# Compact Shear Specimen for Mode II Fracture Tests: Analysis and Fracture Interpretation


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**Keywords:** Compact shear specimen, crack, singular integral equations, asymptotic solution, stress intensity factors, fracture criteria.



**Abstract.** The work presents theoretical analysis and fracture interpretation of Compact Shear (CS) specimen for Mode II fracture testing of materials. Two fracture criteria are used: the maximum circumferential stress criterion and the criterion of minimum strain energy density. The problem for two parallel crack s in an infinite body under shear loading as in the CS specimen is simplest model for this investigation. Previously obtained results for stress intensity factors (Petrova, Sadowski, 2012) are used in this investigation. The interaction of the cracks leads to mixed-mode loading conditions near the crack tips. The dominant stress intensity factor is Mode II $K_{II}$, but $K_I$ is also non-zero. Using the fracture criteria the direction of crack propagation as well as the critical condition under which the crack would initiate is calculated. The influence of the geometry of the problem, i.e. the crack inclination angle to the remote shear loading, distance between the cracks, on the main fracture characteristics is investigated.


## Introduction

Many materials, such as cement pastes, concretes, wood, different composites, are generally weak in shear [1] and also reveal shear fracture under compression [2]. The shear fracture is also observed and intensively investigated for geomaterials [3]. Hence the study of materials response to shear load is important in fracture mechanics of materials. For experimental shear Mode II fracture tests the geometry of the specimen and loading conditions should be chosen so that

- to induce shear with minimal influence of normal stresses,
- to minimize influence of edges of the specimen on the crack tips,
- to prevent friction between the cracked faces.

The friction between the faces behind the crack tips can cause a significant increase in values of Mode II stress intensity factor $K_{II}$ [4,5]. Besides, it was shown in [4] that the compressive stresses ahead of the crack tip significantly affect the Mode II fracture.

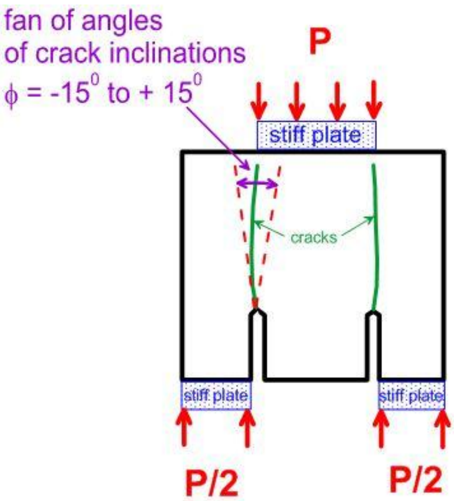
The general expression for the stress intensity factors (Mode I and Mode II) is written as $K_{I,II} = f_{I,II} P\sqrt{a}$ , where $P$ is applied load, $a$ is the crack length and $f_{I,II}$ is the geometric correction factor. All geometrical effects are reflected in $f_{I,II}$. The fracture parameters are measured in a test on any specimen, provided $f_{I,II}$ is known for the configuration. Because of analytical solutions are not available for the real specimen geometries finite element analysis and boundary element analysis are used for determination of $f_{I,II}$.

Different types of Mode II specimens are used, a review of these specimens can be found, for example, in [6] and in the stress intensity factors handbook [7]. One of these specimens, the compact shear (CS) specimen was introduced in [8,9], Fig. 1. This specimen is convenient for Mode II fracture testing, but experimental and numerical investigations show significant differences in the values of the stress intensity factors have resulted from minor geometrical differences [8,9,10]. Therefore the problem for study CS specimens in order to get the optimal specimen geometry remains important for planning the Mode II fracture experiment.

It has been observed that a crack under mixed mode loading and, in particular, under Mode II loading will not follow a straight path, but will deviate at an angle $\phi$ with respect to the crack axis. For mixed mode cases special fracture criteria are formulated, which determine the critical conditions for crack propagation and the direction of this propagation, i.e. the angle $\phi$. Discussion on applicability of existing fracture criteria can be found in [11]. In the present paper two of the most widely used criterion will be considered: the maximum circumferential stress criterion (Cherepanov, 1963; Erdogan and Sih, 1963; Panasyuk and Berezhnitskij, 1964, see for references [12,13,14]) and the criterion of minimum strain energy density [11,15]. Previously obtained results for stress intensity factors [16] are used in this investigation and are cited here for the sake of completeness. The problem for the cracks under shear loading corresponding to the loading in CS specimen in an infinite body was considered in [16] and analytical and semi-analytical solution based on the singular integral equations was obtained. In [17] numerical analysis was done by other approach, Finite Element Method implemented in FRANC2D/L code. Analytical and FEM calculations were also compared and discussed. This work is concentrated on studying of the application of fracture criteria for fracture interpretation of the results that is important for experiments.

## Formulation of the problem and solution

The scheme of the CS specimen for the Mode II fracture test is presented in Fig.1a. The Fig.1b shows the experimental CS specimen made from concrete, the shear fracture experiment has been done in [18].




(a)

(b)

Fig. 1. Compact shear specimen: a) Sketch of the experimental specimen for Mode II fracture test; b) An experimental specimen after the Mode II fracture test [18].

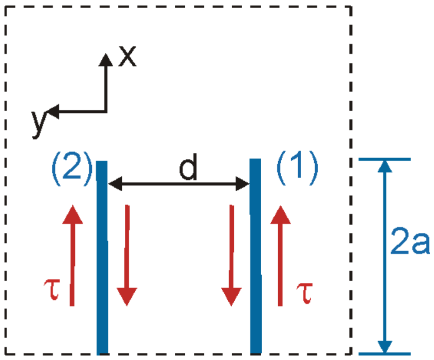



Fig. 2. Two parallel cracks in an infinite plate under the load τ as in the CS specimen.

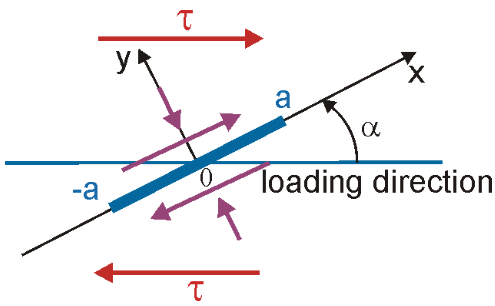



Fig. 3. A crack with the inclination angle α to the direction of the applied loading.

The problems for two parallel cracks in an infinite body (Fig. 2) is the simplest model for this investigation. The cracks (with length $2a$) are under shear loading corresponding to the loading in CS specimen. The shear load τ is $\tau = P/dB$, $P$ is the load and $B$ is thickness of the specimen in the Fig.1a, $d$ – the distance between the cracks (Fig. 2).

Approximate analytical and semi- analytical method (based on the singular integral equations) is used for the problem. Following the procedure presented in [12] the asymptotic solution for two arbitrary oriented cracks (one crack is shown in Fig. 3) was derived. The cracks faces are under shear loading $s_n = i(-1)^{n-1}\tau e^{-2i\alpha_n}$ (n=1, 2), which corresponds to the loading of the considered problem for $\alpha_n = \alpha = 0^0$ (n=1,2) (Figs. 2 and 3). For two parallel cracks the stress intensity factors (SIFs) are written as

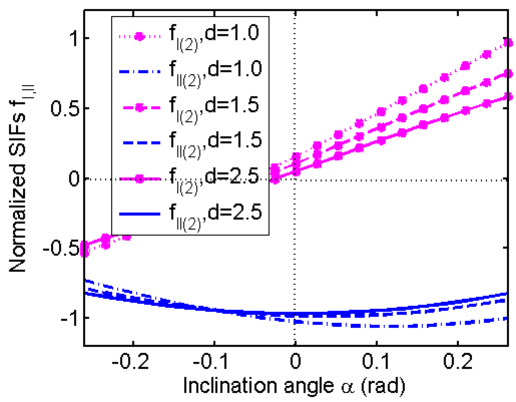
$$K_{II1}^{\pm} = -K_{II2}^{\pm} = -\tau\sqrt{a}\left(1 - \frac{1}{8}\lambda^2 + \frac{29}{128}\lambda^4 + 0(\lambda^6)\right);$$
$$K_{I1}^{\pm} = K_{I2}^{\pm} = \tau\sqrt{a}\left(\pm\frac{3}{16}\lambda^3 + 0(\lambda^5)\right). \qquad (1)$$

This result was obtained in [16,17]. Here the upper part of the "$\pm$ " or "$\mp$ " signs refers to the upper tips and the lower part to the lower tips of cracks (Fig. 2). The small parameter is equal to the ratio of the size of cracks to the distance between the cracks, i.e. $\lambda = 2a/d < 1$.

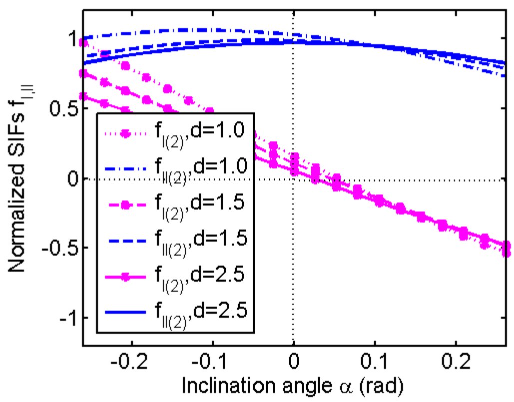
For considered problem, Eq. 1, $K_I$ is positive at the upper crack tips for both cracks and negative for the lower tips. Hence, the direction of loading in the CS specimen (Fig.1) is appropriate for the experimental test. In the specimen, proposed in [8,9], the direction of loading is opposite and $K_I$ is negative in the investigated crack tips, hence, compression is observed near these tips that influences the accuracy of the experimental data [4,5,7].

The asymptotic solution Eq. 1 can serve as first approximation of the SIFs of the considered problem for the non-dimensional parameter $\lambda = 2a/d$ in the range from 0 to 0.6 which corresponds $d > 3.3a$.

A numerical solution of the system of integral equations, which is suitable for small distances between cracks, was obtained by Gauss–quadrature method [12,19]. Fig. 4 presents the normalized SIFs $f_{II} = K_{II}/\tau\sqrt{a}$, $f_I = K_I/\tau\sqrt{a}$ as function of inclination angle α of the parallel cracks to the direction of the applied load (Fig. 3). The angle α varies from –15° to 15° and is shown in radians in the figures. The non-dimensional distance *d/a* equals to 1, 1.5 and 2.5 and the designation for the non-dimensional distance is *d* in the figures. We can see that the value of $f_{II}$ is changed slightly with changing the angle α, but changing of $f_I$ is stronger. For α =0 we have $f_I \neq 0$, and for α close to ±0.05 (±2.9°) – $f_I$ = 0. For some angles α we have physically non-realistic negative values of $f_I$, it indicates possible compression near these tips that should be avoided in experiments for Mode II fracture. Fortunately, for $\alpha$ =0 values of $f_I$ for the upper tips of both cracks are positive and it was also obtained in the analytical solution Eq. 1. The difference in the results of asymptotic solution for $0 < \lambda < 0.6$, Eq. 1, and the present numerical solution is about 3%.



(a)



(b)

Fig. 4. SIFs at the upper crack tips: a) for crack 1 and b) for crack 2. The results are for different distances *d* between the cracks.

**Fracture criteria and direction of crack propagation**

From experimental and theoretical investigations of cracks under mixed-mode loading, it is known that the cracks deviate from their initial propagation direction. For prediction of the crack growth and direction of this growth a fracture criterion should be applied. Two criteria will be considered: the maximum circumferential stress criterion [12,13] and the criterion based on the strain energy density function [11,15].

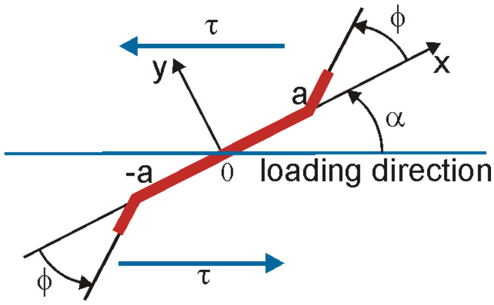



Fig. 5. A crack with the kink angle $\phi$.

**Maximum circumferential stress criterion.** Using the maximum circumferential stress criterion (Cherepanov, 1963; Erdogan and Sih, 1963; Panasyuk and Berezhnitskij, 1964, see for references [12,13,14]) the direction of the initial crack propagation (Fig. 7) is evaluated as

$$\phi = 2\arctan\left[\left(K_I - \sqrt{K_I^2 + 8K_{II}^2}\right)/4K_{II}\right] \tag{2}$$

and the critical stresses can be calculated from the expression

$$\cos^3(\phi_0/2)\left(K_I - 3K_{II}\tan(\phi_0/2)\right) = K_{Ic}/\sqrt{\pi}$$

as

$$p_{cr}/p_0 = 1/[\cos^3(\phi_0/2)\left(f_I - 3f_{II}\tan(\phi_0/2)\right)] \tag{3}$$

Here $K_{Ic}$ is the fracture toughness of the material, $p_0 = K_{Ic}/\sqrt{\pi a}$ and $f_{I,II}$ are the non-dimensional SIFs.

For pure Mode II crack the SIF factor $K_I$ is equal to zero and Eq. 2 gives the fracture angle $\phi_0 \approx \mp 70.5^\circ$ (the upper sign is for the upper crack tip, the lower – for the lower). The initial direction of crack propagation in the general case is determined from Eq. 2 by substitution of the results for SIFs $K_I^\pm$ and $K_{II}^\pm$.

**Strain energy density criterion.** Now the strain energy density criterion is used for fracture interpretation of the results. In [11,15] the minimum strain energy density factor criterion was introduced. The local strain energy density is given by $dW/dV = S/r$. Based on the stress intensity factor solutions $K_I$ and $K_{II}$, the strain energy density (SED) factor S($K_I$, $K_{II}$) is defined as

$$S(K_I, K_{II}) = a_{11}K_I^2 + 2a_{12}K_I K_{II} + a_{22}K_{II}^2, \tag{4}$$

where

$$\begin{aligned} a_{11} &= \frac{1}{16\mu}(1+\cos\phi)(\kappa - \cos\phi), \quad a_{12} = \frac{1}{16\mu}\sin\phi[2\cos\phi - (\kappa - 1)], \\ a_{22} &= \frac{1}{16\mu}[(\kappa+1)(1-\cos\phi) + (1+\cos\phi)(3\cos\phi - 1)] \end{aligned} \tag{5}$$

and $\kappa = 3 - 4\nu$ is for plain strain, μ is the shear modulus and ν is Poisson's coefficient. In Eq. 5 $\phi$ is the polar angle of the polar coordinate system ($r$, $\phi$) with the origin at the crack tip. SED factor determines the mixed mode effects, i.e., the direction of crack initiation as well as the critical condition under which the crack would initiate.

The criterion can be expressed mathematically as

$$\frac{\partial S}{\partial \phi} = 0, \quad \frac{\partial^2 S}{\partial \phi^2} > 0 . \tag{6}$$

The crack growth occurs when the SED factor reaches critical value, i.e. $S = S_{cr}$ for $\phi = \phi_0$. Here $S$ is Eqs. 4 and 5.

In the SED criterion the angle $\phi_0$ depends on Poisson's ratio. For Mode II cracks the maximum stress criterion predicts a fixed angle $\phi_0 \approx \mp 70.5^0$, which corresponds to a material with zero Poisson's ratio in SED criterion. For $\nu = 0.3$ the angle of crack propagation is $\phi_0 \approx \mp 82.3^0$.

**Numerical results**

Figs. 6 and 7 present results based on the maximum stress criterion. Eq. 2 and the values of SIFs obtained in the previous section were used in Fig. 6 and Eq. 3 was used in Fig. 7. Fig. 6 shows the influence of the inclination angle α on the angle $\phi$ at the upper crack tips. The dashed line $\phi_0 = \mp 70.5^0$ is plotted in the Fig. 6. We can see that the fracture angle is very sensitive to the direction of the applied load.

Fig. 7 shows the influence of the inclination angle $\alpha$ on the normalized critical stress $p_{cr} / p_0$ Eq. 3 at upper crack tips for different distances *d* between the cracks. For $\alpha$ = 0 the critical stress at both crack tips is less than 1, i.e. less than the stress for one crack.

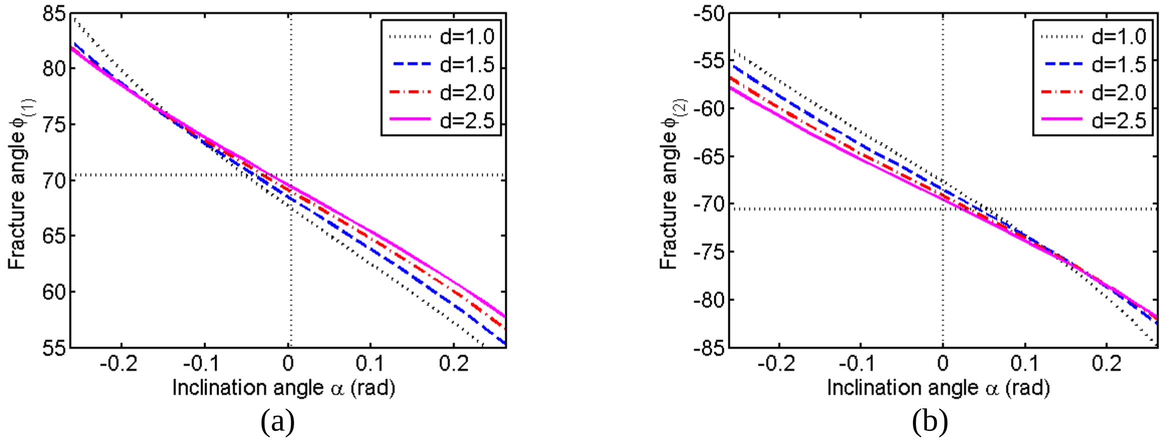



Fig. 6. The angle $\phi$ at upper crack tips verses inclination angle $\alpha$ for different distances *d*. a) for crack 1, b) for crack 2.

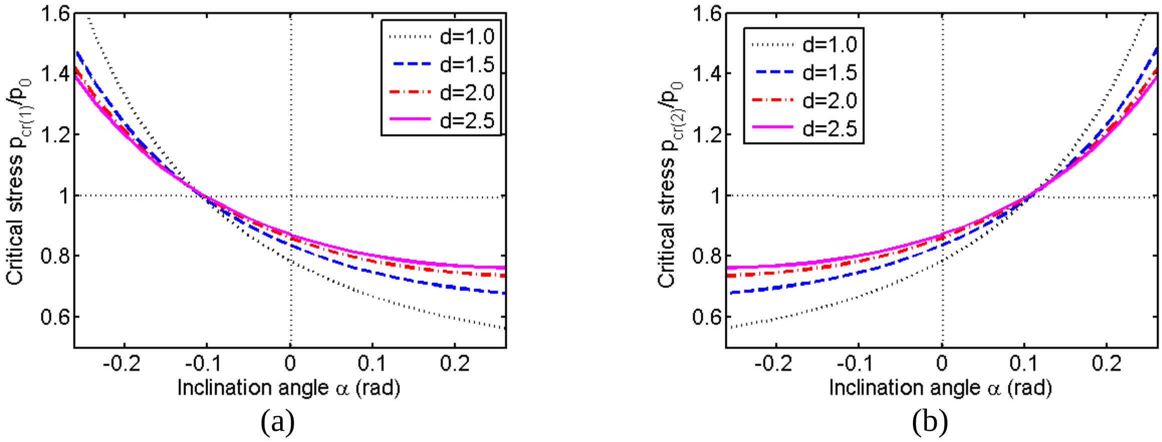
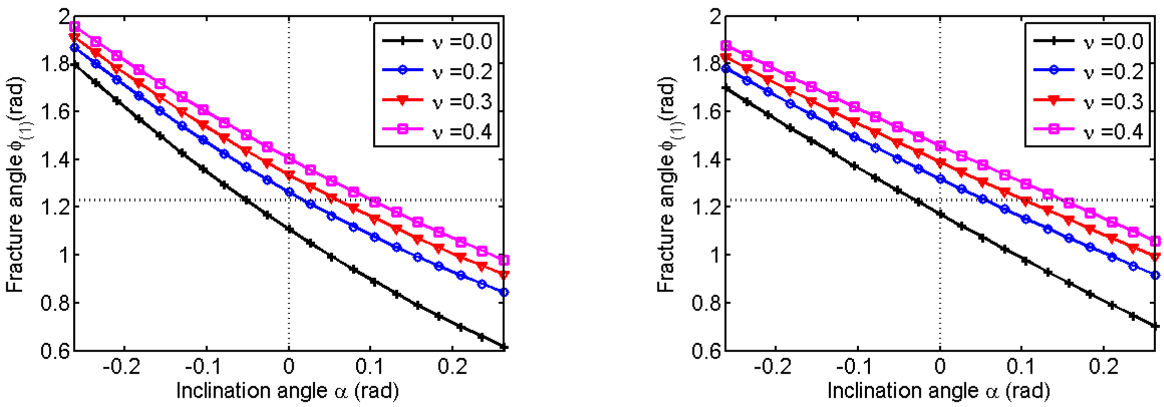



Fig. 7. The critical load at upper crack tips verses inclination angle $\alpha$ for different distances *d*. a) for crack 1, b) for crack 2.

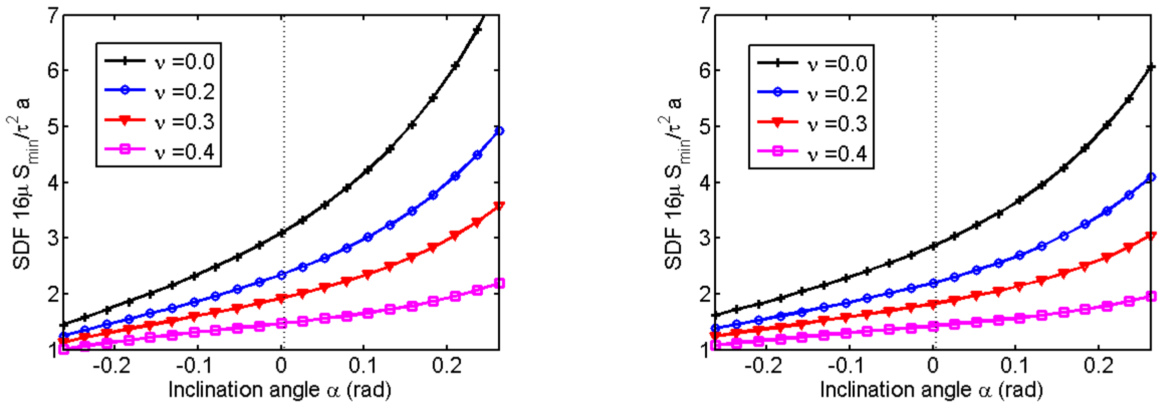
Figs. 8 and 9 present results based on the strain energy density criterion. Eq. 6 with Eqs. 4, 5 and the values of SIFs obtained in the previous section were used in Fig. 8. Fig. 8 shows the influence of the inclination angle $\alpha$ on the fracture angle $\phi$ at the upper tip of the crack 1 for different values of Poisson's ratio ($\nu$ = 0, 0.2, 0.3, 0.4) and for two distances $d$ between the cracks ($d$ = 1, 2). The curve for $\nu$ = 0 (black line) corresponds to the result for the maximum stress criterion. The horizontal dashed line corresponds to $\phi_0 = 70.5^\circ$ (in radians at the figure) for pure Mode II and for $\nu$ = 0.

Fig. 9 shows plot of $16\mu S_{\min}/\tau^2 a$ versus the inclination angle $\alpha$ for $\nu$ varying from 0 to 0.4. The quantity $16\mu S_{\min}/\tau^2 a$ increases with the angle α. As $S_{min}$ will be used as a material constant, the above statement implies that the lowest value of the applied stress (the critical stress) to initiate crack propagation occurs at $\alpha = \pi/12$ for a material with low Poisson's ratio.




(a) (b)

Fig. 8. The angle $\phi$ at upper tip of crack 1 verses inclination angle $\alpha$ for different Poisson's coefficients and distances $d$: a) $d$ = 1, b) $d$ = 2.




(a) (b)

Fig. 9. Normalized SDF for upper tip of crack 1 verses inclination angle $\alpha$ for different Poisson's coefficients and distances $d$: a) $d$ = 1, b) $d$ = 2.

## Conclusions

Theoretical analysis and fracture interpretation of CS specimen for Mode II fracture testing of materials is presented. The problem for two parallel cracks in an infinite body under shear loading as in the CS specimen is simplest model for this investigation. Previously obtained results for stress intensity factors [16,17] are used in this investigation. The present work is concentrated on studying of the application of fracture criteria for fracture interpretation of the results. Two fracture criteria are used: the maximum circumferential stress criterion and the criterion of minimum strain energy density. The direction of crack propagation and the critical condition under which the crack would initiate are obtained. The influence of the geometry of the problem, i.e. the crack inclination angle $\alpha$ to the remote shear loading and distance between the cracks, on the main fracture characteristics is investigated. The value of SIF $K_{II}$ is changed slightly with changing the angle $\alpha$, but changing of SIF Mode I $K_I$ is stronger. At the same time the values of the fracture angle $\phi$, of the critical stresses and of SED $S_{min}$, which are functions of SIFs $K_I$ and $K_{II}$, are sensitive to the crack inclination angle $\alpha$. Comparison between the results for two criteria is presented. The SED criterion includes the material parameter (Poisson's ratio) and can get more results, but comparison with experimental date should be done.

**Acknowledgement.** The research leading to these results has received funding from the European Union Seventh Framework Programme (FP7/2007 – 2013), FP7 - REGPOT – 2009 – 1, under grant agreement No: 245479; CEMCAST. The support by Polish Ministry of Science and Higher Education - Grant No 1471-1/7.PR UE/2010/7 - is also acknowledged.

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