Collective behavior of defects and criticality of damage-failure transitions

Oleg Naimark

Institute of Continuous Media Mechanics UB RAS, 1 Acad.Korolev str., 61013 Perm, Russia

naimark@icmm.ru

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Abstract. Statistical theory of revealed specific type of criticality – structural-scaling transitions in the ensemble of mesoscopic defects. The key results of statistically based phenomenology are the establishment of characteristic multiscale collective modes of defects responsible for relaxation and damage-failure transition.

Introduction

The problem of fracture treated as a critical phenomenon represents one of the key problems of fundamental and applied physics of materials science. Experimental studies of material responses in a large range of loading rates show that the behavior of solids is intimately linked with the evolution of typical mesoscopic defects (microcracks, microshears). This characterizes generically solids under dynamic and fatigue loading, when the internal times of the evolution of ensemble of defects for different structural levels are approaching the characteristic loading times. Statistical theory of typical mesoscopic defects (microcracks, microshears) revealed specific type of criticality – structural-scaling transitions and allowed the development of phenomenology of damage-failure transition based on the definition of non-equilibrium free energy of solid with defects.

Theory

Statistical theory of the evolution of typical mesoscopic defects (microcracks, microshears) allowed us to establish specific type of critical phenomena in solid with defects – structural-scaling transitions and to propose the phenomenology of damage-failure transition [1]. The key results of the statistical theory and statistically based phenomenology are the establishment of two order parameters responsible for the structure evolution – the defect density tensor p_{ik} and the structural scaling parameter $\delta = \left(R/r_0\right)^3$, which represents the ratio of the spacing between defects and characteristic size of defects. Non-equilibrium free energy F represents generalization of the Ginzburg-Landau expansion in terms of mentioned order parameters – the defect density tensor (defect induced deformation $p = p_{zz}$ in uni-axial case) and structural scaling parameter δ:

$$F = \tfrac{1}{2} A(\delta,\delta_*)p^2 - \tfrac{1}{4} Bp^4 - \tfrac{1}{6} C(\delta,\delta_c)p^6 - D\sigma p + \chi(\nabla_l p)^2, \qquad (1)$$

where $\sigma = \sigma_{zz}$ is the stress, χ is the non-locality parameter, A, B, C, D are the material parameters, δ_* and δ_c are characteristic values of structural-scaling parameter (bifurcation points) that define the areas of typical nonlinear material responses on the defect growth (quasi-brittle, ductile and finegrain state) in corresponding δ–ranges: $\delta < \delta_c \approx 1$, $\delta_c < \delta < \delta_*$, $\delta > \delta_* \approx 1.3$. The damage kinetics is determined by the kinetic equations for the defect density p and scaling parameter δ

$$\dot{p} = -\Gamma_p \frac{\Delta F}{\Delta p}, \quad \dot{\delta} = -\Gamma_\delta \frac{\partial F}{\partial \delta} \quad , \qquad (2)$$

where Γ_p, Γ_δ are the kinetic coefficients, $\Delta(...)/\Delta t$ is the variation derivative. Kinetic equations Eq.2 and the equation for the total deformation $\varepsilon = C\sigma + p$ (C is the component of the elastic compliance tensor) represent the constitutive equations of materials with mesodefects. Material responses on the loading realize as the generation of characteristic collective modes – the solitary waves in the range of $\delta_c < \delta < \delta_*$ and the "blow-up" dissipative structure in the range $\delta < \delta_c \approx 1$. The generation of these collective modes under the loading provides the change of the system symmetry and initiates specific mechanisms of the momentum transfer (plastic relaxation) and damage-failure transition on the scales of damage localization with the blow-up kinetics.

The damage-failure scenario includes the "blow-up" kinetics of damage localization as the precursor of crack nucleation according to the self-similar solution:

$$p = g(t) f(\xi), \xi = x/L_H, \; g(t) = G(1 - t/\tau_c)^{-m}, \qquad (3)$$

where τ_c is the so-called "peak time" ($p \to \infty$ at $t \to \tau_c$ for the self-similar profile $f(\xi)$ localized on the scale L_H, $G > 0, m > 0$ are the parameters of non-linearity, which characterise the free energy release rate for $\delta < \delta_c$.

The self-similar solution Eq.3 describes the blow-up damage kinetics for $t \to t_c$ on the set of spatial scales $L_H = kL_c$, $k = 1,2,...K$, where L_c and L_H corresponds to the so-called "simple" and "complex" blow-up dissipative structures. Generation of the complex blow-up dissipative structures appears when the distance L_S between simple structures approaches to the scale L_c. Similar scenario of the "scaling transition" proceeds for the blow-up structures of different complexity to involve in the process of the final stage of damage localization the larger scales of material.

The description of damage kinetics as the structural-scaling transition allowed the consideration of solid with defects as a dynamical system with spatial degrees of freedom (corresponding to the set of blow-up dissipative structures of different complexity) naturally evolve into a self-organized critical points related to critical value δ_c of structural-scaling parameter δ. Stochastic behaviour in this case can be linked with the dynamics of the critical state with the features of flicker noise, or $1/f$ - noise. The systems reveal the so-called self-organized criticality (SOC) with universal behavior that is typical for the late state evolution of dynamic systems when the correlation will appear on all length of scales and the system is critical.

The self-similar nature of mentioned collective modes associated with damage localization zones has the great importance in the case of dynamic loading, when the "excitation" of these modes can lead to the subjection of relaxation and failure to the dynamics of these modes. The examples for this situation are the transition from the steady-state to the branching regimes of crack propagation, qualitative change of the fragmentation statistics with the increase of the energy density imposed into the material, the self-similar features of numerous spall failure (the so-called "dynamic branch"), the delayed failure phenomenon in shocked materials (failure wave).

2. Experiment

2.1. Nonlinear crack dynamics. Crack branching

The understanding of self-similar scenario of damage-failure transition stimulated our experimental study of crack dynamics for the explanation of mechanisms of transition from the steady-state to the branching regime, fragmentation statistics and failure wave phenomenon [2]. The stress field in the area of crack tip in the preloaded (by external stress σ) PMMA plate and the diagram “crack velocity V versus applied stress σ” are presented in Fig.1 according to the data of high speed framing with the usage REMIX REM 10-8 camera (time lag between pictures $10\,\mu s$). Three characteristic regimes of crack dynamics were established in the different ranges of crack velocity: steady-state $V < V_C$, branching $V > V_C$ and fragmenting $V > V_B$, when the multiply branches of main crack have the autonomous behavior (Fig.1, 2). Steady-state regime of crack dynamics is the consequence of the subjection of damage kinetics to the self-similar solution of the stress distribution at the crack tip (mechanically speaking to the stress intensity factor). Bifurcation point V_C ($V_C \approx 0.4 V_R$ where V_R is the Rayleigh wave speed) corresponds to the transition to the regime, when the “second attractor” (with the symmetry properties related to the number of the blow-up dissipative structures) disturbs the steady-state regime due to the excitation of numerous new failure hotspots (the daughter cracks having the image of mirror zones on the fracture surface).

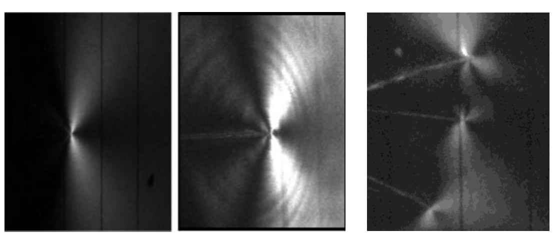


Fig.1. Stress pattern for steady-state $V < V_C$, branching $V > V_C$ and fragmenting $V > V_B$ scenario of crack dynamics.

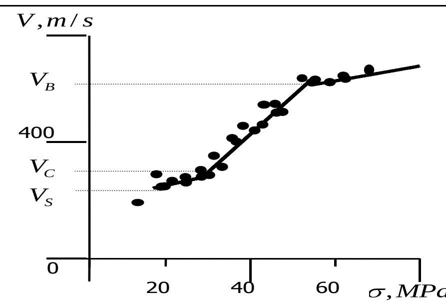


Fig. 2. Crack velocity V versus stress σ.

The morphology of fracture surface corresponding to different regime of crack dynamics is presented in Fig.3. The change of the symmetry properties of nonlinear system were studied under the recording of dynamic stress signal (polarization of laser beam) at the front of propagating crack in the point deviated on 4 mm from the main crack path. The corresponding phase portraits $\dot{\sigma} \sim \sigma$ for steady-state and branching regimes of crack dynamics are presented in Fig. 4 and confirmed the existence of two “attractors”, which subject the crack dynamics. The first attractor is related to the intermediate asymptotic solution for the stress distribution at the crack tip. The second attractor has degrees of freedom corresponding to the set of blow-up dissipative structures of different complexity.

2.2 Self-organized criticality and fragmentation statistics

Fundamental failure and fracture properties of the material are central in determining the nature of the fragment size distribution. Fragment size distributions can range from the relatively tight exponential functions to power-law relations spanning a number of decades in fragment size. Onset of fracture asymptotes to a range of length scales in which destruction is self-similar and fractal, requiring that consequences, including the fragment size distributions, exhibit a power-law dependence on the length scale [3].

The linkage of scenario of crack propagation and symmetry properties of dynamic system “solid with defects” allowed us to propose the interpretation of fragmentation statistics depending on the energy density imposed. A large number of the fragmentation statistics were proposed: log-normal, power-low, exponential, combination of exponential and power laws. These theories have focused on the prediction of mean fragment size through energy and momentum balance principles, and on statistical issues of fragment size distribution. The energy density $E < E_C$ (E_C corresponds to the critical velocity V_C of the steady state – branching transition) provides the stress intensity controlled failure scenario. The transient densities $E_B > E > E_C$ $(V_C < V < V_B)$ lead to the exponential fragmentation statistics that is sensitive to both self-similar solutions: the self-similar stress distribution at the crack tip and collective blow-up modes of damage localization. The power low statistics is characteristic for the self-organized criticality (SOC) scenario [4] that was studied in recovery test for dynamically loaded rods of the fused quartz (Fig.5).

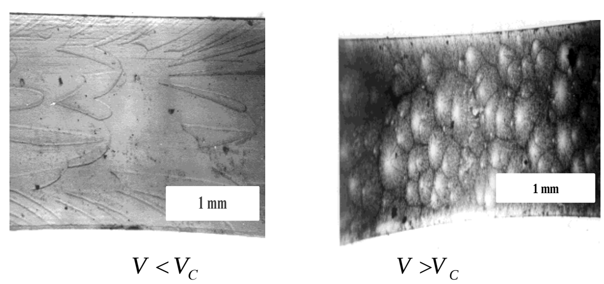


Fig. 3. Fracture surfaces

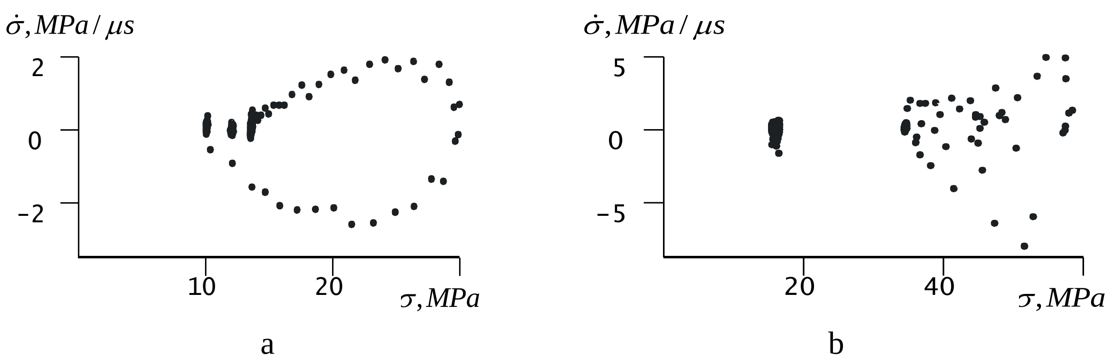


Fig. 4. Stress phase portraits $\dot{\sigma} \sim \sigma$: a - $V = 200 m/s$, b - $V = 615 m/s$

The power low spatial scaling of the fragment distribution was observed under the increase of the energy density $E > E_B$ ($V > V_B$, Fig.2). This statistics corresponds to the multiscale damage localization according to the set of blow-up dissipative structures that have the image of the stochastic “cloud” on the phase portrait (Fig. 4)

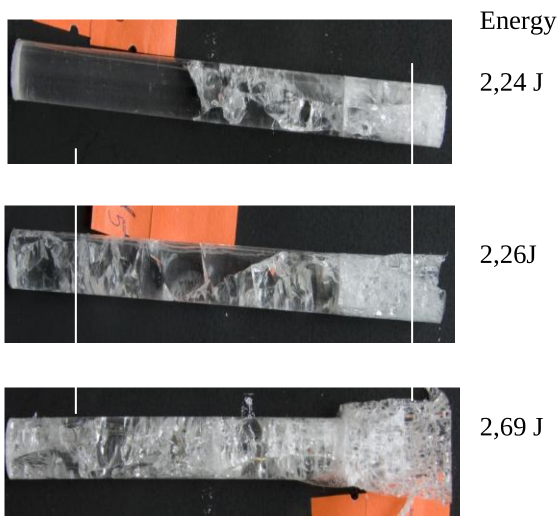


Fig. 5. Fragmentation pattern for different energies *E* in dynamically loaded fused quartz rods

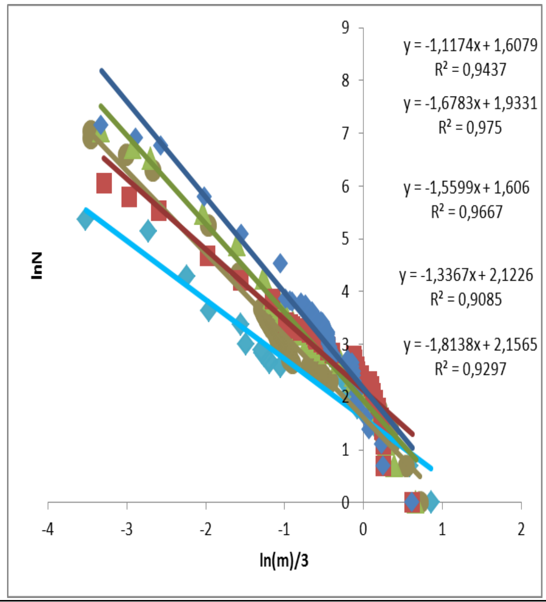


Fig. 6. Double logarithmic plot of cumulative distribution function of fragment sizes

2.3 Resonance excitation of damage localization. Failure waves

The solution Eq. 3 allowed us to link the self-similar features of failure kinetics and generation of blow-up dissipative structures. It was established the correspondence of failure hotspots nucleation having the image of mirror zones in experiments with numerous spall failure in shocked cylindrical rods of PMMA and ultraporcelain [5,6]. The multiple mirror zones with an equal size were excited on different spall cross sections in the shocked rod when the stress wave amplitude exceeded some critical value corresponding to the transition to the so-called “dynamic branch” under spalling (Fig. 7). The constant size of damage localization zones in the sample cross-sections at the dynamic branch corresponds to the regime of the resonance excitation of the blow-up dissipative structures of different complexity depending on the pulse rise time: the smallest mirror zone size nucleates at the high pulse amplitude.

Theoretically predicted low limit of damage localization scale L_C shows the existence of critical energy density, which provides the limit size of fragmented structure close to L_C and the degeneration of the power low statistics into the mono-disperse distribution. Such fragmentation dynamics can be linked to the failure wave phenomenon [7]. The important feature of failure wave phenomenon is that the velocity of failure wave doesn’t depend on the velocity of propagation of the single crack. The stored elastic energy in material is the main factor, which provides the ability of brittle solid to the generation of failure wave.

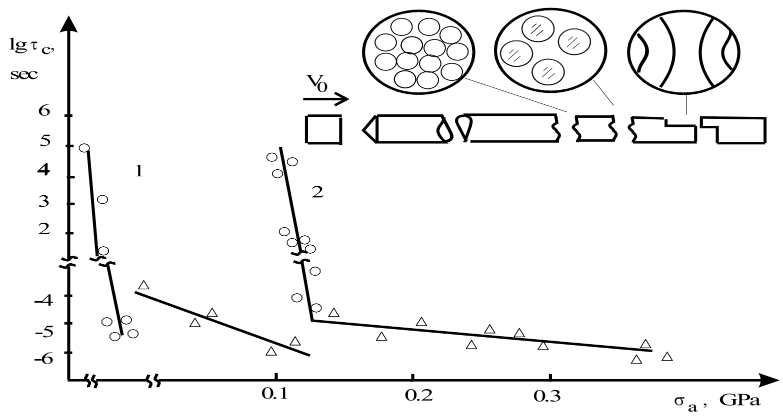
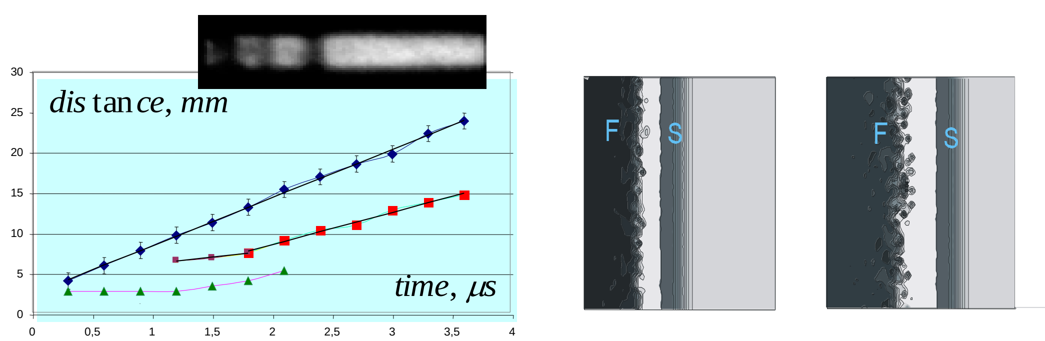


Figure 7.1. Fracture time t_c for shocked rod of PMMA (1) and ultraporcelain (2) versus stress amplitude σ_a. Insert: surface pattern with mirror zones in different spall cross sections [6].

The failure waves represent the specific dissipative structures (the "blow-up" dissipative structures) in the microshear ensemble that could be excited due to the shock wave pass [8]. Experimental study of failure wave generation and propagation was realized for the symmetric Taylor test on fused-quartz rods [9]. Fig. 7a shows the processing of a high-speed photography (upper picture) for the flyer rod traveling at 534 m/s at impact. Three dark zones correspond to the image of impact surface (green triangle), failure wave (red square) and (blue diamond) the shock wave. The initial slope for the failure wave gives the front velocity $V_{fw} \approx 1.57\, km/s$ that is close to traditionally measured in the plate impact test [9]. However, the experiment revealed the increase of failure front velocity up to the value $V_{fw} \approx 4\, km/s$. Approaching of failure wave front velocity to the shock front velocity supports theoretically based result concerning the failure wave nature as “delayed failure” with the limit of “delay time” corresponding to the “peak time” in the self-similar solution [10].



a b

Fig. 7. a – The Taylor test data; b - Simulation of shock wave (S) and failure wave (F) propagation for different time [**].

Numerical simulation (Fig.7b) of damage kinetics describes the self-similar “blow-up” dynamics of damage-failure transition, supports the assumption concerning the failure wave mechanism as delayed failure with the delay time of the development of “peak regime” of “blow-up” dissipative structure. Time of the delay τ_D represents generally the sum of the induction time τ_I - the time of the formation of damage spatial distribution close to the self-similar profile, and the “peak time” τ_C - time of the “blow-up” damage kinetics.

Steady-state regime of failure wave front propagation can be linked with the successive „resonance" activation of "blow-up" dissipative structures in the condition, when $\tau_D \approx \tau_C$.

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