Comparison of the Size-Independent Fracture Energy of Concrete Obtained by Two Main Experimental Methods

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Abstract. The measuring of the fracture energy of concrete was thoroughly analyzed by several researches with regards to obtain a size-independent value. In this work prismatic notched specimens were subjected to three-point bend tests according to the experimental requirements of two main methods. Three different kinds of notched beam were manufactured. Two of the specimens were tested according to the simplified method proposed by Abdalla and Karihaloo. The other type of specimen was subjected to three-point bend tests with the self-weight compensation according to the method proposed by Elices et al. The size-independent fracture energy of concrete obtained with each method was analyzed. The two values were in excellent agreement which can thus be regarded as a true property of the material. It is therefore concluded that either method can be used to obtain a unique value of the size-independent fracture energy of concrete.

Introduction

The most extended method for measuring the fracture energy is the work-of-fracture method recommended by RILEM [1]. The values determined by this method present a dependency on size and shape of the test specimen, demonstrated and analyzed by several authors [2-4]. The reasons for variability of the RILEM fracture energy of concrete with the size of the ligament area are a subject of ongoing investigations [10-15].

Several researches have analyzed this size effect on fracture energy measurements according to RILEM procedure and they have proposed new methods or modifications to obtain a size-independent specific fracture energy of concrete. The two most popular methods for measuring the size-independent fracture energy of concrete are based on the local fracture energy model of Hu et al [8] and the experimental corrections to avoid energy dissipations proposed by Elices et al [9-11].

This paper deals with the comparative experimental analysis on the size-independent fracture energy of concrete determined by these two methods. Therefore three point bend test on notched beams according to each method were carried out. The beams designated TFE05 and TFE005 were used to obtain the true fracture energy of concrete by means of the simplified local fracture energy method of Abdalla and Karihaloo. In contrast, the beams designated SWC05 were used to test according to the method of Elices et al. The TFE05 and TFE005 specimens were tested according to the RILEM procedure with a relative notch to depth ratios of 0.05 and 0.5 respectively, and the self-weight was no compensated. The SWC05 specimens were subjected to three-point bend tests with the mentioned self-weight compensation (among other modifications).

The results attempt to show the relationship between both methods, as well as ensure the existence of a fracture material property independent on any geometrical parameter and the test method.

Theoretical background

Local fracture energy method (LFEM). Hu et al. [8] argued that the effect of the free boundary of the specimen is felt in the fracture process zone (FPZ) of concrete. The energy required to create a fresh crack decreases as the crack approaches to the free boundary [12]. This change in the local fracture energy (g_f) is represented by a bi-linear approximation, as shown in Fig. 1. The transition from the size-independent specific fracture energy of concrete (G_F) to the rapid decrease occurs at the transition ligament length (a_l), which depends on the both the material properties and specimen size and shape [13, 14]. The measured RILEM fracture energy (G_f) represents the average of the local fracture energy function over the ligament area (dashed line in Fig. 1). The relationship between all the involved variables is given by:

$$G_f(a,D) = \frac{\int_0^{D-a} g_f(x)\,dx}{D-a} = \begin{cases} G_F\left[1 - \dfrac{a_l/D}{2(1-a/D)}\right]; & 1-a/D > a_l/D \\ G_F\,\dfrac{1-a/D}{2a_l/D}; & 1-a/D \le a_l/D \end{cases} \qquad (1)$$

Where D is the total depth of the specimens and a is the initial notch depth.

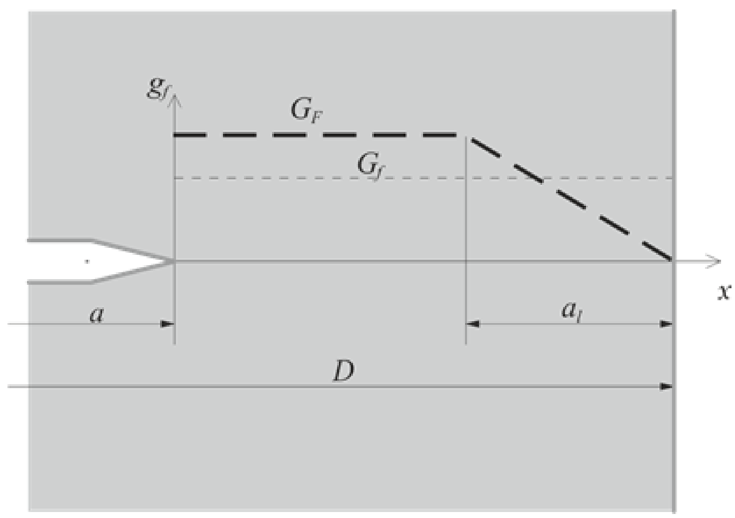

Fig.1. Local fracture energy model of Hu et al.

To obtain the values of G_F and a_l for each mix of concrete, the RILEM fracture energy of at least four specimens with the same size and different notch depths must be previously determined [14]. Applying Eq. 1 to each different notch depth specimen, it is obtained an overdetermined system of equations. This system must be solved by a least squares method for obtaining the best estimates of G_F and a_l [12, 14].

Abdalla and Karihaloo [21] showed that the same size-independent specific fracture energy can also be obtained by testing a single size specimen with only two notch to depth ratios, provided they are well separated (*a/D* = 0.05 and 0.50). Thus greatly simplifies the determination of G_F, especially when large specimens are required for testing [12]. This simplified method is one of the methods analyzed in the present paper.

Method proposed by Elices et al. (*P-δ* tail) Elices, Planas and Guinea [9-11] identified several sources of energy dissipation that may influence the measurement of G_F. The most important of them is the curtailment of the tail part of the load-displacement curve (*P-δ*) in a three-point bend test. They proposed some corrections in order to avoid several sources of energy dissipation as the adjustment of the initial stiffness of the *P-δ* curve, the adequate design of supports and the system load and the determination of the non-measured energy dissipation at the very end of the test. The last energy dissipation corresponds to the curtailment of the tail of the *P-δ* curve at the end of the test.

To estimate this non-measured energy when the test is interrupted ($W_{nm1} + W_{nm2}$) at very low loads it is necessary to model the beam behavior when the cohesive crack closely approaches to the free surface [11]. For cohesive materials and specimens where weight is compensated, the last phase of a stable three-point bend test can be modeled following the rigid-body kinematics (Fig. 2) used by Petersson [15].

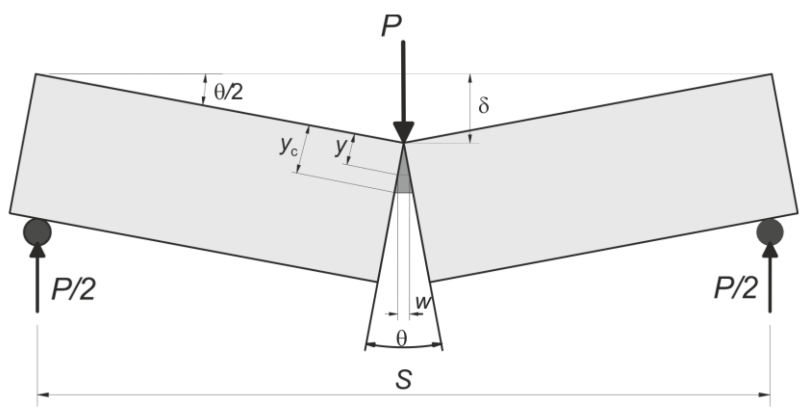

Fig.2. Rigid-body model of the behavior of the specimen at the end of the test.

Taking into account the geometrical relationships given by the rigid-body model, the non-measured fracture energy of the three-point bend test (Fig. 3) must be estimated by [11]:

$$W_{nm} = W_{nm1} + W_{nm2} = \frac{2A}{\delta_u} \quad (2)$$

where A is the experimental coefficient of adjustment of the *P-δ* tail and δ_u is the last recorded midspan deflection of the specimen at the end of the test (Fig. 3).

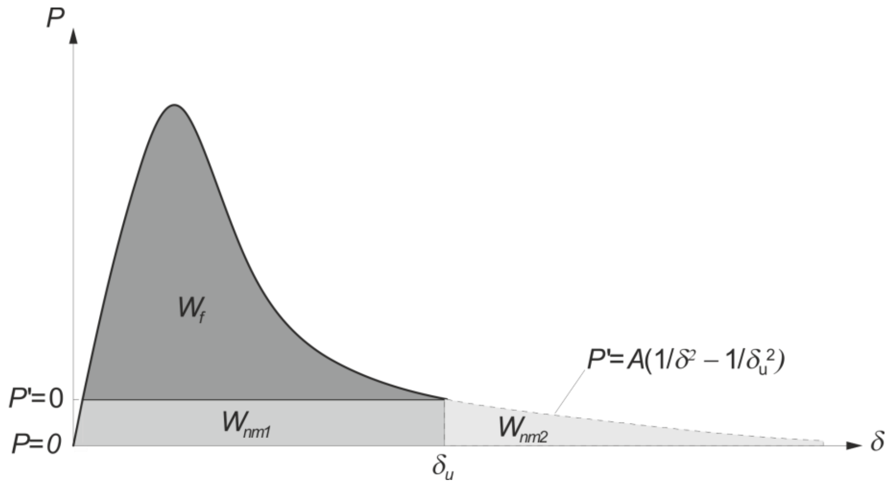

Fig.3. *P-δ* curve in a three-point bend test and the measured (W_f) and non-measured (W_{nm1}+W_{nm2}) fracture energy.

Once the non-measured energy has been estimated, the size-independent fracture energy of concrete can be obtained as [11]:

$$G_F = \frac{\int_0^{\delta_u} P d\delta + 2A/\delta_u}{b(D-a)}$$

Comparison of the Two Main Methods for measuring the Size-Independent Fracture Energy of Concrete.

Experimental Procedure. To compare the measured values of the size-independent fracture energy given by application of the two methods described above, an experimental campaign was carried out. Prismatic notched specimens were subjected to three-point bend tests according to the experimental requirements of each method. Three kinds of notched beams were manufactured. The TFE05 and TFE005 specimens were used to obtain the true fracture energy of concrete by means of the simplified local fracture energy method [12]. In contrast, SWC05 specimens were used to the adjustment of the tail of the *P-δ* curve method. Table 1 shows the geometrical dimensions of all specimens according to Fig. 4.

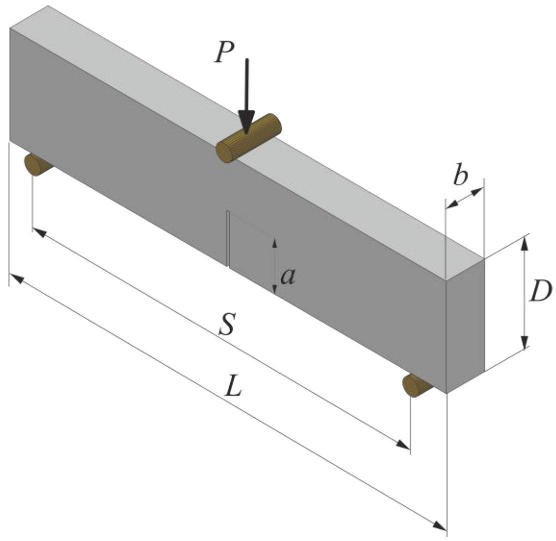

Fig.4. *P-δ* curve in a three-point bend test and the measured *(W_f)* and non-measured

Table 1. Geometrical properties of the notched specimens

Specimen	***D* [mm]**	***b* [mm]**	***S* [mm]**	***L* [mm]**	***a* [mm]**	***S/D***	***α = a/D***
TFE005	120	60	480	540	6	4	0.05
TFE05	120	60	480	540	60	4	0.5
SWC05	120	60	480	540	60	4	0.5

Four samples were tested for each type of specimen. The initial correction by crushing of concrete in supports was made for all specimens. The TFE05 and TFE005 specimens were tested according to the RILEM procedure and the self-weight was no compensated. Fig. 5 shows a picture of a three-point bend test of these specimens (left) and its instrumentation (right).

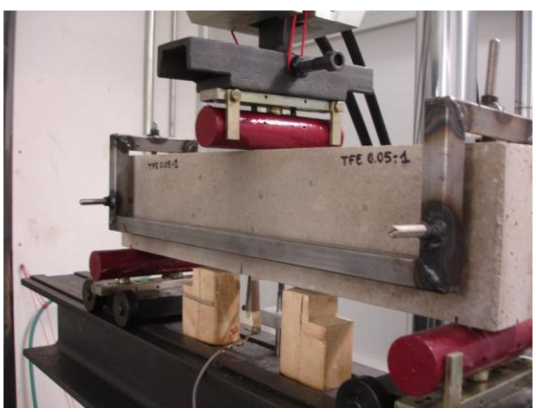

Fig.5. Three-point bend test of the TFE05 specimens (left) and its instrumentation (right)

The SWC05 specimens were subjected to three-point bend tests with the indicated self-weight compensation (Fig. 6). In these specimens the ultimate displacement at which the test was stopped was 3.5 millimeters.

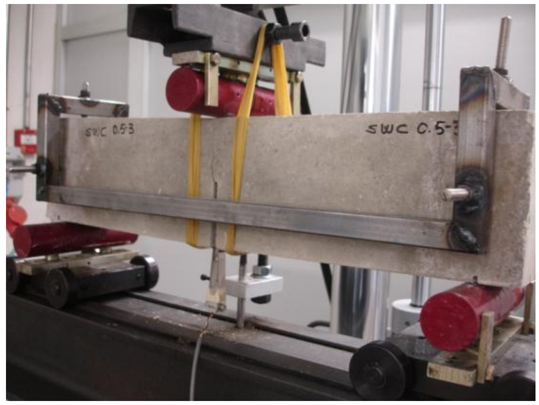

Fig.6. Three-point bend test of SWC05 specimens with weight compensation

All tests were driven in a closed-loop servo-hydraulic dynamic testing machine with crack mouth opening displacement (CMOD) control. The CMOD displacement was measured with a clip gauge transducer and a LVDT linear displacement transducer was used to measure the vertical displacement at midpoint (Fig. 5.right). A reference frame was used to fix the LVDT transducer to the top of the specimen so only the vertical displacement by deformations of the specimen was measured. Therefore, the load-CMOD and load-displacement curves for all specimens have been recorded. Loading was conducted according to CMOD control, at a rate as long as the total time of the tests was at least twenty minutes.

Concrete. All specimens have been manufactured with a single mixed concrete. Concrete mix proportions by weight of sand/gravel/cement/water were 1.4/3.5/1/0.4. The sand and gravel were siliceous aggregates with a maximum size of 8 millimeters mixed according to the Fuller method. Initial notches were made by cutting with a diamond saw blade 3 millimeters in thickness. There was a strict control of the specimen-making process to minimize scatter in test results.
Compressive tests were carried out on cylindrical specimens of 150×300 millimeters (diameter×height). The young's modulus was estimated from the $P-CMOD$ curve, according to the procedure indicated in [33]. Brazilian tests were also carried out on cylindrical specimens of 150×300 millimeters to obtain the split tensile strength of concrete. Table 2 shows the mechanical properties of concrete.

Table 2. Mechanical properties of concrete

Compressive strength, f_c [MPa]	36.9 ±6%
Splitting tensile strength, f_{ti} [MPa]	3.1 ±13%
Modulus of rupture, f_f [MPa]	4.6 ±1%
Young's modulus, E_c [GPa]	28.2 ±11%

Results.
Table 3 shows the average values obtained from the three-point bend tests of TFE005 and TFE05 specimens according to the RILEM procedure [1], such as: the maximum load obtained from tests (P_{max}), the self-weight of specimens (m), the vertical displacement at the end of the test (δ_u), the maximum load corrected according to the self-weight of the specimen (P'_{max}), the work of fracture measured from the P-δ curve (W_f), the total work of fracture considering the self-weight correction (W_{fT}), the ligament area (A_{lig}) and the RILEM specific fracture energy (G_f).

Table 3. Determination of RILEM fracture energy in TFE005 and TFE05 specimens

Specimen	TFE005	TFE05
P_{max} [N]	4733 ±4%	1332 ±4%
m [kg]	9.6 ±2%	9.6 ±1%
δ_u [mm]	1.41 ±4%	1.35 ±1%
P'_{max} [N]	4775 ±4%	1358 ±4%
W_f [Nmm]	797.7 ±12%	333.5 ±5%
W_{fT} [Nmm]	915.4 ±11%	448.3 ±4%
A_{lig} [mm^2]	6840	3600
G_f [N/m]	133.8 ±11%	124.5 ±4%

The comparison of G_f values obtained for TFE005 and TFE05 specimens reveals the size dependency of the specific fracture energy according to RILEM procedure. Applying the simplified method of Abdalla and Karihaloo [22] to these results, the size-independent fracture energy of concrete and the transition length were obtained (Table 5).

Table 4 shows the SWC05 specimen results for determining G_F according to the adjustment of the tail method where: δ_0 is the displacement considered as the initial point for adjusting the tail of the curve, A is the constant of adjustment, W_{nm} is the non-measured work of fracture and G_F is the size-independent specific fracture energy.

Table 4. Determination of size-independent fracture energy of concrete in SWC05 specimens

Specimen	SWC05-1	SWC05-2	SWC05-3	SWC05-4
P_{max} [N]	1444	1585	1410	1547
m [kg]	9.4	9.8	9.7	9.7
δ_0 [mm]	2.1	2.1	2.1	2.1
δ_u [mm]	3.5	3.4	3.6	3.5
A [Nmm^2]	84.4	172.3	98.1	113.8
W_f [Nmm]	518.6	484.4	390.3	430.3
W_{nm} [Nmm]	48.3	99.9	56.6	65.5
A_{lig} [mm^2]	3600	3600	3600	3600
G_F [N/m]	157.5	162.3	123.9	137.7

The specific fracture energy (G_F) determined with this method is directly the size-independent fracture energy of concrete. So the value of the true fracture energy obtained with this method is the average value for all specimens SWC05 (Table 5).

Table 5. Values of the size-independent fracture energy of concrete obtained by each method

Method	G_F [N/m]	a_l [mm]
LFEM	144.2	16.4
P-δ tail	145.4 ±12%	-

Comparing G_F results, a great accuracy between them is observed. Consequently, the two methods used in this paper to measure the size-independent fracture energy of concrete show the same value, as corresponds to a property of the material. Although these two methods have different experimental procedures, they are interrelated. Both of them consider some corrections corresponding to the final part of the work-of-fracture test. The local fracture energy model considers the influence when the crack approaches to the back free boundary surface of the specimen that corresponds to the end of the test [8]. On the other hand, the method of Elices et al. consists in determining the non-measured work of fracture by adjusting the tail of the *P*-δ curve that also corresponds to the final part of the test [11].

Conclusions

An experimental comparative analysis of the two main methods used to measure the size-independent fracture energy of concrete has been carried out. This analysis has not been done before in the literature. The value of the fracture energy obtained with both methods was practically identical. Both methods are interrelated as had been anticipated by Abdalla and Karihaloo. Both procedures apply some corrections to the final part of the *P*-δ diagram in the work-of-fracture test. The local fracture energy model considers the influence when the crack approaches the back-face free boundary surface of the specimen towards the end of the test. On the other hand, the method of Elices et al. consists in determining the non-measured work of fracture by adjusting the tail of the *P*-δ curve that also corresponds to the final part of the test. It is therefore concluded that either method can be used to obtain a unique value of the size-independent fracture energy of concrete.

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