Durability estimations for in-service titanium compressor disks subjected to multiaxial cyclic loads in low- and very-high-cycle fatigue regimes

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Abstract. Present research is devoted to criteria and models of multiaxial fracture for cases of low-cycle fatigue (LCF) and very-high-cycle fatigue (VHCF). The model parameters are determined by using the data of uniaxial fatigue tests for various stress ratios. The procedure for stress state calculating of a compressor disk of a gas turbine engine subjected to simultaneously flight cycle loading conditions and blades vibrations is outlined. Discussed models of multiaxial fatigue fracture, based on calculated various stress states, are used to estimate the durability of the compressor disk under LCF and VHCF conditions. The stress state simulation results are compared with observed in-service data collected from fractographic investigations of failed disks.

Introduction

The phenomenon of in-service gas turbine engine (GTE) compressor disks fatigue fracture is well-known [1]. Usually compressor disks are manufactured from Ti-based alloy Ti-6Al-4V. According to fractured disk analysis in most cases the fatigue fracture is observed near the contact zone of disk and blade.

Because of fatigue failure of the disks of the I-stage low-pressure-compressor (LPC) one of the GTE was repeated it was developed numerical approach to estimate the disk durability. The finite element (FE) model of real GTE disk with shrouded blades and pins is created and 3D strain-stress state was calculated under multiaxial loading conditions and under centrifugal, aerodynamic and nonlinear contact loads. Peak stress-strain values correspond to flight velocity 200 m/s and rotation frequency 3000 rpm. Aeroelastic effects due to airflow and deformable structure interactions are also were taken into account.

Durability estimations were found as a limit of safety flight loading cycles number for the structure. Several multiaxial fatigue criteria were considered and results of simulated durability were compared with flight service data. In addition the very-high-cycle fatigue (VHCF) due to observed high frequency axial vibrations of shroud ring is studied. Maximal amplitude of vibrations is assumed to be equal to ±1mm under frequency 3000 rpm. Durability estimations are found also as vibration cycles number safety limit according to VHCF criteria. Because of absence of experimentally proved VHCF multiaxial criteria the known low-cycle fatigue (LCF) criteria are applied. Parameters of these criteria were determined on the basis of not numerous uniaxial VHCF experimental data for investigated material.

Statement of the problem

Estimation Models Based on the Stress-Strain State. The determination of multiaxial fatigue fracture parameters is based on experimental curves of uniaxial cyclic tests for different values of stress ratio $R=\sigma_{\min}/\sigma_{\max}$, where $\sigma_{\max}$ and $\sigma_{\min}$ are the maximal and minimal stresses during the cycle. The stress amplitude is denoted as $\sigma_a=(\sigma_{\max}-\sigma_{\min})/2$ and the stress range is $\Delta\sigma=\sigma_{\max}-\sigma_{\min}$.

The experimental data of uniaxial tests are described by Weller curves, which can analytically be represented by the Baskin relation [2]

$$\sigma=\sigma_u+\sigma_c N^{\beta} \tag{1}$$

where σ_u is the fatigue limit, σ_c is the fatigue strength factor, β is the fatigue strength exponent, and N is the number of cycles to fracture. Typical curve is shown in Fig. 1, in this regime we are interested in the left branch of the curve for durability $N<10^7$.

The problem of studying fatigue fracture implies that the spatial function of durability N distribution must be determined from equations in the form (1) generalized to the case of multiaxial stress state and containing the calculated stresses in the structure under study.

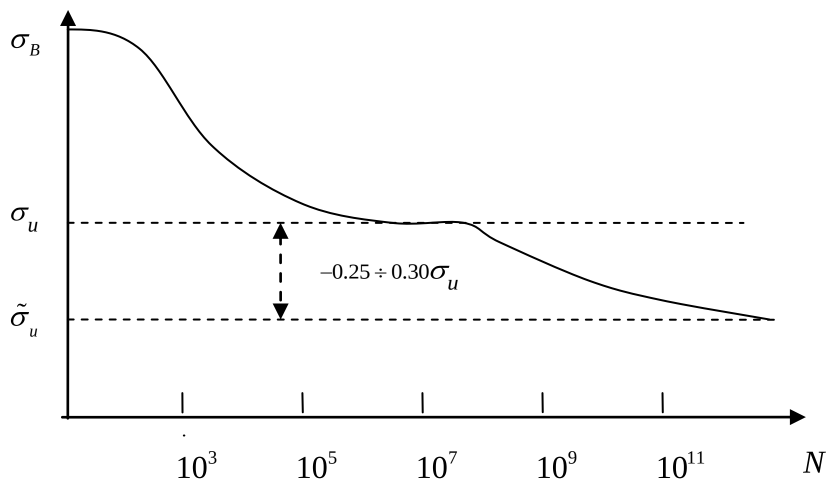


Figure 1. Scheme of S-N curve for fatigued metals.

Let us consider the basic ways of generalizing the results of uniaxial tests to the case of multiaxial stress state [2].

Sines's model. According to [3], the uniaxial fatigue curve (1) can be generalized to the case of multiaxial stress state as

$$\Delta\tau/2+\alpha_s\sigma_{\text{mean}}=S_0+AN^{\beta},\quad \sigma_{\text{mean}}=(\sigma_1+\sigma_2+\sigma_3)_{\text{mean}}, \tag{2}$$
$$\Delta\tau=\sqrt{(\Delta\sigma_1-\Delta\sigma_2)^2+(\Delta\sigma_1-\Delta\sigma_3)^2+(\Delta\sigma_2-\Delta\sigma_3)^2}/3$$

where σ_{mean} is the mean sum of principal stresses over a loading cycle, $\Delta\tau$ is the change in the octahedral tangent stress per cycle, $\Delta\tau/2$ is the octahedral tangent stress amplitude, and α_s, S_0, A and β are parameters to he determined from experimental data.

The model parameters from uniaxial fatigue curves are determined in [2]:

$$S_0=\sqrt{2}\sigma_u/3,\quad A=10^{-3\beta}\sqrt{2}(\sigma_B-\sigma_u)/3,\quad \alpha_s=\sqrt{2}(2k_{-1}-1)/3,\quad k_{-1}=\sigma_u/(2\sigma_{u0}) \tag{3}$$

where σ_u and σ_{u0} are the fatigue limits according to the curves $\sigma_a(N)$ for $R = -1$ and $R = 0$, respectively.

Crossland's model. According to [4], the uniaxial fatigue curve can be generalized to the case of multiaxial stress state as

$$\Delta\tau/2 + \alpha_c(\overline{\sigma}_{\max} - \Delta\tau/2) = S_0 + AN^{\beta}, \quad \overline{\sigma}_{\max} = (\sigma_1 + \sigma_2 + \sigma_3)_{\max} \qquad (4)$$

where $\overline{\sigma}_{\max}$ is the maximum sum of principal stresses in a loading cycle; the parameters α_c, S_0, A and β are to be determined. The final expressions relating the parameters of the multiaxial model to those of the uniaxial fatigue curves for $R = -1$ and $R = 0$ read [2]:

$$S_0 = \sigma_u\left[\sqrt{2}/3 + (1-\sqrt{2}/3)\alpha_c\right], \quad A = 10^{-3\beta}\left[\sqrt{2}/3 + (1-\sqrt{2}/3)\alpha_c\right](\sigma_B - \sigma_u) \qquad (5)$$
$$\alpha_c = (k_{-1}\sqrt{2}/3 - \sqrt{2}/6)\Big/\left[(1-\sqrt{2}/6) - k_{-1}(1-\sqrt{2}/3)\right]$$

Findley's model. The form of this model for the case of multiaxial stress state is proposed in [5]:

$$(\Delta\tau_s/2 + \alpha_F\sigma_n)_{\max} = S_0 + AN^{\beta} \qquad (6)$$

where τ_s, σ_n are the modules of the tangent stress and normal stress for the plane with normal n_i, for this plane combination $\Delta\tau_s/2 + \alpha_F\sigma_n$ takes a maximum value. Parameters of the model read

$$S_0 = \sigma_u\left(\sqrt{1+\alpha_F^{\,2}} + \alpha_F\right)/2, \qquad A = 10^{-3\beta}(\sqrt{1+\alpha_F^{\,2}} + \alpha_F)(\sigma_B - \sigma_u)/2 \qquad (7)$$
$$\alpha_F = \frac{\sqrt{5k_{-1}^{\,2} - 2k_{-1}}/2 - k_{-1}(1-k_{-1})}{k_{-1}(2-k_{-1})}$$

Here are approximate values of the parameters for titanium alloy Ti-6Al-4V [2] for a specific computational example that will be considered below: the limit strength is $\sigma_B = 1100$ MPa, the fatigue limits according to the curves $\sigma_a(N)$ for $R = -1$ and $R = 0$ are equal $\sigma_u = 450$ MPa and $\sigma_{u0} = 350$ MPa, respectively, the exponent in the power-law dependence on the number of cycles is $\beta = -0.45$, Young's modulus is $E = 116$ GPa, the shear modulus is $G = 44$ GPa, and Poisson's ratio is $\nu = 0.32$.

Models Based on the Strain State Estimation. The classical Coffin—Manson relation [1] describing the uniaxial fatigue fracture on the basis of the strain change per loading cycle read

$$\frac{\Delta\varepsilon}{2} = \frac{\sigma_c}{E}(2N)^b + \varepsilon_c(2N)^c$$

where σ_c is the (axial) fatigue strength coefficient ε_c is the (axial) fatigue plasticity coefficient, and b and c are the fatigue strength and fatigue plasticity exponents. Briefly outlined below are models generalizing the Coffin-Manson relation to the case of multiaxial fatigue fracture.

Brown-Miller model. The model is proposed in [6], this model takes into account the influence of the normal strains to the plane of maximum shear strains:

$$\frac{\Delta\gamma_{\max}}{2} + \alpha_{bm}\Delta\varepsilon_{\perp} = \beta_1\frac{\sigma_c - 2\sigma_{\perp\text{mean}}}{E}(2N)^b + \beta_2\varepsilon_c(2N)^c \qquad (8)$$

where $\gamma_{ij} = 2\varepsilon_{ij}$, ε_{ij} are the strain tensor components, $\Delta\gamma_{\max}/2$ is the range of the maximum shear strains attained on a plane, $\Delta\varepsilon_{\perp}$ is the range of the normal strains on this plane, and $\sigma_{\perp\text{mean}}$ is the cycle-average normal stress on this plane. Approximate values of the coefficients are given in [2]: $\alpha_{bm} = 0.3$, $\beta_1 = (1+\nu)+(1-\nu)\alpha_{bm}$, $\beta_2 = 1.5+0.5\alpha_{bm}$.

Fatemi-Socie's model. The model is proposed and developed in [7]. This model takes into account the influence of the normal stresses to the plane of maximum shear strains:

$$\frac{\Delta\gamma_{\max}}{2}(1+k\frac{\sigma_{\perp\max}}{\sigma_y}) = \frac{\tau_c}{G}(2N)^{b_0} + \gamma_c(2N)^{c_0} \tag{9}$$

Here $\sigma_{\perp\max}$ is the cycle-maximum normal stress on the plane where $\gamma_{\max}$ is attained, σ_y is the material yield strength, τ_c is the fatigue (shear) strength coefficient, γ_c is the fatigue (shear) plasticity coefficient, and b_0 and c_0 are the fatigue strength and fatigue plasticity exponents. The coefficient k is approximately equal to $k = 0.5$ [2].

Smith-Watson-Topper's model. The model is proposed in [8]. This model takes into account the influence of the normal stress to the plane of maximum tensile strains:

$$\frac{\Delta\varepsilon_1}{2}\sigma_{\perp 1\max} = \frac{\sigma_c^{\;2}}{E}(2N)^{2b} + \sigma_c\varepsilon_c(2N)^{b+c} \tag{10}$$

where $\Delta\varepsilon_1$ is the change in the maximum principal strain per cycle and $\sigma_{\perp 1\max}$ is the maximum normal stress on the plane of maximum tensile strains. The fatigue parameters of titanium alloys for this class of models are chosen from data [2]: $\sigma_c = 1445$ MPa, $\varepsilon_c = 0.35$, b = -0.095, c = -0.69, $\tau_c = 835$ MPa, $\gamma_c = 0.20$, b_0 = -0.095, c_0 = -0.69, σ_y = 910 MPa.

Models of Fatigue Fracture with Damage. *Lemaitre-Chaboche's model*. In [9], the differential equation for the damage D accumulated under multiaxial cyclic loading is suggested. Integrating yields

$$N = \frac{1}{(1+\beta)a_M}\left[\frac{(1-3b_2\bar{\sigma})}{A_{IIa}}\right]^{\beta}\left\langle\frac{(\sigma_u-\sigma_{VM})}{(A_{IIa}-A^*)}\right\rangle \tag{11}$$

where the notation from [9] is preserved:

$$A_{IIa} = 0.5\sqrt{1.5\left(S_{ij,\max}-S_{ij,\min}\right)\left(S_{ij,\max}-S_{ij,\min}\right)} \quad , \quad \sigma_{VM} = \sqrt{0.5S_{ij,\max}S_{ij,\max}}$$

$$\bar{\sigma} = (\sigma_1+\sigma_2+\sigma_3)_{\text{mean}}/3, \quad A^* = \sigma_{10}\left(1-3b_1\bar{\sigma}\right), \quad a_M = a/M_0^{\beta}$$

where $S_{ij,\max}$ and $S_{ij,\min}$ are the maximum and minimum values of the stress deviator in the loading cycle; the angle brackets are defined as: $\langle X\rangle = 0$ for $X < 0$ and $\langle X\rangle = X$ for $X \geq 0$. The model parameters for a titanium alloy are given in [9]: $\beta = 7.689$, $b_1 = 0.0012$, $b_2 = 0.00085$ 1/MPa, $a_M = 4.1\cdot 10^{-28}$, $\sigma_{10} = 395$ MPa, and $\sigma_u = 1085$ MPa.

LU's model (Liege University). This model is proposed and validated in [9]. In this case, the integrated differential equation for the damage read

$$N = \frac{\gamma+1}{C}\left\langle \frac{\sigma_u - \theta\cdot\sigma_{VM}}{A_{IIa} - A^*} \right\rangle f_{\mathrm{cr}}^{-(\gamma+1)} \tag{12}$$

where the notation of [9] is preserved:

$$f_{\mathrm{cr}} = \frac{1}{b}\left(A_{IIa} + a\sigma_H - b\right)\ , \quad f_{\mathrm{cr}} > 0\ \ , \ A^* = \sigma_{-1}\left(1 - 3s\sigma_H\right)\ , \qquad \sigma_H = (\sigma_1 + \sigma_2 + \sigma_3)_{\max} / 3$$

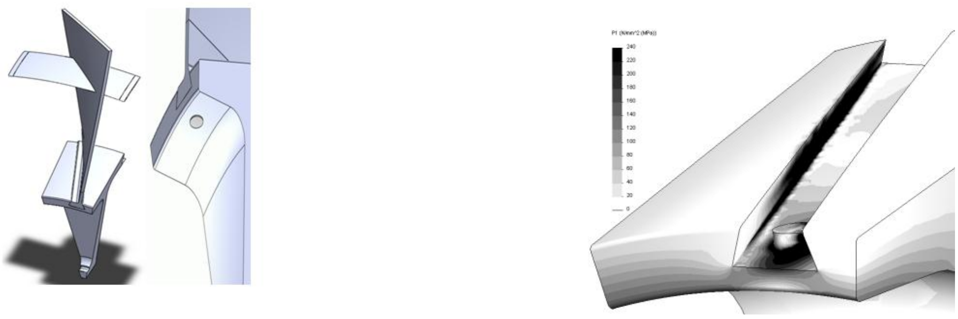
The model parameters are taken from [9]: a =0.467, b=*220* MPa, γ =*0.572*, $C = 7.12\cdot 10^{-5}$, θ =*0.75*, *s*=*0.00105* 1/MPa, σ_{-1} =*350* MPa, σ_u =*1199* MPa.

Example of multiaxial stress state calculation and durability estimation for structural elements in the flight cycle of loading

Computational Model of the Compressor Disk. As an example, we consider the problem of fatigue fracture of the GTE compressor disk under low-cycle-fatigue (LCF) conditions. It is assumed that the cycle of multiaxial loading of the disk-blade system is the flight loading cycle (FLC), in which the maximal loads at the aircraft cruising speed and the corresponding angular velocities of the compressor disk rotation are attained. The problem is to determine the disk service life N (the number of FLCs before fracture) from relations (2), (4), (6), and (8)-(12). To this end, it is necessary to calculate the stress state of the disk-blade system under the combined action of the external loads, represented by the centrifugal forces, the distributed aerodynamic pressures on the blades, and the forces of nonlinear contact interaction between the disk, blades, and any other additional structural elements that are taken into account; these elements will be discussed below.

In the present paper, the three-dimensional stress-strain state of the contact system of the compressor disk and blades (Fig. 2a) is analyzed numerically using finite-element software package [10], and the distributed aerodynamic loads are determined approximately by analytical methods based on the use of classical solutions to the problem of flow about a grid of plates at arbitrary angle of attack; the solutions are obtained by the complex variable analysis methods on the basis of the isolated profile hypothesis with the blade strain state taken into account (see [11] for details).

The input parameters are the following: the angular velocity of rotation ω = 314 rad/s (3,000 revolutions per minute), the dynamic pressure at infinity $\rho v_\infty^2 / 2$ = 26,000 N/m^2, what corresponds to the flow velocity 200 m/s and density 1.3 kg/m^3. The total number of finite elements does not exceed 100,000, and it is quite acceptable for computations on a personal computer. The material properties are as follows: E=116 GPa, ν =0.32, and ρ = 4370 kg/m^3 for the disk (titanium alloy), E = 69 GPa, ν = 0.33, and ρ = 2700 kg/m^3 for the blades (aluminum alloy), and E = 207 GPa, ν = 0.27, and ρ = 7860 kg/m^3 for the fixing pins (steel).

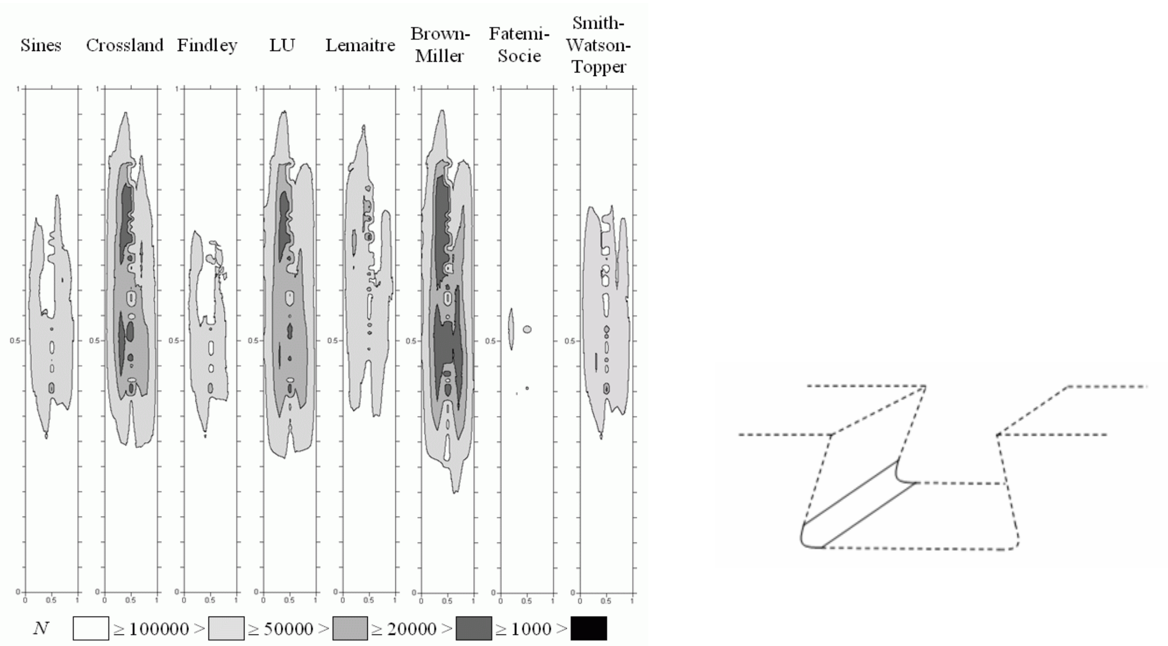


a) b)

Figure 2. Examples (a), (b) of considered in-blades and in disks stress-states respectively.

The computations [11] show that the most dangerous areas are situated near the "swallow tail" contact regions between the disk and the blades. The computations [11] also show that the best correspondence between computational and experimentally observable stress concentration regions is provided when the possibility of detachment and slip of the disk and blade contact boundaries is taken into account. At the boundary of the fixing pin (Fig. 2-a), the conditions of complete adhesion are posed from technological considerations. Figure 2-b shows the zone of maximum tensile stresses concentration at the left (rounded) corner of the groove where the blade is inserted. One can see that the stress concentration increases from the front to the rear part of the groove, which coincides with the data on the location of fatigue crack nucleation regions in the rear part of the disk [1].

Service Life Estimation of Structural Elements. In Fig. 3a, the computed values of the number *N* of flight cycles before fracture for the chosen criteria and models of multiaxial fatigue fracture are displayed near the left "swallow tail" disk-blade contact joint (in the regions of maximum stress concentration). In Fig. 3b, the neighborhood of the disk contact groove left corner is shown by solid lines.



a) b)

Figure 3. Examples of stress-strain (a) distributions estimated by the different models and (b) shape of slot for investigated titanium disk with indication area for crack initiation.

In Fig. 3a, the horizontal axis represents the dimensionless coordinate of the rounding of the groove's left corner; the vertical axis represents the dimensionless coordinate across the groove depth. The Sines, Lemaitre-Chaboche, Brown-Miller, and Smith-Watson-Topper criteria provide estimates of the service life of gas turbine engine disks of about 20,000-50,000 cycles. The Crossland and LU criteria predict the possibility of fatigue fracture after less than 20,000 flight cycles. On the whole, all these criteria give similar pictures of the fatigue fracture regions location. The Fatemi-Socie criterion gives a service life prediction of about 100,000 cycles. The deviation of the Fatemi-Socie estimate from the results based on the other criteria may testify that the shear mechanism of multiaxial fatigue fracture, which is reflected in this criterion, is not purely realized in flight loading cycles.

Example of multiaxial stress state calculation and durability estimations for structural elements under low amplitude axial loading.

Alternative fatigue mechanism. Multiaxial very-high-cycle fatigue regime. In addition the alternative fatigue mechanism is studied for observed high frequency axial vibrations of shroud ring. The amplitudes of vibrations and stress state disturbances near stress concentrators are relatively small, but the number of high frequency vibrations may be about 10^9 - 10^{10}. Therefore the

investigation of very-high-cycle fatigue (VHCF) is necessary because fatigue may take place even if stress level is less than classical fatigue limits [1].
At present there is no experimentally proved multiaxial VHCF theory. Therefore in order to get durability estimations the known multiaxial LCF models (2), (4) и (6) are used taking into account general assumptions about VHCF curves. Typical fatigue curve is presented in Fig.1, in case of VHCF the right part of the curve for $N > 10^8$ is of interest.
Generalization of Sines model (2). The VHCF parameters are detected by using one-dimensional fatigue curves in the same way as in the LCF case. The similarity between left and right parts of fatigue curve is taken into account by substitutions $\sigma_B \to \sigma_u$, $\sigma_u \to \tilde{\sigma}_u$, $\sigma_{u0} \to \tilde{\sigma}_{u0}$, where $\tilde{\sigma}_u$ and $\tilde{\sigma}_{u0}$ are «new» fatigue limits for right part of fatigue curve for asymmetry factors $R = -1$ и $R = 0$. VHCF parameter values for generalized of Sines model (2) read:

$$S_0 = \sqrt{2}\tilde{\sigma}_u / 3,\; A = 10^{-8\beta}\sqrt{2}(\sigma_u - \tilde{\sigma}_u) / 3,\; \alpha_s = \sqrt{2}(2k_{-1} - 1)/3,\quad k_{-1} = \tilde{\sigma}_u / \tilde{\sigma}_{u0} / 2$$

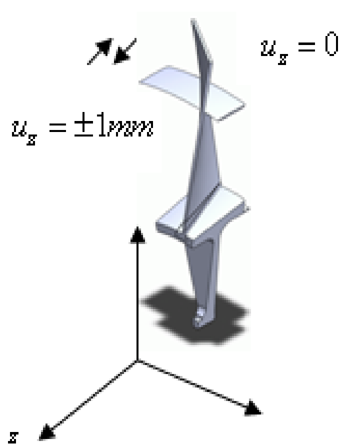
Generalization of Crossland model (4). By analogy the VHCF parameters for generalized of Crossland model (4) read:

$$S_0 = \tilde{\sigma}_u\left[\sqrt{2}/3 + (1 - \sqrt{2}/3)\alpha_c\right],\quad A = 10^{-8\beta}\left(\sigma_u - \tilde{\sigma}_u\right)\left[\sqrt{2}/3 + (1 - \sqrt{2}/3)\alpha_c\right]$$

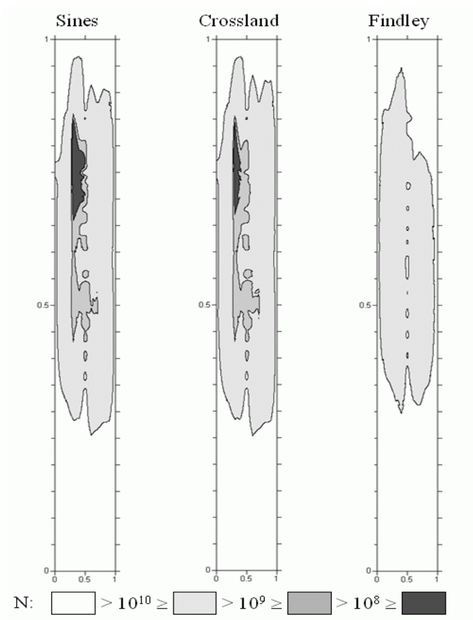
Generalization of Findley model (6). The VHCF parameters for generalized of Findley model read:

$$S_0 = \tilde{\sigma}_u\left(\sqrt{1 + \alpha_F^{\,2}} + \alpha_F\right) / 2,\; A = 10^{-8\beta}\left(\sigma_u - \tilde{\sigma}_u\right)(\sqrt{1 + \alpha_F^{\,2}} + \alpha_F) / 2$$

where for titanium alloy the following parameter values are used σ_u=450 MPa, $\tilde{\sigma}_u$=250 MPa, $\tilde{\sigma}_{u0}$=200 MPa, β = -0.3.



a)



b)

Figure 4. Scheme (a) of blade deformation in the case of introducing in-calculation vibrations

and (b) examples of stress-state estimation in this case by different criteria

Numerical model for low amplitude of axial vibrations of shroud ring. Axial displacements of shroud ring are caused by its vibrations. The form of vibrations along the ring contains about 12-16 half waves. For disc-blade sector calculation the right side of shroud ring displacement is equal to zero while the left side displacement is equal to maximal vibration amplitude $\pm 1mm$ (Fig. 4-a) for frequency of 3000 rpm. Low amplitude vibrations are imposed on the basic stress state.

Structure element durability estimation according to VHCF criteria. Maximal stress concentration is observed near the rounding of the groove's left corner (Fig. 3b). On Fig. 4b for selected area of left corner the calculated limits of N (safety flight loading cycles number) are shown. The results are obtained by using VHCF generalized criteria of Sines, Crossland and Findley. In spite of rather low stress amplitude level in this case fatigue zones are possible also. The fatigue zones are situated near the rear part of the groove's left corner (in the same place as in the case of LCF). The safety flight loading cycles number N is approximately equal to 10^9 - 10^{10} what corresponds to exploitation time of 50 000 hours.

Though these durability estimations are rather approximate they point onto possibility of fatigue development in considered structure elements for both cases of LCF (flight cycles) and VHCF (high frequency low amplitude vibrations). The most serious danger may happen due to mutual action of mentioned mechanisms because they may develop almost simultaneously in one and the same place.

Summary

In present research the procedure of structure elements durability estimation for two alternative LCF (flight cycles) and VHCF (vibrations) fatigue mechanisms is developed. The comparative study of durability estimation of the GTE compressor disk-blade contact structure is performed on the basis of Sines, Crossland and Findley fatigue models. Obtained results indicate very close durability estimations for LCF and VHCF with in-service time for titanium compressor disk one of the GTE.

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References

[1] A.A. Shanyavskiy: *Modeling of Metal Fatigue fracture*. (Monograph, Ufa, Russia 2007).
[2] N.G. Burago, A.B. Zhuravlev, I.S. Nikitin: Mechanics of Solids, Vol. 46 (2011), p. 828
[3] G. Sines: Metal fatigue, McGraw-Hill, (1959), p. 145
[4] B. Crossland: In: *Proceedings Int. Conf. on Fatigue of Metals*, London (1956), p.138
[5] W.N. Findley: J. of Eng. for Industry, (1959), p. 301
[6] M. Brown, K.J. Miller: Institute of Mech. Engineers, Vol. 187 (1973), p. 745
[7] A. Fatemi, D.F. Socie: Fatigue Fract. Engng Mater. Struct. Vol. 11 (1988), p.149
[8] R.N. Smith, P. Watson, T.H. Topper: J. of Materials, Vol. 5 (1970), p. 767
[9] A.K. Marmi, A.M.Habraken, L. Duchene: In: *Proceedings of 9 Int. Conf. of Multiaxial Fatigue and Fracture* (ICMFF9), edited by A. Carpintery, L. Pook, A. Spagnoli, Parma, Italy (2010), p. 559
[10] A.A. Alyamovskii, A.A. Sobachkin, E.V. Odintsov, et al. *SolidWorks: Computer Simulation in Engineering Practice* (BKhv-Peterburg, St. Peterburg, Russian, 2005)
[11] N.G. Burago, A.B. Zhuravlev, I.S. Nikitin: Vych. Mech. Sploshn. Sred. Vol. 4 (2011), p. 5