DEFORMATION AND FRACTURE OF FRAGILE ANISOTROPIC MATERIALS AND DESIGNS UNDER DYNAMIC LOADING

Pavel Radchenko[1,a], Andrey Radchenko[2,b]

[1]Institute of Strength Physics and Materials Science of SB RAS, 2/4, pr. Akademicheskii, Tomsk, 634021, Russia

[2]Tomsk State University of Architecture and Building, sq. Solyanaya, Tomsk. 634003, Russia

[a]radchenko@live.ru, [b]andrey-radchenko@live.ru

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Abstract. The problem of the normal interaction of an isotropic compact cylindrical steel projectile with an orthotropic plate in the limit of penetration in the range of velocities from 50m/s to 400m/s. The material of a barrier is orthotropic organoplastic with initial orientation of the mechanical properties and material, whose properties are rotated on 90° relative to the axis OY. Investigated the fracture of the barrier, a comparative analysis of the effectiveness of their protective properties, depending on the orientation of the elastic and strength properties of anisotropic material. The problem is solved numerically by the finite element method in three-dimensional formulation. The behavior of the projectile is described by elastic-plastic material, anisotropic material behavior of the barriers described in terms of elastic-brittle model with different limits of material strength on compression and tension.

Introduction. At present, the wide application of the materials with preset directivity of properties in various fields of engineering defines the increased interest to the investigations of anisotropic materials behaviour under various conditions. But in Russia as well as abroad such investigations are conducted mainly for static conditions. The behaviour of anisotropic materials under dynamic loads is practically not investigated. This is especially the case with experimental investigations as well as with mathematical and numerical modelling. The impact interaction of the solids in a wide range of kinematic and geometric conditions is the complex problem of mechanics. The difficulties, connected with the theoretic study of the fracture and deformation of materials on impact by analytical methods force to introduce member of simplifying hypothesis which distort the real picture in majority of cases. In this connection it should be accepted that the leading role in the investigation of phenomena, connected with high-speed interaction of solids belongs to the experimental and numeric investigations at present. The investigations of the material damage under impact show that the fracture mechanisms change with the interaction conditions. The experiments strongly testifiers that in a number of case the resulting fracture is determined by the combination of several mechanisms. But in the experiments we fail to trace sequence, operation time and the contribution of various fracture mechanisms. Besides, the distractions, obtained at the initial stages of the process can't always be identified in the analysis of the resulting fracture of the materials. For anisotropic materials the strength itself is multivalued and uncertain notion due to the polymorphism of behaviour of these materials under the load. The limiting state of anisotropic bodies may be of different physical nature in dependence on load orientation, stressed state type and other factors. The dependence of the physical nature of limiting states is revealed in the sturdy of the experimental data. The investigations of hydrostatic pressure effect upon the strength of isotropic materials show that comprehensively the compression exerts a weak action on the resistance of isotropic materials under static loads. Therefore, the classic theories of strength, plasticity and creep are based on assumption about the lack of the effect of fall stress tensor upon strength isotropic materials. In the experiments with anisotropic materials it was state that flow phenomenon may arise only under the action of hydrostatic pressure. Upon the materials, strength is due to the anisotropy. The shape of anisotropic bodies changes under the action of hydrostatic pressure. If these changes reach such values that, they don't disappear under relief, the limiting state should come. Therefore, the postulate of classic strength that hydrostatic pressure can't transfer the material to the dangerous state is not valid for anisotropic materials. The phenomenological approach to the investigation of the dynamics of deformation and fracture of anisotropic and isotropic materials is used in the project. The phenomenological approach to the materials strength requires that the conditions of the transition into the limiting state of various physical natures should be determined by one equation (criterion). The necessity of such an approach results from fracture polymorphism, being deduced experimentally. For anisotropic bodies the phenomenological approach has many advantages, since there appears the possibility to use general condition of strength for the material different in composition and technology but similar in symmetry of properties, and also for the materials with substantial anisotropy, for which one and the same stressed state can result in limiting conditions, different in physical nature.

Basic equations of mathematical model. The system of the equations describing non-stationary adiabatic movements of the compressed media in the Cartesian coordinate system XYZ, includes following equations:
– continuity equation

$$\dot{\rho} + \operatorname{div} \rho \vec{\upsilon} = 0; \tag{1}$$

– motion equation

$$\begin{aligned} \rho \dot{u} &= \sigma_{xx,x} + \sigma_{xy,y} + \sigma_{xz,z} \\ \rho \dot{\upsilon} &= \sigma_{yx,x} + \sigma_{yy,y} + \sigma_{yz,z}; \\ \rho \dot{w} &= \sigma_{zx,x} + \sigma_{zy,y} + \sigma_{zz,z} \end{aligned} \tag{2}$$

– energy equation

$$\dot{E} = \frac{1}{\rho} \sigma_{ij} e_{ij}; \quad i, j = x, y, z. \tag{3}$$

Here ρ – density of media; $\vec{\upsilon}$ – velocity vector, u, υ, w – components of velocity vector on axes x, y, z accordingly; σ_{ij} – components of a symmetric stress tensor; E – specific internal energy; e_{ij} – components of a symmetric strain rate tensor; the point over a symbol means a time derivative; a comma after a symbol – a derivative on corresponding coordinate.
The behavior of the aluminum isotropic cylinder at high-velocity impact is described by elastic-plastic media, in which communication between components of strain velocity tensor and components of stress deviator are defined by Prandtl-Reuss equation:

$$2G\left(e_{ij} - \frac{1}{3} e_{kk} \delta_{ij}\right) = \frac{DS^{ij}}{Dt} + \lambda S^{ij}, \ (\lambda \geq 0); \quad \frac{DS^{ij}}{Dt} = \frac{dS^{ij}}{dt} - S^{ik} \omega_{jk} - S^{jk} \omega_{ik}, \tag{4}$$

where $\omega_{ij} = \frac{1}{2}\left(\nabla_i \upsilon_j - \nabla_j \upsilon_i\right)$, G – shear modulus. Parameter $\lambda = 0$ at elastic deformation, and at elastic ($\lambda > 0$) is defined by means of a Mises condition:

$$S^{ij} S_{ij} = \frac{2}{3}\sigma_d^2, \tag{5}$$

where σ_d – dynamic yield point. The ball part of stress tensor (pressure) is calculated on the Mi-Gruneisen equation as function of specific internal energy E and density ρ:

$$P = \sum_{n=1}^{3} K_n \left(\frac{V_0}{V} - 1\right)^n \left[\frac{1 - K_0\left(\frac{V_0}{V} - 1\right)}{2}\right] + K_0 \rho E, \tag{6}$$

where $K_0,\ K_1,\ K_2,\ K_3$ – constants of material.

The behavior of an anisotropic material of targets is described within the limits of elastic-fragile model. Before fracture components of a stress tensor in a target material were defined from equations of the generalized Hooke's law which have been written down in terms of strain rate:

$$\dot{\sigma}_{\mathrm{ij}} = C_{ijkl} e_{kl}, \tag{7}$$

where C_{ijkl} – elastic constants.

Thus components of a tensor of elastic constants possess, owing to symmetry of stress tensors and strain tensors and presence of the elastic potential, following properties of symmetry:

$$C_{ijkl} = C_{jikl} = C_{ijlk} = C_{ijlk}; \quad C_{ijkl} = C_{klij}. \tag{8}$$

At transition to another, also orthogonal, coordinate system, elastic constants will be transformed by equations:

$$C'_{abcd} = C_{ijkl} q_{ia} q_{jb} q_{kc} q_{ld}, \tag{9}$$

where q_{ij} – cosine of the angle between corresponding axes i and j. In three-dimensional space transformation of the component of a tensor of the fourth rank demands summation of the compositions, containing as multipliers 4 cosines of angles of rotation of axes.

Fracture of an anisotropic material is described within the limits of model with use of Tsai-Wu fracture criterion with various ultimate strengths of pressure and tension [1]. This criterion, which has been written down by scalar functions from components of a stress tensor, has the following appearance:

$$f(\sigma_{ij}) = F_{ij}\sigma_{ij} + F_{ijkl}\sigma_{ij}\sigma_{kl} + \ldots \geq 1; \quad i, j, k, l = 1, 2, 3. \tag{10}$$

Here F_{ij} and F_{ijkl} are components of tensor of the second and the fourth rank respectively, and obey transformation laws:

$$F'_{ab} = F_{\mathrm{ij}} q_{ia} q_{jb}; \quad F'_{abcd} = F_{\mathrm{ijkl}} q_{ia} q_{jb} q_{kc} q_{ld} . \tag{11}$$

Components of tensors of strength for criterion are defined by following equations:

$$F_{ii} = \frac{1}{X_{ii}} - \frac{1}{X'_{ii}}; \quad F_{iiii} = \frac{1}{X_{ii} X'_{ii}}; \quad F_{ij} = \frac{1}{2}\left(\frac{1}{X_{ij}} - \frac{1}{X'_{ij}}\right); \quad F_{ijij} = \frac{1}{4 X_{ij} X'_{ij}}; \quad i \neq j , \tag{12}$$

where X_{ii}, X'_{ii} – limits of strength on pressure and tension along the direction i; X_{ij}, X'_{ij} – shear strength along the two opposite directions with $i \neq j$. Coefficients F_{1122}, F_{2233}, F_{3311} are defined at carrying out the experiments on biaxial tension in planes 1–2, 2–3, 1–3 accordingly. The remained coefficients are defined similarly at combined stressing in corresponding planes (Radchenko et al., 2012).

It is supposed that fracture of anisotropic materials in the conditions of intensive dynamic loads occurs as follows:

– if strength criterion is violated in the conditions of pressure $(e_{kk} \leq 0)$, the material loses anisotropy of properties, and its behaviour is described by hydrodynamic model, thus the material keeps its strength only on pressure; the stress tensor becomes in this case spherical ($\sigma_{ij} = -P$);

– if the criterion is violated in the conditions of tension $(e_{kk} > 0)$, the material is considered completely fractured, and components of a stress tensor are appropriate to be equal to zero ($\sigma_{ij} = 0$).

Pressure in orthotropic materials of targets is calculated by means of the equation of a condition [2]:

$$P = \left[\exp\left(4\beta \frac{V_0 - V}{V_0}\right) - 1\right] \frac{\rho_0 \alpha^2}{4\beta} . \tag{13}$$

Here ρ_0 is initial density of a material; V_0, V – relative initial and current volumes. Coefficients of the given equation are calculated from a shock adiabat: $D = \alpha + \beta u$, where α =1400 m/s, β =2.25, and u – mass velocity.

Initial and boundary conditions. It is considered (Fig. 1) a three-dimensional task of high-speed interaction of compact (diameter of the projectile is equal to its height) cylindrical projectile (area D_1) with one or several targets (areas D_2, D_3, D_4). In this paper we consider the materials with the following mechanical characteristics [3]: Steel St3 with ρ_0 =7850 kg/m^3, E =204 GPa, μ =0.3, G =79 GPa, $\sigma_{0.2}$ =1.01 GPa, K_0 =1.91, K_1 =153 GPa, K_2 =176 GPa, K_3 =53.1 GPa; aluminum with ρ_0 =2700 kg/m^3, E =70 GPa, μ =0.3, G =27 GPa, $\sigma_{0.2}$ =310 MPa, K_0 =2.13, K_1 =74.4 GPa, K_2 =53.2 GPa, K_3 =30.5 GPa; organoplastic with ρ_0 =1350 kg/m^3, E_1 =48.6 GPa, E_2 =21.3 GPa, E_3 =7.1 GPa, μ_{12} =0.28, μ_{23} =0.26, μ_{31} =0.037, G_{12} =930 GPa, G_{23} =900 GPa, G_{31} =850 GPa, X_{11} =2.67 GPa, X_{22} =1.18 GPa, X_{33} =0.395 GPa, X'_{11} =0.37 GPa, X'_{22} =0.5 GPa, X'_{33} =1.94 GPa, X_{12} =0.975 GPa, X_{23} =0.8 GPa, X_{31} =0.607 GPa, $X_{11}^{(12)}$ =2.3 GPa, $X_{11}^{(31)}$ =2 GPa, $X_{22}^{(12)}$ =1 GPa,

$X_{22}^{(23)}$ =0.9 GPa, $X_{33}^{(23)}$ =0.35 GPa, $X_{33}^{(31)}$ =0.31 GPa, c_x =6000 m/s, c_y =3970 m/s, c_z =2300 m/s. The meeting corner (between a normal to a target and a longitudinal axis of the projectile) made a corner $\alpha = 0^\circ$ (normal impact).

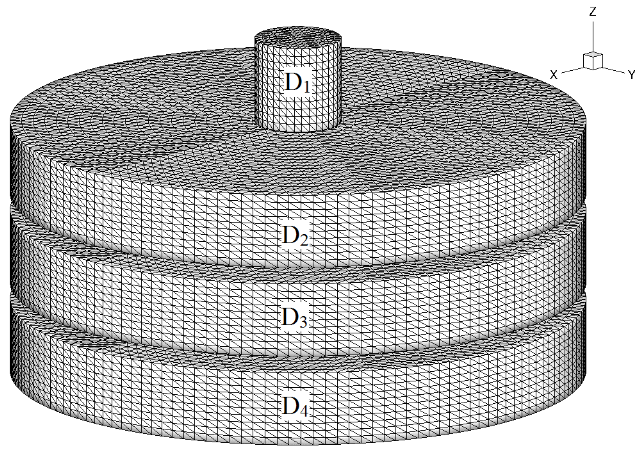


Fig. 1. Three-dimensional formulation of the task

Initial conditions ($t = 0$):

$$\begin{aligned}
&\sigma_{ij} = E = u = v = 0, \quad w = \upsilon_0, \quad i, j = x, y, z, \quad x, y, z \in D_1 \\
&\sigma_{ij} = E = u = v = w = 0, \quad i, j = x, y, z, \quad x, y, z \in D_2, D_3, D_4 \\
&\rho = \rho_i, \quad x, y, z \in D_i, \quad i = 1, 2, 3, 4
\end{aligned} \tag{14}$$

Boundary conditions [4,5]:
On free surfaces conditions of free border are realized:

$$\overline{T}_{nn} = \overline{T}_{n\tau 1} = \overline{T}_{n\tau 2} = 0. \tag{15}$$

On contact surface sliding condition without a friction is realized:

$$\overline{T}_{nn}^{+} = \overline{T}_{nn}^{-}, \quad \overline{T}_{n\tau}^{+} = \overline{T}_{n\tau}^{-} = \overline{T}_{ns}^{+} = \overline{T}_{ns}^{-} = 0, \quad \overline{\upsilon}_n^{+} = \overline{\upsilon}_n^{-}. \tag{16}$$

Here $\overline{n}$ – a unit vector of a normal to a surface in a considered point, $\overline{\tau}$ и $\overline{s}$ – unit vectors, tangents to a surface in this point, $\overline{T}_n$ – a force vector on a platform with a normal $\overline{n}$, $\overline{\upsilon}$ – a velocity vector. The subscripts at vectors $\overline{T}_n$ and $\overline{\upsilon}$ also mean projections on corresponding basis vectors; the badge plus "+" characterizes value of parameters in a material on the top border of a contact surface, a badge a minus "–" – on bottom.

Discussion of the results. We studied the penetration of a barrier with the initial orientation of the properties, as well as penetration of a barrier with properties reoriented by 90° about the axis OY [6]. In the direction of the axis Z initial material has the highest strength on compression and the lowest strength on tension. Refocused the material, on the contrary, along the Z axis has the lowest compressive strength and the highest tensile strength. In addition to the various limits of the tensile

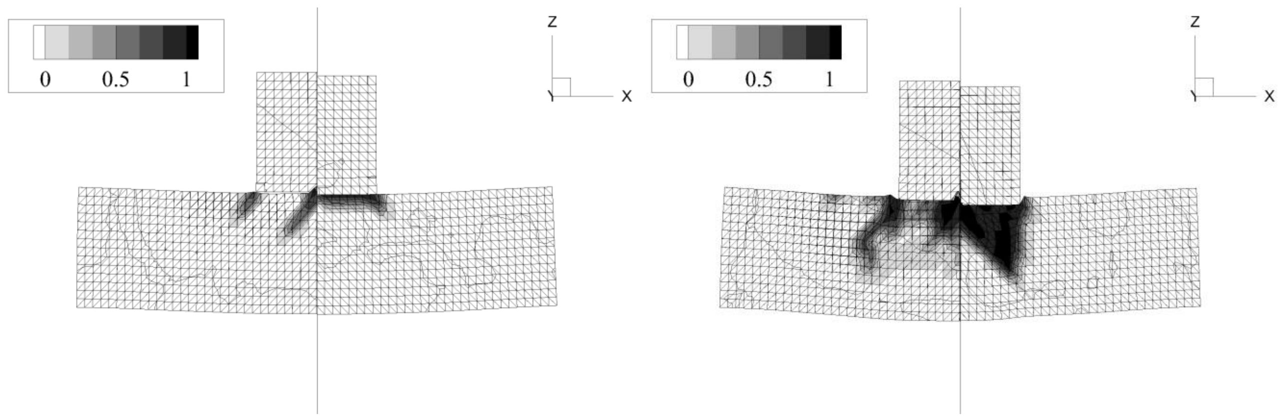
and compression on the dynamics of fracture will affect significantly the velocity of propagation of waves of compression and unloading, which in an anisotropic material depend on the direction.
A material of the projectile is isotropic steel, a material of targets – orthotropic organoplastic. Orientation of properties of orthotropic material changes by turn of axes of symmetry of an initial material round an axis on an angle β=90°.

On Fig. 2 and Fig. 3 configurations of the projectile and targets with distribution of isolines of relative volume of fractures for various velocities of interaction at the moment of time t=40 μsec are presented. To the left of a symmetry axis configurations for initial orientation of a material of a target, to the right – for the reoriented material are given.
For a case of initial orientation of properties of organoplastic at velocity 50 m/s (Fig. 2a, to the left of a symmetry axis) on an obverse surface of a target on perimeter of the projectile and on a contact surface in the target center the conic zones of fracture focused at an angle 45° to a direction of impact are formed. These zones arise in an initial stage of interaction at the expense of action of tensile stress in the unloading waves extending from an obverse surface of a target and a lateral surface of the projectile. The further development of these zones of fracture is caused by action of tensile stress as a result of introduction of the projectile. At initial velocity 50 m/s there is no perforation of a target. To t=30 μsec velocity of the projectile reduces to zero and the kickback of the projectile from a target is observed. Values of vertical component of the velocity of the center of projectile weights υ_z and a part of the fractured material of a target are presented in Table 1 (at tension D_t and pressure D_p at the moment of time t=50 μsec).

Table 1. Velocity of the center of projectile weights and part of the break material in targets

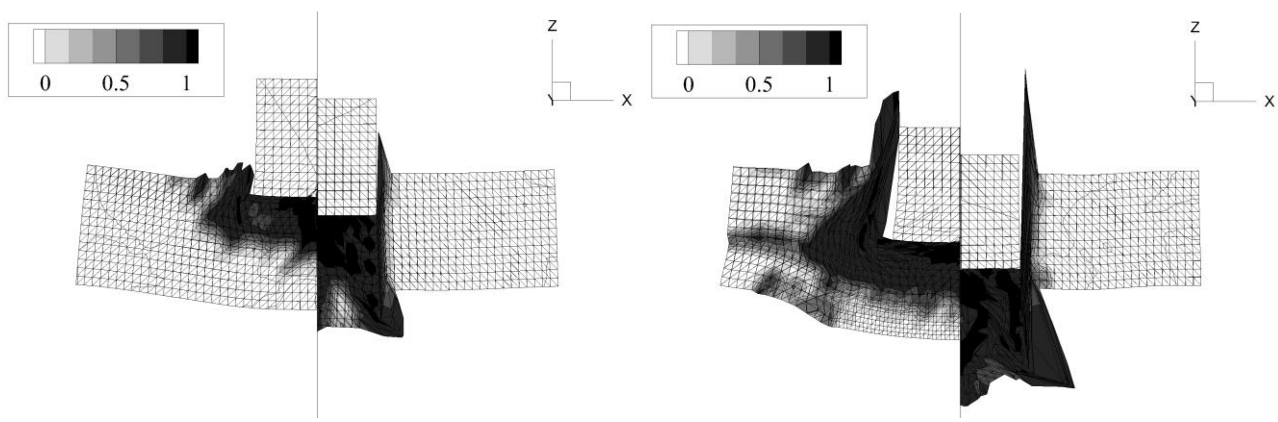
υ_0, m/s	50		100		200		400	
β	0°	90°	0°	90°	0°	90°	0°	90°
υ_z, m/s	–5.17	4.08	9.05	12.97	49.97	127.6	191.55	303.69
D_t	0.012	0.005	0.056	0.011	0.162	0.128	0.502	0.282
D_p	0.006	0.002	0.043	0.029	0.104	0.021	0.112	0.019

In case of the reoriented material (Fig. 2a, to the right of a symmetry axis) a picture of development of fracture is qualitative other. In this case, strength of a material on pressure in a direction of axis Z (an impact direction) is minimal. It leads to that the material break in the wave of pressure formed at the moment of impact and extending on a thickness of a target. Penetration of the projectile thus occurs in already weakened material. Though perforation in this case also isn't present, the projectile gets deeply, and its full braking is observed in 50 μsec. With increase in velocity of impact the volume of areas of fracture grows. At velocity 100 m/s (Fig. 2b) fracture areas extend to a greater depth on a thickness of a target. And for an initial material of a target the marked orientation (45°) was kept only by a crack extending from an obverse surface on perimeter of the projectile. The crack located near to an axis of symmetry isn't identified any more. It is caused by that with increase in velocity of impact the amplitude of the pressure wave grows – its size is already sufficient for material fracture in the top half of target.



a b

Fig. 2. Relative volume of fractures in target. υ_0 =50 m/s (a) and υ_0 =100 m/s (b), t=40 μsec.



a b

Fig. 3. Relative volume of fractures in target. υ_0 =200 m/s (a) and υ_0 =400 m/s (b), t=40 μsec.

In case of the reoriented material the unloading wave extending from a back surface of a barrier, lowers level of compressive stresses that leads to smaller distribution of fracture area on a thickness near to a symmetry axis (Fig. 2b). For velocity 100 m/s also it is not observed perforation of targets, thus in case of an initial material velocity of the projectile reduced to zero at 45 μsec, in case of the reoriented material – at 60 μsec.

For velocities of impact 200 m/s and above (Fig. 3) it is already observed perforation of targets from both types of materials. But thus the plate from an initial material greater maintains resistance to penetration of the projectile in comparison with a plate from the reoriented material. For example, at initial velocities 200 m/s (Fig. 3a) and 400 m/s (Fig. 3b) post-perforation velocity of the projectile after perforation of plates from an initial material makes 37 m/s and 187 m/s accordingly, and post-perforation velocity after perforation plates of the reoriented material 125 m/s and 300 m/s. Greater resistance to penetration of the projectile in plates from an initial material is caused by a various picture of fracture which is defined by orientation of elastic and strength properties in relation to external loading. For velocities of impact above 200 m/s there is fracture of the reoriented material in the unloading wave extending from a back surface of a target (Fig. 3), that increases volume of the break material in front of the projectile, essentially reducing resistance to penetration. Such dynamics of fracture become clear by various velocities of wave distribution in the initial and reoriented materials.

In an initial material velocity of wave distribution the greatest in a direction of an axis X – perpendicular to an impact direction, therefore unloading waves from an obverse surface of a target and a lateral surface of the projectile lower stresses in a pressure wave to its exit on a back surface that doesn't lead to material fracture in a pressure wave in the bottom half of plate and a unloading wave from a back surface of the barrier having small amplitude at the expense of easing of a pressure wave. In the reoriented material velocity of distribution of waves is maximal in a direction of axis Z, therefore the pressure wave loses energy only on fracture of a material and being reflected from a back surface by an intensive unloading wave breaking a material.

Conclusion. The offered model allows to describe adequately main laws of the fracture processes of anisotropic materials under dynamic loads. The carried out researches have shown, that anisotropy of properties is the essential factor which is necessary for taking into account for the adequate description and the prediction of development of shock-wave processes and fracture in the materials under dynamic loadings. The influence of anisotropic properties orientation increases with decrease in the velocity interaction. The qualitative and quantitative discrepancies in the fracture of isotropic and anisotropic materials under the dynamic loads are defined not only by strength parameters but either by the interaction of the compression and tension waves. Different velocities of waves propagation along the directions in anisotropic barriers provide the discharge of the impact wave and the narrowing in the fracture region. It is established that at low-velocity impact formation and a direction of development of fracture zone in a target is defined by orientation of elastic and strength properties of an anisotropic material in relation to an impact direction. Depending on orientation of properties development of the conic cracks caused by combined action of tensile stresses in waves of unloading and at the expense of penetration of the projectile, or fracture of material in a pressure and unloading wave is probable.

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