

Now it is necessary to take into account the volume of pores (contrary to subsection A), since the crack volume may be neglected). When bringing in the next in turn portion of pores (being of larger-scale than those having been brought in earlier) of concentration $d\tilde{\nu}$ the porosity is increased by $dm = (4/3)\pi(1-m)d\tilde{v}$. Here factor $(1-m)$ takes into account the relative decrease of the volume resulted from the smaller-scale pores having been brought in previously (the effect of intersecting of the pores belonging to their portion under consideration with the boundaries of the pores having been brought in previously may be neglected because of much smaller radii of the latter ones). Thus, upon integrating, the following result is obtained:

$$\tilde{v} = -\frac{3}{4\pi}\ln(1-m) \qquad (5)$$

Although the systems of differential equations Eqs. 2 and 4 have been derived assuming that the cracks and pores are widely enough distributed on their radii, the experiments have shown that the results of calculations based on the solutions of these systems of equations with the above initial conditions yield approximately true results for quite considerable values of the concentration $\tilde{v}$ even for the cases, when the radii of cracks are practically the same as those of pores, provided that the cracks and pores are arranged randomly enough [7,8].

If there are both cracks and pores in the material, the effective properties, i.e. E and ν, may be calculated (at finite values of their concentration) assuming that at first pores and then cracks are brought in the material, and it is supposed that the effective properties calculated, as above, for pores are taken as those appearing in the initial conditions taken to calculate the contribution of cracks to the effective properties resulted from both pores and cracks. Alternatively, a similar procedure is applied to the reverse situation: pores are brought in after cracks. Certainly, it is assumed that in the first case, pores are smaller than cracks and in the second case cracks are smaller than pores.

Now analyze the systems of differential equations Eq. 2 and 4. The Poisson's ratio values lie within the interval from 0 to 0.5; so the variations of the functions in the right hand sides of Eqs. 2 and 4 are small enough to make it acceptable replacement of these functions by their mean values within the above interval, the maximum error associated with this replacement being $\varepsilon = \frac{|\bar{f} - f_{\min}|}{\bar{f}}$. Then the above systems of equations take form:

$$\begin{cases} \dfrac{d\ln E}{d\tilde{v}} = -\dfrac{16}{45}(10-3\nu)\dfrac{1-\nu^2}{2-\nu} \approx \text{-}1.72, \varepsilon = 4.3\% \\ \dfrac{d\ln \nu}{d\tilde{v}} = -\dfrac{16}{15}(3-\nu)\dfrac{1-\nu^2}{2-\nu} \approx \text{-}1.53, \varepsilon = 4.8\% \end{cases} \qquad (6)$$

$$\begin{cases} \dfrac{d\ln E}{d\tilde{v}} = -2\pi\dfrac{(1-\nu)(9+5\nu)}{7-5\nu} \approx \text{-}8.29, \varepsilon = 1,4\% \\ \dfrac{d\ln \nu}{d\tilde{v}} = -2\pi\dfrac{(1-\nu^2)(1-5\nu)}{7-5\nu} \approx -1.01(1-5\nu), \varepsilon = 4.8\% \end{cases} \qquad (7)$$

Let's solve Eqs. 6 and 7 taking into account the above initial conditions.
For cracks we obtain:

$$E = E_0 e^{-1.72\tilde{v}}, \upsilon = \upsilon_0 e^{-1,53\tilde{v}} \qquad (8)$$

For pores we obtain:

$$E = E_0 e^{-8.29\tilde{v}}, \upsilon = \frac{\upsilon_0}{5\upsilon_0 - (5\upsilon_0 - 1)e^{1.01\tilde{v}}}, \tag{9}$$

where the Poisson's ratio of the material by itself is assumed exceeding 0.2. Taking into account Eq. 5, we obtain:

$$E = E_0(1-m)^{1.98}, \upsilon = \frac{\upsilon_0}{5\upsilon_0 - (5\upsilon_0 - 1)(1-m)^{-0.24}} \tag{10}$$

Experimental data analysis

Analyze now, with help of Eqs. 8 and 10, the experimental data presented in works [1-4].

In Table 1 the data on porosity and mechanical properties of SiC ceramics specimens, described in [1-3], are listed.

Table 1. Mechanical properties of SiC ceramics specimens [1-3]

Specimen	Density ρ, [g/cm^3]	Porosity m, [%]	Young's modulus E, [GPa]	Poisson's ratio υ
SiC1	3.28	1.1	380	0.22
SiC2	3.26	2.4	375	0.21
SiC3	3.22	3.0	360	0.21
SiC4	3.05	4.5	360	0.21
SiC5	2.93	10.3	255	0.21

Using: (i) estimation for Young's modulus relating to practically "non-porous" SiC ceramics E_0=400 GPa [2] , and (ii) the calculation results for effective Young's modulus obtained with help of Eq. 10 for the porosity values given in Table 2, the porosity values have been found (see the third column of Table 2). From Table 2 it is seen that using Eq. 10 to calculate E gives an overestimated value of it. Hence it is seen that taking into account only the porosity is not sufficient to explain the obtained reduction of Young's modulus of the ceramics under consideration as compared to the value of this modulus that would be in the absence of any pores in it. Now calculate by means of Eq. 8 the concentration of cracks that would be sufficient to give the value of the calculated Young's modulus reduction enabling one to fully explain the experimental results presented in [1-3] (cf. second and third columns of Table 2). The results of the aforementioned calculation are presented in fourth and fifth columns of Table 2. In the fifth column of Table 2, calculated mean value of the ratio of crack radius to the distance between cracks, i.e. $\tilde{v}^{1/3}$, is presented. These calculations have been performed assuming the radii of cracks being much less than those of pores.

Table 2. Results of effective modulus calculation for SiC ceramics specimens [1-3]

Specimen	Young modulus E, [GPa] (experimental [1-3])	Young modulus E, [GPa] (calculation)	Crack concentration $\tilde{v}$	Mean value of the ratio of crack radius to the distance between cracks ($\tilde{v}^{1/3}$)
SiC1	380	391	0.017	0.26
SiC2	375	381	0.009	0.21
SiC3	360	377	0.026	0.3
SiC4	360	365	0.008	0.2
SiC5	255	323	0.137	0.52

Next, let us analyze the data presented in [4]. Based on generalization of vast amount of experimental data obtained for SiC ceramics in nuclear industry, the following empirical formula for the dependence of the Young's modulus on the porosity has been proposed [4] :

$$E = E_0 e^{-3.57m}, \tag{11}$$

where E_0=400 GPa is the modulus for the assumed to be pore-free SiC material, the uncertainty of the elastic modulus E of SiC being 10% for porosity m<1% and 15% for m>1%. Eq. 11 implies that porosity is the only cause for Young's modulus decrease, no effect of cracks being present. The results of calculations of the effective elastic modulus (similar to the above calculations used to analyze the data presented [1-3]) are listed in Table 3.

Table 3. Results of effective modulus calculation for SiC ceramics [4]

Porosity m, [%]	Young's modulus E, [GPa] (empirical [4])	Young's modulus E, [GPa] (calculation)	Crack concentration $\tilde{\nu}$	Mean value of the ratio of crack radius to the distance between cracks ($\tilde{\nu}^{1/3}$)
1	444	451	0.009	0.21
5	385	416	0.045	0.36
10	322	373	0.086	0.44
15	269	333	0.12	0.50

It is seen that the difference between the empirical and the calculated Young's modulus exceeds markedly the uncertainty of the empirical formula if the porosity m>10%. So, this difference can't be explained as that resulted from statistical scatter.

Difference between the values of concentration of cracks presented in fourth columns of Tables 2 and 3 leads to the conclusion that Young's modulus in [1-3] for specimens 1 and 4 may be underestimated and overestimated respectively.

Analysis of the experimental data presented in works [1-4] leads to the conclusion, that considering only porosity as the cause of decreasing elastic modulus in ceramics is not sufficient. The calculations based on the differential scheme has shown that the effect of cracks should also be taken into account.

Some prospects for experimental research on damage and strength of ceramics

The above analysis has two lacks. Firstly, it remains unknown, whether: (i) the radii of pores exceed those of cracks and then one needs to consider the pores as being situated in the solid possessing the deformation properties that are the effective ones determined by the cracks, or (ii) the radii of cracks exceed those of pores and then one needs to consider the cracks being situated in the solid possessing the deformation properties that are the effective ones determined by the pores. However, the experiments have shown [7, 8] that even if the radii of pores are close to those of cracks the difference in the calculation results obtained for the overall deformation properties in these two cases turns out to be small enough, so that frequently it is possible to neglect this difference. Secondly, the above analysis is only estimative. As to this lack, it seems that in order to make such analysis considerably more exact, one need not to confine oneself to using only the overall deformability and porosity, as it has been done above.

There are the following two ways that may considerably improve the analysis as compared to the one performed above.

The first way is to prepare thin sections as it is accepted, say, in petro physics, and to analyze the cracks and pores, like those considered above, in situ. Note at this juncture the work [12] where the role of pores as fracture origins in ceramics is considered.
The second way is to apply ultrasonic methods to investigate the ceramics specimens, using the expressions for the velocities of elastic waves:

$$c_1 = \sqrt{\frac{E(1-\upsilon)}{(1+\upsilon)(1-2\upsilon)\rho}}\,,\ c_2 = c_1\sqrt{\frac{1-2\upsilon}{2(1-\upsilon)}}\,, \qquad (12)$$

Here c_1 and c_2 are the velocities of longitudinal and shear waves respectively. Note that this way was applied in [10].
As it's seen from Eq. 12, elastic modulus and porosity both determine velocities of elastic waves, but only porosity affect the density, so that there is possibility to estimate separately the effect of pores and that of cracks. Such possibility was used in [10] by means of the approach based on applying the differential scheme. An additional way helpful to raise the reliability of solving this problem is the experimental one: a specimen should be compressed to get full closure of the cracks and then compare velocities of acoustic waves for this specimen in its compressed state and its state before it has been compressed. Such experimental results were analyzed in [8], where it was found that these results are in good agreement with respective theoretical results obtained using the differential scheme [5].
Sounding a specimen with ultrasonic waves one can found their effective modulus, provided that these elastic waves are not too short in comparison with the characteristic lengths of cracks and pores present within the specimen; so, varying the typical length of these waves one can obtain valuable information on length-scales of the cracks and pores.

Summary and conclusions

The idea of the presented study was to use the experimentally determined dependence of Young's modulus for porous material (ceramics) on its porosity and by means of comparison of this dependence with that found theoretically (with help of applying the differential scheme, assuming that only pores present in the material are responsible for the above dependence), to learn whether the material contains also a sufficiently great number of cracks capable considerably affect overall deformability of the material. The performed analysis including also the use of the theoretical dependence (found with help of application of the differential scheme) of the overall deformability on the concentration of multitude of cracks presumably present in porous material has shown that such cracks indeed are present in the material under study.
For the effective modulus calculations differential scheme proposed in [5-7] and approved on model specimens and in situ [6-9] was used. Analysis of experimental data [1-4] on the dependence of Young's modulus for SiC ceramics on their porosity has shown that taking into account the effect of cracks is necessary to explain the above experimental dependence. Making use of ultrasonic sounding may by highly instrumental for performing further experimental research on strength of ceramics.
The results of the presented study have shown their attractive potentialities for further studying ceramics and ceramic-like materials (like rock).
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