Delamination of the Soft Coatings of Complicated Structure from the Hard Substrate Caused by Torsion or Indentation

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Abstract. Stresses at the interface between the functionally graded coating and the elastic half-space are of particular interest because of their influence on the propagation of cracks and other defects on this interface. Shear stresses at this interface associated with rapid variation in elastic properties with depth are particularly dangerous because of potential delaminating.

Method of construction an approximate analytical solution of the dual integral equations, generated by contact problems of the elasticity theory for inhomogeneous coatings is developed. New effective method of construction an analytical approximation of kernel transforms by the function of a special kind is obtained. Using approximation of kernel transforms of high accuracy, analytical solutions of contact problems for materials with coatings of complicated structure are constructed. Coatings which elastic properties are much smaller than elastic properties of the substrate are considered. This formulation corresponds to contact problems for inhomogeneous layer on a non-deformable substrate. Influence of the structure of inhomogeneous coatings on a distribution of contact stresses acting under a stamp, as well as the influence on the distribution of stresses, deformations and displacement inside the elastic half-space is investigated.

1 Introduction

The problem about torsion of a circular punch attached to homogeneous isotropic elastic half space, to the best of our knowledge, was first formulated and solved by Reissner and Sagoci [1]. Rostovtsev [2] and Sneddon [3] independently developed an alternative technique to solve this problem by reduction to a dual integral equation using Hankel integral transform.

Problem about a punch attached to inhomogeneous isotropic half-space was first addressed in works of Kassir [4]. Aizikovich [5] proposed an approximated analytical solution for the problem about torsion of a punch attached to a half-space coated by a layer with arbitrary variation of the shear modulus through its thickness. Grilitsky [6] addressed the static problem about torsion of a punch attached to an isotropic two-layer media and orthotropic layer. The solution was given in the form of series expansion with respect to the relation of the punch radius to the thickness of the upper layer.

More recently, Liu and Wang [7] considered functionally-graded half-space using piecewise-linear approximation of kernel's transform to reduce the problem to an integral equation.

In the present work, we make the next step and consider a circular punch attached to a elastic half space coated with a layer when both the half-space and the coating are transversely-isotropic. Properties of the layer vary through the thickness arbitrarily. It is assumed that the axes of material symmetry of the half-space and coating coincide with the axis of symmetry of the punch.

For a transversely-isotropic material the number of independent elastic constants is five and the Hooke's law in cylindrical coordinate system has the following form:

$$\begin{cases} \varepsilon_r = \dfrac{1}{E_r}\sigma_r - \dfrac{\nu_{\varphi r}}{E_\varphi}\sigma_\varphi - \dfrac{\nu_{zr}}{E_z}\sigma_z, & \tau_{r\varphi} = G_{r\varphi}(z)\varepsilon_{r\varphi} \\ \varepsilon_\varphi = -\dfrac{\nu_{r\varphi}}{E_r}\sigma_r + \dfrac{1}{E_\varphi}\sigma_\varphi - \dfrac{\nu_{z\varphi}}{E_z}\sigma_z, & \tau_{rz} = G_{rz}(z)\varepsilon_{rz} \\ \varepsilon_z = -\dfrac{\nu_{rz}}{E_r}\sigma_r - \dfrac{\nu_{\varphi z}}{E_\varphi}\sigma_\varphi + \dfrac{1}{E_z}\sigma_z, & \tau_{\varphi z} = G_{\varphi z}(z)\varepsilon_{\varphi z} \end{cases} \tag{1.1}$$

with $E_\varphi=E_r$, $\nu_{\varphi z}=\nu_{z\varphi}$, $G_{rz}=G_{\varphi z}$.

2 Formulation of the problem and reduction to the integral equation.

We consider a rigid punch with flat circular base $r\le a$ attached without slipping to the boundary Γ of elastic inhomogeneous transversely isotropic half-space Ω. The shear moduli $G_{r\varphi}$, G_{rz} of the half-space vary with depth as

$$G_{r\varphi} = \begin{cases} f_{r\varphi}(z) & -H \le z \le 0 \\ f_{r\varphi}(-H) & -\infty < z < -H \end{cases}, \quad G_{\varphi z} = \begin{cases} f_{\varphi z}(z) & -H \le z \le 0 \\ f_{\varphi z}(-H) & -\infty < z < -H \end{cases} \tag{2.1}$$

Here $f_{r\varphi}$, $f_{\varphi z}$ are certain functions that determine the law of variation of shear modulus inside the coating. Cylindrical coordinate system r, φ, z with the origin at the center of the punch is chosen. Direction of z-axis coincides with the transverse axis of the half-space and axis of symmetry of the punch (Fig.1). The punch is subjected to torque M. Under the action of this torque, the punch twists about z-axis on angle ε, which leads to the torsion strain in Ω. Outside of the punch, surface Γ is traction-free:

$$z=0,\ \sigma_z = \tau_{rz} = 0, \ \begin{cases} \tau_{\varphi z} = 0, & r > a \\ u_\varphi = r\varepsilon, & r \le a \end{cases} \tag{2.2}$$

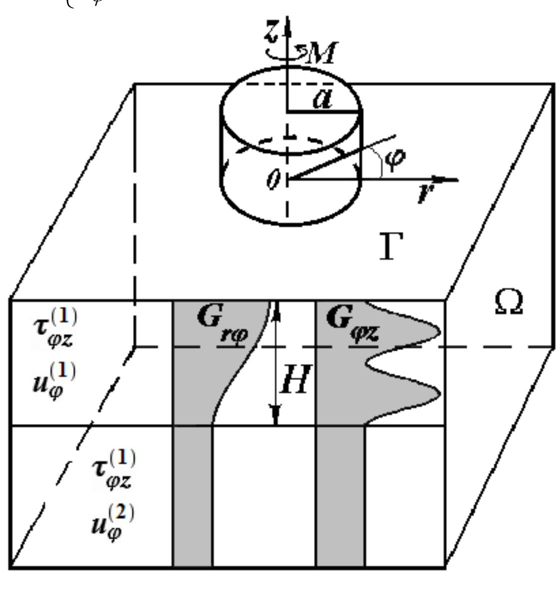


Fig. 1. Scheme of the contact problem about torsion.

The stresses vanish at $r\to\infty$ and $z\to\infty$.

The coating and the substrate are assumed to be glued without sliding, so that the continuity condition

$$z=-H,\ \tau_{\varphi z}^{(1)}=\tau_{\varphi z}^{(2)},\ u_{\varphi}^{(1)}=u_{\varphi}^{(2)} \tag{2.3}$$

is satisfied. Hereafter, superscripts (1) and (2) correspond to the coating and to the substrate, respectively. The quantity of primary interest is contact tangential stress under the punch

$$\tau_{\varphi z}\big|_{z=0}=\tau_a(r),\quad r\le a \tag{2.4}$$

Let us now reduce the formulated problem to integral equation for function $\tau_a(r)$. For this purpose, we need first to solve an auxiliary problem on equilibrium of a transversely-isotropic elastic half-space under the action of a tangential loading at $z=0$:

$$\sigma_z=\tau_{rz}=0;\quad \tau_{\varphi z}=\tau(r) \tag{2.5}$$

Taking into account that the layer is subjected to torsion loading only, we look for the solution of the auxiliary problem in form:

$$u_z=u_r=0,\ u_{\varphi}=u(r,z) \tag{2.6}$$

According to the Hooke's law, the stresses in the half-space take the form:

$$\begin{cases}\sigma_r=\sigma_{\varphi}=\sigma_z=\tau_{rz}=0;\\ \tau_{\varphi z}=G_{\varphi z}(z)\dfrac{\partial u_{\varphi}}{\partial z};\\ \tau_{r\varphi}=G_{r\varphi}(z)\left(\dfrac{\partial u_{\varphi}}{\partial r}-\dfrac{u_{\varphi}}{r}\right);\end{cases} \tag{2.7}$$

Out of the three equilibrium equations, two are satisfied identically and one can be written in terms of the displacements as follows:

$$G_{r\varphi}(z)\left(\frac{1}{r}\frac{\partial u_{\varphi}^{(1)}}{\partial r}+\frac{\partial^2 u_{\varphi}^{(1)}}{\partial r^2}-\frac{1}{r^2}u_{\varphi}^{(1)}\right)+G'_{\varphi z}(z)\frac{\partial u_{\varphi}^{(1)}}{\partial z}+G_{\varphi z}(z)\frac{\partial^2 u_{\varphi}^{(1)}}{\partial z^2}=0,\ -H\le z\le 0 \tag{2.8}$$

$$G_{r\varphi}(z)\left(\frac{1}{r}\frac{\partial u_{\varphi}^{(2)}}{\partial r}+\frac{\partial^2 u_{\varphi}^{(2)}}{\partial r^2}-\frac{1}{r^2}u_{\varphi}^{(2)}\right)+G_{\varphi z}(z)\frac{\partial^2 u_{\varphi}^{(2)}}{\partial z^2}=0,\quad -\infty<z<-H \tag{2.9}$$

Thus, we need to find the solution of the equations (2.8), (2.9) under boundary conditions (2.5) and continuity conditions (2.3). Hankel integral transform can be effectively used for this goal. Let us denote:

$$u_{\varphi}^{(j)}(r,z)=\int_0^{\infty}U_j(\gamma,z)\,\mathrm{J}_1(r\gamma)\gamma d\gamma,\quad j=1,2. \tag{2.10}$$

Substituting (2.10) into (2.8) and (2.9), and equating to zero the expressions under the integral sign in the received parities, we will get the system of the ordinary differential equations relating functions $U_j(\gamma,z)$, $\mathrm{j}=1,2.$:

$$G'_{\varphi z}(z)U'_1(\gamma,z)+G_{\varphi z}(z)U''_1(\gamma,z)-\gamma^2G_{r\varphi}(z)U_1(\gamma,z)=0,\ -H\le z\le 0 \tag{2.11}$$

$$G_{\varphi z}(-H)\cdot U''_2(\gamma,z)-\gamma^2G_{r\varphi}(-H)\cdot U_2(\gamma,z)=0,\ -\infty<z<-H \tag{2.12}$$

Introducing notations

$$\kappa(z)=\frac{G_{r\varphi}(z)}{G_{\varphi z}(z)},\quad \nu(z)=\frac{G'_{\varphi z}(z)}{G_{\varphi z}(z)}, \tag{2.13}$$

Formulated problem can be reduced to the following integral equation:

$$\int_0^1\tau(\rho)\rho\int_0^\infty L(u)\,\mathrm{J}_1(ur\lambda^{-1})\,\mathrm{J}_1(u\rho\lambda^{-1})du\,d\rho=\lambda\sqrt{G_{r\varphi}(0)\cdot G_{\varphi z}(0)}\cdot r\varepsilon,\ \ r\le 1$$

Where $\lambda=H/a;\, r'=r/a;\, \tau(\rho)=\tau_a(\rho a)$ (2.14)

L(u) is the kernel transform of integral equation. In general case of arbitrary law of variation of shear moduli across the coating, the kernel transform can be calculated numerically.

3 Problem solution in the general case.

As it shown in [8], kernel transform can be represented as:

$$L(u)=\Pi_N(u)+\Sigma_\infty(u) \tag{3.1}$$

where the following notations are used:

$$\Pi_N(u)=\prod_{i=1}^{N}\frac{u^2+A_i^2}{u^2+B_i^2}; \tag{3.2}$$

$$\Sigma_M(u)=\sum_{k=1}^{M}\frac{c_k|u|}{u^2+D_k^2}$$

and $A_i, B_i, D_k\in C;\quad C_k\in R$ are certain constants. For the kernel transform $L_N(u)$ in the form (3.2), contact tangential stress can be written as (see [8] for detail):

$$\tau(r)=\frac{4\varepsilon\sqrt{G_{r\varphi}(0)\cdot G_{\varphi z}(0)}}{\pi}\left\{L_N^{-1}(0)\frac{r}{\sqrt{1-r^2}}+\sum_{i=1}^{N}C_iZ(r,A_i\lambda^{-1})\right\} \tag{3.3}$$

where constants C_i must be determined from the system of linear algebraic equations:

$$\sum_{i=1}^{N}C_i\,p\left(\frac{A_i}{\lambda},\frac{B_k}{\lambda}\right)+\frac{1+B_k\lambda^{-1}}{L_N(0)B_k{}^2\lambda^{-2}}=0 \tag{3.4}$$

and the following notations are used:

$$Z(r,A)=\frac{\operatorname{sh}Ar}{r}+\frac{r\operatorname{sh}A}{S(1,r)}-Ar\int_r^1\frac{\operatorname{ch}At\,dt}{S(t,r)}\ ;$$

$$p(A,B)=\left(A\operatorname{ch}A+B\operatorname{sh}A\right)\cdot\left(B^2-A^2\right)^{-1};$$

$$S(t,r)=\sqrt{t^2-r^2}\left(t+\sqrt{t^2-r^2}\right) \tag{3.5}$$

Solution (3.3) is asymptotically exact at both large and small values of characteristic geometrical parameter λ of the problem. To construct approximation of function $L(u)$ by expression (3.2) and to get coefficients A_i, B_i, $(i=1,\ldots,N)$ we can use the algorithm described in [8].

4 Numerical example.

As an application of the obtained solution, we consider a stainless steel substrate (shear module G^{steel}=142*GPa*) coated with transversely isotropic Yttria Stabilized Zirconia (YSZ) plasma-sprayed layer (shear moduli $G_{r\varphi}^{YSZ}=45GPa$; $G_{\varphi z}^{YSZ}=38GPa$) ([9], [10]) and compare stresses induced by torsion of the punch attached to the top of the coating. We consider two types of variation of the elastic properties of the substrate-coating system:

- material 1 with the properties exponentially varying across the coating (functionally graded coating, figure 2)

$$G_{r\varphi}(z)=\begin{cases}142{,}66-97{,}66\cdot e^{5z/H}, & -H\le z\le 0\\ 142, & -\infty<z<-H\end{cases}\tag{4.1}$$

$$G_{\varphi z}(z)=\begin{cases}142{,}71-104{,}71\cdot e^{5z/H}, & -H\le z\le 0\\ 142, & -\infty<z<-H\end{cases}\tag{4.2}$$

- material 2 with the perfect substrate-coating interface.

$$G_{r\varphi}(z)=\begin{cases}45, & -H\le z\le 0\\ 142, & -\infty<z<-H\end{cases}\quad G_{\varphi z}(z)=\begin{cases}38, & -H\le z\le 0\\ 142, & -\infty<z<-H\end{cases}\tag{4.3}$$

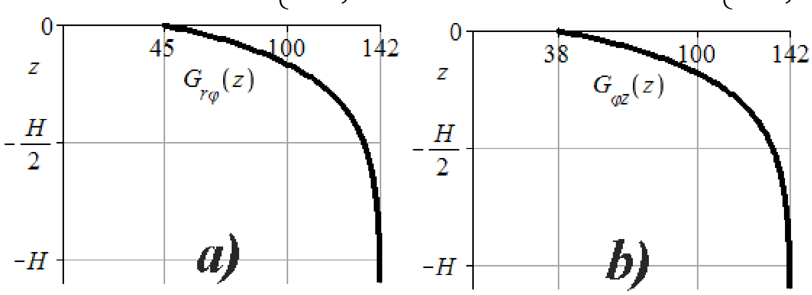


Figure 2. Variation of shear modules in depth, material 1.

Kernel transforms for materials 1 and 2 are shown in Figure 3. The horizontal coordinate axis is in logarithmic scale. Transform for material 2 converges to 1 as $u\to\infty$ faster than those for material 1.

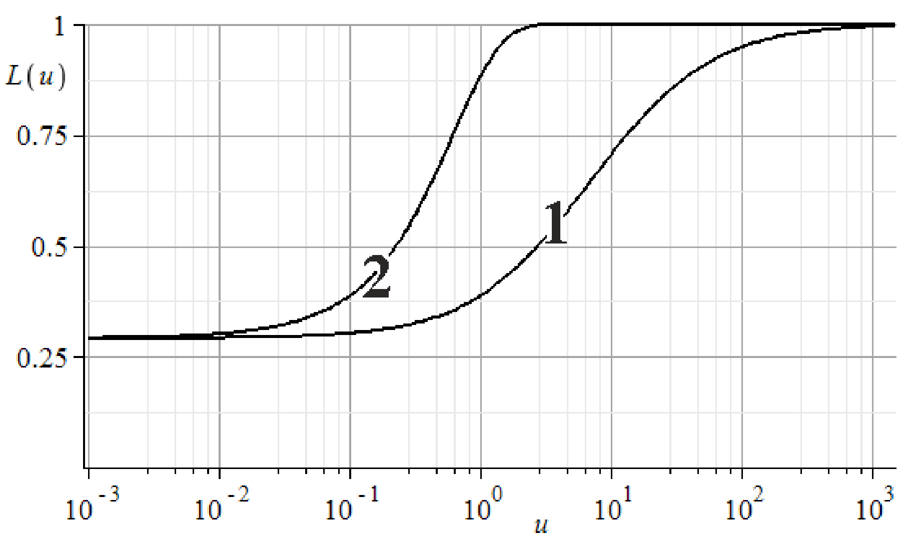


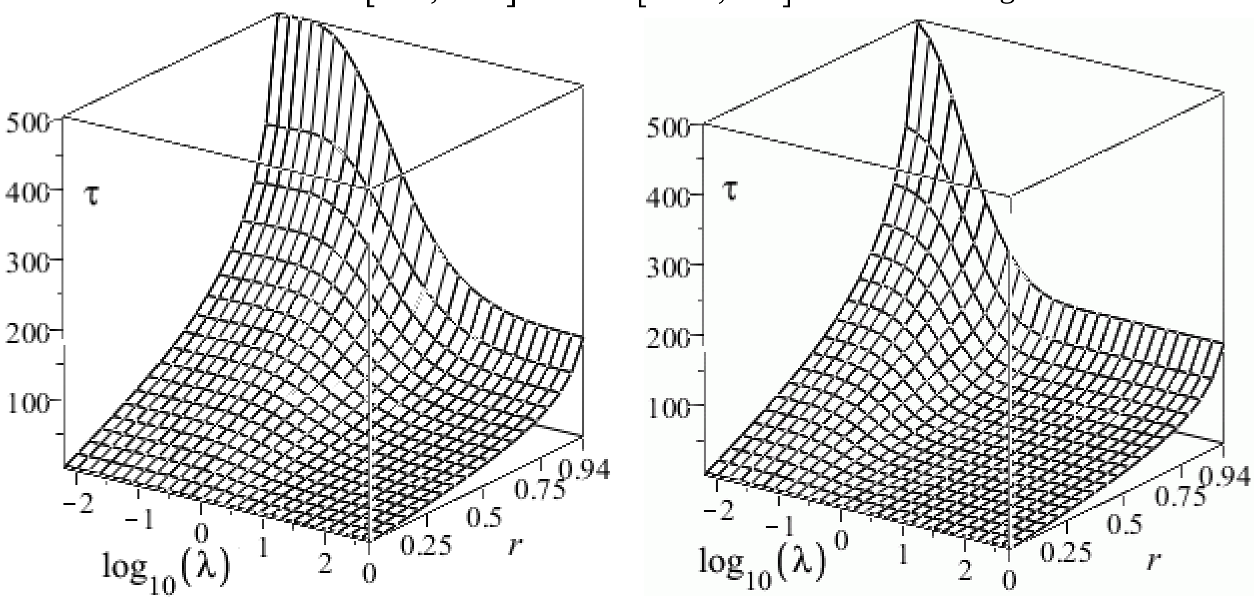
Figure 3. Transform of integral equation for materials 1, 2.

Function

$$\Delta_L(u) = \left| \frac{L_N(u)}{L(u)} - 1 \right| \cdot 100\% \tag{4.4}$$

represents the relative error of approximation of kernel transform by expression (3.2). Approximations of kernel transforms for materials 1 and 2 are constructed. The errors of approximation of kernel transform are: $\max\limits_{u>0} \Delta_L(u) = 0.3\%$ for material 1, $\max\limits_{u>0} \Delta_L(u) = 0.5\%$ for material 2.

The contact stresses for $r \in [0.01, 0.94]$ and $\lambda \in [0.006, 540]$ are shown in Figure 4.



a) material 1. *b) material 2.*

Figure 4. Contact stresses on the surface (z=0), $r \in [0, 0.94], \lambda \in [0.006, 540]$.

To compare contact stresses for inhomogeneous material system 1 or 2 with the contact stresses appearing in a homogeneous media we introduce a dimensionless parameter

$$\tau_{rel}(\lambda, r) = \frac{\tau(\lambda, r)}{\tau_{\text{hom}}(r)}, \tag{4.5}$$

where $\tau(\lambda, r)$ is contact tangential stresses under the punch for inhomogeneous substrate/coating system (materials 1 or 2) and $\tau_{\text{hom}}(r)$ is contact stresses for homogeneous isotropic half-space with shear modulus equal to shear modulus of the substrate $G=G^{steel}$ under the same loading.

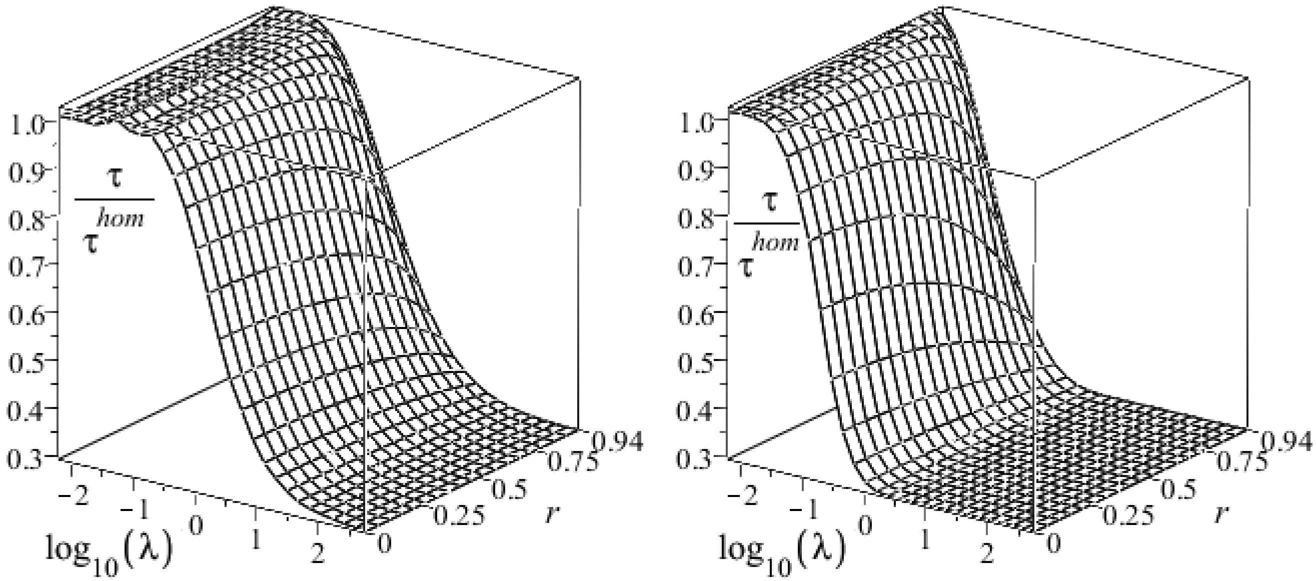
The value of $\tau_{rel}(\lambda, r)$ is illustrated in Figure *5*. It is seen that

1) for λ<0.02 for material 1 and λ<0.008 for material 2, contact stresses are almost equal to those appearing in the homogeneous half-space with the shear modulus of the substrate.

2) for λ>100 for material 1 and λ>4 for material 2 contact stresses almost coincide with the ones in the homogeneous half-space with shear modulus $G = \sqrt{G_{r\varphi}(0) G_{\varphi z}(0)}$;

3) the range of λ for which the distribution of contact stresses is significantly different from that for the homogeneous half-space is much wider for material 1 (0.02<λ <100) than for material 2 (0.008<λ<4). It means that, for material 1, substantial redistribution of contact stresses takes place for a wider range of the punch size. In these ranges reduction of stress concentration near $r=1$ can be observed as compare to the case of the homogeneous half-space. In other words, it means that for a coating with functionally graded shear moduli, redistribution of contact stresses and reduction of

stress concentration takes place in much wider range of the punch size than for a coating having strict interface with the substrate.



a) material 1. *b) material 2.*

Fig. 5. Relative contact stresses ($\tau_{rel}(\lambda, r)$) on the surface(z=0)), $r \in [0, 0.94], \lambda \in [0.006, 540]$

5 Summary

The approximate analytical solutions for the contact problem about torsion of a flat circular punch attached to a transversely isotropic elastic half-space with inhomogeneous coating is derived. The class of kernel transforms of the integral equations for a case of transversely isotropic half-space has no significant differences with the case of an isotropic half-space.

The distribution of displacements and contact stresses in depth inside and outside of the punch is obtained for the example of YSZ coating on a stainless steel substrate. The case of continuous exponential variation of elastic properties across the coating and the case of homogeneous coating are compared. It is shown that, for functionally-graded coating, there is a substantial redistribution of contact stresses on the surface and in the depth of material.

The same technique is used to solve contact problems about indentation the stamp into the functionally-graded inhomogeneous media.

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References

[1] Reissner E., Sagoci H. F., Forced torsional oscillations of an elastic half-space, J. Appl. Phys. 15 (1944) 652-654.
[2] Rostovtsev N.A., On the problem of torsion of an elastic halfspace, Prikl. Math. Mekh. 19 (1955) 55-60.
[3] Sneddon I.N., The Reissner–Sagoci problem, Proc. Glasgow Math. Assoc. 7 (1966) 136–144.
[4] Kassir M.K., The Reissner-Sagoci problem for a non-homogeneous solid, Int.J. Engng Sci. 8 (1970) 875-885.
[5] Aizikovich S.M., Torsion of a non-homogeneous half-space by a circular indenter, In a book: Analysis of shells and plates, Published by Rostov. Ing. Stroit. Inst., Rostov-on-Don, 1978, pp. 156-169.

[6] Grilitskii D. V., Torsion of a two-layer elastic medium, Prikl. Mekh. Vol. 7, Iss.1 (1961) 89-94.
[7] Tie-Jun Liu, Yue-Sheng Wang., Reissner–Sagoci problem for functionally graded materials with arbitrary spatial variation of material properties, Mechanics Research Communications. 36 (2009) 322–329.
[8] Aizikovich S.M., Asymptotic solutions of contact problems of elasticity theory for media non-homogeneous with depth, J.Appl. Math. Mech. 46 (1982) 116-124.
[9] Sevostianov, I. and Kachanov, M. Elastic and conductive properties of plasma-sprayed ceramic coatings in relation to their microstructure – an overview. Journal of Thermal Spray Technology 18 (2009), 822-834.
[10] Sevostianov, I., Kachanov, M., Ruud, J., Lorraine, P., Dubois, M. Quantitative characterization of microstructures of plasma-sprayed coatings and their conductive and elastic properties. Materials Science and Engineering-A, 386 (2004), 164-174.