Damage Evolution in Pure Bending of Viscoplastic Sheets

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Abstract. An analysis of plane-strain bending at large strains for a rigid viscoplastic incompressible material model including a damage evolution law is performed. The Mises-type yield criterion is adopted. The yield stress depends on the equivalent strain rate (the quadratic invariant of the strain rate tensor). The fracture criterion is based on a critical value of the damage parameter. For reasons of space, the present paper is restricted to analytical treatment of the boundary value problem which enables the original system of equations to be reduced to two simple hyperbolic equations whose numerical solution can be found with no difficulty and with a high accuracy.

Introduction

Pure plane-strain bending at large strains is one of the classical problems in plasticity theory. A number of analytical and semi-analytical solutions have been proposed for various rigid- and elastic- plastic models in the literature [1-7]. A unified method for isotropic incompressible materials has been proposed in [8]. The method has been extended to a class of anisotropic materials in [9] and has been successfully used for springback calculation in the case of elastic-plastic non-linear hardening materials in [10]. In particular, it has been shown in [9, 10] that an effect of elasticity at large strains is negligible, unless the stage of unloading is of interest. Therefore, a rigid viscoplastic model is adopted in the present paper. The paper deals with an extension of the approach to analysis of plane-strain pure bending proposed in [8] to include a damage evolution equation in the case of rigid viscoplastic incompressible materials. The effect of viscosity is introduced assuming that the yield criterion depends on the equivalent strain rate (the quadratic invariant of the strain rate tensor). The damage evolution equation used is similar to those proposed in [11-14] among others. A similar approach for strain-hardening materials has been developed in [15]. An advantage of the approach chosen is that the original boundary value problem is reduced to rather a simple system of two partial hyperbolic differential equations written in characteristic coordinates. The key point of this successful transformation is a simple mapping between Lagrangian and Eulerian coordinate systems found in [8]. Numerical methods for such systems are well documented and the numerical solution can be found with a high accuracy.

Kinematics

The approach proposed in [8] is based on the mapping between Eulerian Cartesian coordinates (x, y) and Lagrangian coordinates (ζ, η) in the form

$$\frac{x}{H} = \sqrt{\frac{\zeta}{a} + \frac{s}{a^2}}\cos(2a\eta) - \frac{\sqrt{s}}{a} \quad \text{and} \quad \frac{y}{H} = \sqrt{\frac{\zeta}{a} + \frac{s}{a^2}}\sin(2a\eta) \tag{1}$$

where H is the initial thickness of the sheet, s is an arbitrary function of a, a is a function of the time, t, and $a = 0$ at $t = 0$. At the initial instant, $a = 0$,

$$s = 1/4. \tag{2}$$

Substituting Eq. 2 into Eq. 1 and applying l'Hospital's rule gives $x = \zeta H$ and $y = \eta H$ at the initial instant when the shape of the specimen is the rectangle defined by the equations $x = -H$, $x = 0$ and $y = \pm L$. The initial shape and the Cartesian coordinate system are shown in Fig. 1. It is possible to assume, with no loss of generality, that the origin of this coordinate system is located at the intersection of the axis of symmetry and surface *AB* throughout the process of deformation. An intermediate shape is also shown in Fig. 1. It is obvious that $\zeta = 0$ for *AB* and $\zeta = -1$ for *CD* throughout the process of deformation. According to Eq. 1, any intermediate shape is determined by two circular arcs, *AB* and *CD*, and two straight lines, *AD* and *CB*. These circular arcs coincide with coordinate curves of the plane polar coordinate system $r\theta$ defined by the following transformation equations

$$\frac{r}{H} = \sqrt{\frac{\zeta}{a} + \frac{s}{a^2}} \quad \text{and} \quad \theta = 2a\eta. \tag{3}$$

Geometric parameters of the shape at any instant are given by (Fig. 1)

$$\frac{R_{AB}}{H} = \frac{\sqrt{s}}{a}, \quad \frac{R_{CD}}{H} = \sqrt{\frac{s}{a^2} - \frac{1}{a}}, \quad \frac{h}{H} = \frac{\sqrt{s} - \sqrt{s-a}}{a} \tag{4}$$

where R_{AB} is the radius of surface *AB*, R_{CD} is the radius of surface *CD*, and h is the current thickness of the sheet.

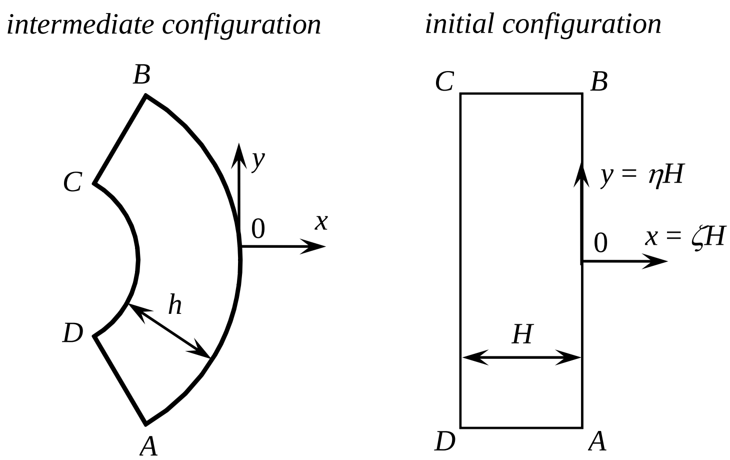

Fig. 1. Coordinate systems, initial shape and intermediate shape in pure bending.

It is possible to verify by inspection that the Lagrangian coordinates coincide with trajectories of the principal strain rates and that the mapping given by Eq.1 satisfies the equation of incompressibility at any instant. It will be shown in the next section that the assumption that the Lagrangian coordinates coincide with the trajectories of the stress tensor allows one to solve the stress equations. In the case under consideration these two conditions (coincidence of the trajectories for the principal stresses and principal strain rates and the equation of incompressibility) are equivalent to the associated flow rule of the classical rate formulation of plasticity theory.

The strain rate components can be found from Eq. 1 and, then, the position of the neutral line is determined by

$$\zeta = \zeta_n = -ds/da \ , \tag{5}$$

the equivalent strain rate by

$$\xi_{eq} = \frac{|\zeta + \mathrm{d}s/\mathrm{d}a|}{\sqrt{3}(\zeta a + s)} \frac{\mathrm{d}a}{\mathrm{d}t} \tag{6}$$

and the equivalent strain by

$$\varepsilon_{eq} = \frac{1}{\sqrt{3}} \ln\left[4(\zeta a + s)\right], \varepsilon_{eq} = \frac{1}{\sqrt{3}} \ln\left\{\frac{\zeta a + s}{4\left[\zeta a_c(\zeta) + s_c(\zeta)\right]^2}\right\}, \varepsilon_{eq} = -\frac{1}{\sqrt{3}} \ln\left[4(\zeta a + s)\right] \tag{7}$$

in regions 1, 2 and 3, respectively. In region 1, $0 \geq \zeta \geq -1/2$, the principal strain rate $\xi_{\zeta\zeta} < 0$ (and $\xi_{\eta\eta} > 0$) during the entire process. In region 3, $-1 \leq \zeta \leq \zeta_n^f$, the principal strain rate $\xi_{\zeta\zeta} > 0$ (and $\xi_{\eta\eta} < 0$) during the entire process. A property of all curves $\zeta = const$ in region 2, $\zeta_n^f \leq \zeta \leq -1/2$, is that each of these curves coincides with the neutral line at one time instant. Consider any ζ-curve of this class and denote a_c the value of a at which the curve coincides with the neutral line. Then, $\xi_{\zeta\zeta} < 0$ ($\xi_{\eta\eta} > 0$) at $a < a_c$ and $\xi_{\zeta\zeta} > 0$ ($\xi_{\eta\eta} < 0$) at $a > a_c$ for this curve. Obviously, the time instant at which the sign is changed depends on the curve such that $a_c = a_c(\zeta)$. The corresponding value of s will be denoted by $s_c(\zeta)$ where $s_c(\zeta) = s\left[a_c(\zeta)\right]$. These values of $a_c(\zeta)$ and $s_c(\zeta)$ are involved in Eq. 7. Also, ζ_n^f is the ζ - coordinate of the neutral surface at the end of the process. If $s(a)$ were known, Eq. 5 would determine $a_c(\zeta)$ and, therefore, $s_c(\zeta)$. Thus, $s(a)$ is the only unknown function in the analysis of kinematics and this function should be found from the analysis of stress and damage.

Stress Analysis and Damage Evolution

The only non-trivial equilibrium equation in the plane polar coordinate system (r, θ) in terms of the radial and circumferential stresses has the form

$$\frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \, . \tag{8}$$

It is obvious that $\sigma_r \equiv \sigma_{\zeta\zeta}$ and $\sigma_\theta \equiv \sigma_{\eta\eta}$. The plane-strain yield condition in the case under consideration is

$$\sigma_r - \sigma_\theta = \pm \frac{2}{\sqrt{3}} \sigma_0 \Phi\left(\xi_{eq}\right)(1 - D) \tag{9}$$

where the upper sign corresponds to the region $-1 \leq \zeta \leq \zeta_n$ and the lower sign to the region $\zeta_n \leq \zeta \leq 0$. Also, the function $\Phi\left(\xi_{eq}\right)$ satisfies the condition $\Phi(0) = 1$, σ_0 is the yield stress in tension at $\xi_{eq} = 0$, and D is the damage parameter. Using Eq. 3 it is possible to replace r and differentiation with respect to *r* with ζ and differentiation with respect to ζ in Eq. 8. Then, using Eq. 9,

$$\frac{\partial \sigma_r}{\partial \zeta} = \mp \frac{a\sigma_0 \Phi\left(\xi_{eq}\right)(1-D)}{\sqrt{3}(\zeta a + s)}. \tag{10}$$

The function $\Phi\left(\xi_{eq}\right)$ should be prescribed and ξ_{eq} can be excluded by means of Eq. 6. The boundary conditions on the radial stress are

$$\sigma_r = 0 \tag{11}$$

for $\zeta = -1$ and $\zeta = 0$. Since there are the two boundary conditions for the differential equation of first order, the function $s(a)$ and, consequently, the neutral line position (see Eq. 5) should be found from the solution to Eq. 10 simultaneously with constant of integration. Also, the radial stress must be continuous across the boundary of the aforementioned regions 1, 2, and 3. Equation (10) should be supplemented with a damage evolution law. A wide class of phenomenological damage evolution laws can be written in the form

$$\dot{D} = \Lambda\left(\frac{\sigma}{\sigma_{eq}}, \varepsilon_{eq}, D\right)\xi_{eq} \tag{12}$$

where the overdot denotes the convected derivative. In the case of plane strain deformation, the flow rule associated with the yield condition Eq. 9 gives $\sigma_z = \sigma = (\sigma_r + \sigma_\theta)/2$. Since the shear stresses in the cylindrical coordinate system vanish, the equivalent stress involved in Eq. 12 is given by $\sigma_{eq} = \left(\sqrt{3}/2\right)\left|\sigma_r - \sigma_\theta\right|$. In the Lagrangian coordinates, Eq. 12 can be rewritten, with the use of Eq. 6, as

$$\frac{\partial D}{\partial a} = \frac{1}{\sqrt{3}}\Lambda\left(\frac{\sigma}{\sigma_{eq}}, \varepsilon_{eq}, D\right)\frac{\left|\zeta + \mathrm{d}s/\mathrm{d}a\right|}{(\zeta a + s)}. \tag{13}$$

The initial distribution of the damage parameter should be prescribed. A widely used assumption is

$$D = D_0 \tag{14}$$

for $a = 0$. Here D_0 is constant. Using Eqs. 6, 7, and 9 the right hand sides of Eqs. 10 and 13 can be represented as functions of *a*, ζ , σ_r and *D*. Therefore, using the boundary conditions (11) the solution to these equations can be in general found numerically. Once the solution for the damage parameters and stress components has been found, the bending moment per unit length is determined by integration [8]

$$M = \frac{H^2}{2a}\int_{-1}^{0} \sigma_{\theta\theta} d\zeta. \tag{15}$$

At the initial instant, the polar coordinate system (r,θ) transforms to the Cartesian coordinate system (x,y). In order to facilitate numerical solution of Eqs. 10 and 13 for the initial stage of the process, the second derivative d^2s/da^2 at the initial instant can be found analytically.

Solution for the Initial Stage of the Process.
Since the distribution of the damage parameter is uniform at the initial instant, $\zeta_n = -1/2$ at $a=0$. Then, it follows from Eq. 5 that

$$ds/da = 1/2 \tag{16}$$

at $a=0$. Moreover, at the initial instant

$$\begin{aligned} &\sigma_r = 0 \qquad \text{everywhere,} \\ &\sigma_\theta = \frac{2}{\sqrt{3}}\sigma_0 \Phi_0(\zeta)(1-D_0) \quad \text{in the range} \quad -1 \le \zeta < -1/2, \\ &\sigma_\theta = -\frac{2}{\sqrt{3}}\sigma_0 \Phi_0(\zeta)(1-D_0) \quad \text{in the range} \quad -1/2 < \zeta \le 0. \end{aligned} \tag{17}$$

The function $\Phi_0(\zeta)$ determines the through-thickness distribution of the function $\Phi(\xi_{eq})$ at the initial instant. Thus, it follows from Eqs. 2, 6 and 16 that

$$\Phi_0(\zeta) = \Phi\left(\left|\zeta + \frac{1}{2}\right| \frac{4}{\sqrt{3}} \frac{da}{dt}\right). \tag{18}$$

It is seen from this equation that the function $\Phi_0(\zeta)$ is symmetric relative to the neutral line. Therefore, it follows from Eq. 17 that $\int_{-1}^{0} \sigma_\theta d\zeta = 0$, as it should be in pure bending. The solution of Eq. 10 in the range $-1 \le \zeta \le \zeta_n$ satisfying Eq. (11) at $\zeta = -1$ can be written in the form

$$\sigma_r = -\frac{a\sigma_0}{\sqrt{3}} \int_{-1}^{\zeta} \frac{\Phi(\xi_{eq})(1-D)}{(za+s)} dz \tag{19}$$

where *z* is a dummy variable of integration. Then, the radial stress acting at $\zeta = \zeta_n$ is

$$\sigma_{32} = -\frac{a\sigma_0}{\sqrt{3}} \int_{-1}^{\zeta_n} \frac{\Phi(\xi_{eq})(1-D)}{(za+s)} dz. \tag{20}$$

The solution of Eq. 10 in the range $\zeta_n \le \zeta \le -1/2$ satisfying the boundary condition $\sigma_r = \sigma_{32}$ at $\zeta = \zeta_n$ can be written in the form

$$\sigma_r = \sigma_{32} + \frac{a\sigma_0}{\sqrt{3}} \int_{\zeta_n}^{\zeta} \frac{\Phi(\xi_{eq})(1-D)}{(za+s)} dz. \tag{21}$$

Then, the radial stress acting at $\zeta = -1/2$ is

$$\sigma_{21} = \sigma_{32} + \frac{a\sigma_0}{\sqrt{3}} \int_{\zeta_n}^{-1/2} \frac{\Phi\left(\xi_{eq}\right)(1-D)}{(za+s)} dz \,. \tag{22}$$

Finally, the solution of Eq. 10 in the range $-1/2 \le \zeta \le 0$ satisfying the boundary condition $\sigma_r = \sigma_{21}$ at $\zeta = -1/2$ can be written in the form

$$\sigma_r = \sigma_{21} + \frac{a\sigma_0}{\sqrt{3}} \int_{-1/2}^{\zeta} \frac{\Phi\left(\xi_{eq}\right)(1-D)}{(za+s)} dz \,. \tag{23}$$

Substituting the boundary condition (11) at $\zeta = 0$ into Eq. 23 gives

$$\sigma_{21} + \frac{a\sigma_0}{\sqrt{3}} \int_{-1/2}^{0} \frac{\Phi\left(\xi_{eq}\right)(1-D)}{(za+s)} dz = 0 \,. \tag{24}$$

Using Eqs. 5, 20 and 22 it is possible to transform Eq. 24 to

$$I_1 + I_2 - I_3 = 0 \,, \tag{25}$$

where

$$I_1 = \int_{-ds/da}^{-1/2} \frac{\Phi\left(\xi_{eq}\right)(1-D)}{(za+s)} dz, \quad I_2 = \int_{-1/2}^{0} \frac{\Phi\left(\xi_{eq}\right)(1-D)}{(za+s)} dz, \quad I_3 = \int_{-1}^{-ds/da} \frac{\Phi\left(\xi_{eq}\right)(1-D)}{(za+s)} dz \,. \tag{26}$$

Differentiating each of these integrals with respect to a and, then, putting $a = 0$ and taking into account Eqs. 2, 14, and 16 give

$$\begin{aligned}
&\left.\frac{\partial I_1}{\partial a}\right|_{a=0} = 4(1-D_0)\left.\frac{d^2 s}{da^2}\right|_{a=0}, \\
&\left.\frac{\partial I_2}{\partial a}\right|_{a=0} = 4(1-D_0)\int_{-1/2}^{0}\left.\left(\frac{d\Phi}{d\xi_{eq}}\frac{\partial \xi_{eq}}{\partial a}\right)\right|_{a=0} dz - 4\int_{-1/2}^{0}\Phi_0\left.\frac{\partial D}{\partial a}\right|_{a=0} dz - 16(1-D_0)\int_{-1/2}^{0}\Phi_0\left(z+\frac{1}{2}\right)dz, \\
&\left.\frac{\partial I_3}{\partial a}\right|_{a=0} = -4(1-D_0)\Phi_0\left.\frac{d^2 s}{da^2}\right|_{a=0} - \\
&-4(1-D_0)\int_{-1/2}^{-1}\left.\left(\frac{d\Phi}{d\xi_{eq}}\frac{\partial \xi_{eq}}{\partial a}\right)\right|_{a=0} dz + 4\int_{-1/2}^{-1}\Phi_0\left.\frac{\partial D}{\partial a}\right|_{a=0} dz + 16(1-D_0)\int_{-1/2}^{-1}\Phi_0\left(z+\frac{1}{2}\right)dz.
\end{aligned} \tag{27}$$

Since $\Phi\left(\xi_{eq}\right)$ is a prescribed function of ξ_{eq}, the derivative $d\Phi/d\xi_{eq}$ at $a = 0$ can be found as a function of ζ by means of Eqs. 2, 6 and 16. The derivative $\partial\xi_{eq}/\partial a$ at $a = 0$ can be evaluated using Eq. 6. It is convenient to introduce the new variable γ by the equation $\gamma = \zeta + 1/2$. Then,

$$\left.\frac{\partial\xi_{eq}}{\partial a}\right|_{a=0}=\mp\mathrm{E}_0(\gamma),\quad \mathrm{E}_0(\gamma)=\frac{4}{\sqrt{3}}\left[\left.\frac{d^2s}{da^2}\right|_{a=0}-16\gamma^2\right]\frac{da}{dt}. \tag{28}$$

It is also convenient to introduce the following notation

$$\Omega(\gamma)=\mathrm{E}_0(\gamma)\left.\left(\frac{d\Phi}{d\xi_{eq}}\right)\right|_{a=0}. \tag{29}$$

Differentiating Eq. 25 with respect to a, using Eqs. 27 and Eq. 29, and replacing ζ with γ yield

$$\begin{aligned}&2\left.\frac{d^2s}{da^2}\right|_{a=0}+\int_0^{1/2}\Omega(\gamma)d\gamma-\int_0^{-1/2}\Omega(\gamma)d\gamma-\\&-\frac{1}{(1-D_0)}\left[\int_0^{-1/2}\Phi_0\left.\frac{\partial D}{\partial a}\right|_{a=0}d\gamma+\int_0^{1/2}\Phi_0\left.\frac{\partial D}{\partial a}\right|_{a=0}d\gamma\right]-4\left[\int_0^{-1/2}\gamma\Phi_0 d\gamma+\int_0^{1/2}\gamma\Phi_0 d\gamma\right]=0\end{aligned}. \tag{30}$$

It is seen from Eq. 28 that $\mathrm{E}_0(\gamma)$ is an even function of γ. It follows from Eq. 6 that ξ_{eq} at $a=0$ is also an even function of γ. Therefore, $d\Phi/d\xi_{eq}$ being a function of ξ_{eq} is also an even function of γ at $a=0$. Finally, the definition for the function $\Omega(\gamma)$ given in Eq. 29 shows that it is an even function of γ. Then,

$$\int_0^{1/2}\Omega(\gamma)d\gamma-\int_0^{-1/2}\Omega(\gamma)d\gamma=\int_0^{1/2}\Omega(\gamma)d\gamma+\int_{-1/2}^{0}\Omega(\gamma)d\gamma=2\left[\Sigma_1\left(\frac{1}{2}\right)-\Sigma_1(0)\right] \tag{31}$$

where $\Sigma_1(\gamma)$ is the anti-derivative of $\Omega(\gamma)$. Analogously, it is seen from Eq. 18 that the function $\Phi_0(\zeta)$ involved in Eq. 30 can be replaced with an even function of γ, say $\Phi_1(\gamma)$. Then, the anti-derivative of the function $\gamma\Phi_1(\gamma)$ is an even function of γ, say $\Sigma_2(\gamma)$. Therefore, $\Sigma_2(1/2)=\Sigma_2(-1/2)$ and

$$\int_0^{-1/2}\gamma\Phi_0 d\gamma+\int_0^{1/2}\gamma\Phi_0 d\gamma=\Sigma_2\left(-\frac{1}{2}\right)-2\Sigma_2(0)+\Sigma_2\left(\frac{1}{2}\right)=2\left[\Sigma_2\left(\frac{1}{2}\right)-\Sigma_2(0)\right]. \tag{32}$$

Substituting Eqs. 31 and 32 into Eq. 30 leads to

$$\left.\frac{d^2s}{da^2}\right|_{a=0}=\frac{1}{2(1-D_0)}\left[\int_0^{-1/2}\Phi_1\left.\frac{\partial D}{\partial a}\right|_{a=0}d\gamma+\int_0^{1/2}\Phi_1\left.\frac{\partial D}{\partial a}\right|_{a=0}d\gamma\right]+4\left[\Sigma_2\left(\frac{1}{2}\right)-\Sigma_2(0)\right]-\Sigma_1\left(\frac{1}{2}\right)+\Sigma_1(0). \tag{33}$$

In general, the derivative $\partial D/\partial a$ at $a=0$ can be found from Eq. 13 with no difficulty since the distribution of stresses is given by Eq. 17, $\varepsilon_{eq}=0$ and $D=D_0$ at $a=0$. Therefore, the right hand side of Eq. 33 can be evaluated and, then, using Eqs. 2 and 16 the function $s(a)$ at the initial stage of the process can be approximated by

$$s(a) = \frac{1}{4} + \frac{a}{2} + \frac{1}{2}\left(\left.\frac{d^2 s}{da^2}\right|_{a=0}\right)a^2. \qquad (34)$$

In the case of many damage evolution laws the function $\Lambda\left(\sigma/\sigma_{eq}, \varepsilon_{eq}, D\right)$ involved in Eq. 12 vanishes at $\varepsilon_{eq} = 0$ (i.e. at $a = 0$). For such laws Eq. 33 simplifies to

$$\left.\frac{d^2 s}{da^2}\right|_{a=0} = 4\left[\Sigma_2\left(\frac{1}{2}\right) - \Sigma_2(0)\right] - \Sigma_1\left(\frac{1}{2}\right) + \Sigma_1(0). \qquad (35)$$

Eq. 34 is still valid but its right hand side should be determined by means of Eq. 35.

Conclusions

The general solution proposed describes the process of pure bending of incompressible, rigid viscoplastic material al large strains. The constitutive equations include quite an arbitrary law of damage evolution. The dependence of the yield stress on the equivalent strain rate is also quite arbitrary. An advantage of the method used is that the boundary value problem has been reduced to two partial differential hyperbolic equations for the damage parameter and the radial stress written in characteristic coordinates (Eqs. 10 and 13). In general, these equations can be solved by standard and well documented numerical methods, though some difficulty can appear because these equations involve the function $s(a)$ which should be found along with solution to Eqs. 10 and 13. In order to determine this function, a non-standard condition in integral form should be used (Eq. 25). Of special importance is the behaviour of the function $s(a)$ at the initial stage of the process, $a \to 0$, because the general stress solution reduces to Eq. 17 at $a = 0$. In order to facilitate numerical treatment of the initial stage of the process, the second derivative of s with respect to a at $a = 0$ has been found analytically (Eqs. 33 and 35).

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