Analysis of Two Nonlocal Criteria of Bridged Cracks Growth

M.N. Perelmuter

A.Yu. Ishlinksky Institute for Problems in Mechanics of RAS
Pr. Vernadskogo 101-1, 119526, Moscow, Russia

perelm@ipmnet.ru

Keywords: crack, growth, bridging, criteria.

Abstract. The comparative analysis of two nonlocal criteria of quasi-static bridged crack growth is considered. These criteria are based on two conditions of fracture: a) the condition for the crack tip advancing; b) the condition of bonds rupture at the trailing edge of bridged zone. The first conditions in these criterions are different and they are based on the consideration of the energy release and consumption rates or on the force approach. The second conditions in both criteria are the same – it is the condition of the limit stretching at the trailing edge of the bridging zone. Based on these two fracture conditions the regimes of the bridged zone and the crack tip equilibrium and growth are considered. Analytical application of these criterions is performed for the problem of the straight crack in homogeneous plane with the rectilinear law of bond stresses.

Two-parametric fracture criterions

The criterion based on the energy condition. The state of the crack with bridged zone (see Fig.1) limit equilibrium corresponds to the following condition [1-2]

$$-\frac{\partial \Pi}{\partial \ell} = \underbrace{-\frac{\partial}{\partial \ell}\left[\int_{v} w(\varepsilon_{ij})dv - \int_{s_e} t_i u_i ds\right]}_{G_{tip}(d,\ell)} \underbrace{-\frac{\partial}{\partial \ell}\int_{s_i} \Phi(u)ds}_{G_{bond}(d,\ell)} = 0 \qquad (1)$$

where $w(\varepsilon_{ij})$ is the density of the deformation energy in the body volume v, ε_{ij} are the components of the strain tensor; t_i, u_i are the tractions and displacements at the body boundary and (or) crack surfaces s_e, $\Phi(u)$ is the density of the strain energy of the bonds in the crack bridging zones, u is the crack opening in the bridging zones of area s_i.

The terms in the brackets in the relation (1) represent the strain energy release rate at creation of a new crack surface and the last term is the rate of the energy absorption in the crack bridging zone. Note, that within the framework of the model the strain energy release rate and the rate of the energy absorption by bonds depend on the bridging zone size and bonds characteristics. The equilibrium bridging zone size is not assumed to be constant during a quasi-static crack growth. It can be determined from condition (1) while the searching for the critical load needs additional conditions of the bond rupture.

In the case of an interface crack the strain energy release rate $G_{tip}(d,\ell)$ can be defined through the stress intensity factors [1, 2] as follows

$$G_{tip}(d,\ell) = \mathbb{Z}K_B^2 \tag{2}$$

where the parameter $\mathbb{Z}$ depends on the elastic properties of joined materials and $K_B = \sqrt{K_I^2 + K_{II}^2}$ is the modulus of the stress intensity factors due to the external loads and the stresses in the crack bridging zone.

The stress intensity factors (SIF) $K_{I,II}$ for the interface bridged crack were determined in [1].

The expression for the rate of the energy absorption by bonds $G_{bond}\left(d,\ell\right)$ can be written as [2]

$$G_{bond}(d,\ell) = \int_{\ell-d}^{\ell} \left(\frac{\partial u_y(x)}{\partial \ell} q_y(u) + \frac{\partial u_x(x)}{\partial \ell} q_x(u) \right) dx - \int_0^{u(\ell-d)} \sigma(u)du + G_c \tag{3}$$

where the second term is the density of deformation energy allocated at break of the bond at the trailing edge of the crack bridging zone.

For a homogeneous material or an adhesion layer connecting different materials the following relations are held [2]

$$G_c = G_b = \int_0^{u(\ell-d)} \sigma(u)du \tag{4a}$$

In this case the expression (3) completely coincides with similar expressions from the Ref. [1].

For a weak matrix material $\left(G_c \ll G_b\right)$ we suppose $G_c = 0$ in (3). In this case $G_{bond}\left(d,\ell\right) \to 0$ if $d/\ell \to 0$ and therefore this approach coincide with the Barenblatt's model in this limit, see details in [1-2].

Conclusively, in the general case the following values of G_c for different types of materials are used in (3)

$$G_c = \begin{cases} G_b - adhesion\ layer \\ 2c_m\gamma_m - composites \\ 0 \quad - weak\ \ junction \end{cases} \tag{4b}$$

where c_m is the volume fraction of the matrix material and $2\gamma_m$ is the matrix toughness.

The condition of the crack tip limit equilibrium (1) can be rewritten as follows taking into account the notation from the formulae (2) and (3)

$$G_{tip}(d,\ell) = G_{bond}(d,\ell) \tag{6}$$

Condition (6) is necessary but insufficient for searching for a limit equilibrium state of the crack tip and the bridging zone. This condition enables us to determine the bridging zone size, d_{cr} such that the crack tip is in an equilibrium state at the given level of the external loads. To search for the limit state of both the crack tip and bridging zone within the framework of the model one should introduce an additional condition, e.g., the condition of bond limit stretching at the trailing edge of the bridging zone $x_0 = \ell - d_{cr}$

$$u(x_0) = ([u_x(x_0)]^2 + [u_y(x_0)]^2)^{1/2} = \delta_{cr} \tag{6}$$

where δ_{cr} is the bond rupture length.

Solving simultaneously equations (5)-(6) we can determine the critical external loads σ_0, the critical bridging zone size d_{cr} and the adhesion fracture resistance at the crack limit equilibrium state for the given crack length and bond characteristics.

The criterion based on the force condition. This criterion is also based on the two fracture conditions. The first of these two conditions is the force condition [3]

$$K_\infty - K_{br} = K_{Ic}, \quad K_{Ic} = \sqrt{EG_{Ic}} \tag{7}$$

where K_∞ is the SIF due to the external loading, K_{br} is the SIF due to the bridged stresses applied to the crack surfaces in the bridged zone, K_{Ic} is the fracture toughness of a matrix, G_{Ic} - fracture energy of a matrix.

The second condition of the force criterion is coincided with eq. (6).

Crack with uniform bridging stresses

Analytical consideration of the criterions is performed for the problem of the straight crack in a homogeneous plane with the rectilinear law of the constant bond stress. In this very simple case the normal bridging stresses in the crack bridging zone are prescribed, uniformly distributed along the bridging zone and independent on the crack opening. The normal displacements of an upper crack surface under an external stress applied normal to the crack plane are given by (plane stress state) [4]

$$u_0(x) = \frac{1}{E}\left(2\sigma_0 - \frac{4P_0}{\pi}\arccos\frac{h}{\ell}\right)\sqrt{\ell^2 - x^2} + \frac{P_0}{\pi E}\left[\left(x-h\right)F(\ell,x,h) - \left(x+h\right)F(\ell,x,-h)\right] \tag{8}$$

where $h = \ell - d$, E is Young modulus of material, σ_0 is an external stress applied normal to the crack plane, P_0 is the normal bridge stress in the crack bridging zone of the size d, and $F\left(\ell,x,h\right)$ is the source function given by [4]

$$F(\ell,x,\xi) = \ln\frac{\ell^2 - x\xi - \sqrt{(\ell^2 - x^2)(\ell^2 - \xi^2)}}{\ell^2 - x\xi + \sqrt{(\ell^2 - x^2)(\ell^2 - \xi^2)}} \tag{9}$$

After some calculations (see the details in [2]) the following relationships for the strain energy release rate and for the rate of the energy absorption by bonds can be obtained from the equations (2) and (3)

$$G_{tip}(d,\ell) = G_f\left(1 - Z_0\,A(t)\right)^2, \quad G_{bond}(d,\ell) = 2G_f\varphi(t) + G_c \tag{10}$$

where

$$t=\frac{d}{\ell},\quad Z_0=\frac{2P_0}{\pi\sigma_0},\quad G_f=\frac{\sigma_0^2\pi\ell}{E} \tag{11}$$

$$A(t)=\arccos(1-t),\quad B(t)=\sqrt{2t-t^2},\quad C(t)=(1-t)\ln(1-t) \tag{12}$$

and

$$\varphi(t)=Z_0\left\{A(t)-B(t)-Z_0\left[A(t)\left[A(t)-2B(t)\right]-2C(t)\right]\right\} \tag{13}$$

The first fracture condition (5) can be written using the equations (10)-(13) as follows

$$2Z_0\left\{\left[2A(t)-B(t)\right]-Z_0\left[1.5A^2(t)-2A(t)B(t)-2C(t)\right]\right\}+2\eta R_0 Z_0^2-1=0 \tag{14}$$

where

$$\eta=\frac{G_c}{G_b},\quad G_b=P_0\delta_{cr},\quad R_0=\frac{\pi E\delta_{cr}}{8P_0\ell} \tag{15}$$

The second fracture condition (6) in this case is

$$\frac{B(t)}{Z_0}-\left[A(t)B(t)+C(t)\right]=R_0 \tag{16}$$

The equations (14) and (16) is the nonlinear algebraic system and the solution of this system (if it exists) gives us the external critical stress and the size of the crack bridging zone in the crack limit equilibrium state. The nonlinear algebraic system is solved numerically and the main parameters governing the solution of the system are η and R_0. If the solution of the system does not exist (under the fixed values of) then, from mechanical point of view, the size of the crack is less than the crack size associated with initiation of quasi-static crack growth. In this case the sub-critical crack growth is observed [2].

Comparison of the energy and force fracture criterion

We will now carry out a comparative analysis of the fracture criterions considered above for a crack with bonds in the bridged zone: the fracture criterion based on the eqs. (14) and (16) (subsequently referred to as the energy criterion) and the fracture criterion with a force condition for the advance of the crack tip (subsequently referred to as the force criterion) for a problem with constant stresses in the bridged zone.

We shall assume that the conditions for the rupture of the bonds at the trailing edge of the bridged zone are identical in both criteria and are determined by the equation (6). The equation for the force fracture criterion when there are constant stresses in the bridged zone, which is analogous to the first of the condition (5), will be based on the eqs. (7).

Using expressions (4.3) and (4.6) from [2] we convert condition (7) for the case of the constant bridged stresses to the dimensionless form

$$\frac{1}{Z_0} - A(t) = \sqrt{2\eta R_0}, \quad t = d/\ell \tag{17}$$

We obtain the equation for determining the length of the bridged zone of the crack in the limit equilibrium state in accordance with the force fracture criterion from eqs. (17) and (16), on eliminating the parameter Z_0 from them. We have

$$C(t) - B(t)\sqrt{2\eta R_0} + R_0 = 0, \quad t = \frac{d_{cr}}{\ell} \tag{18}$$

After solving this equation, the critical external load can be determined, for example, from an expression analogous to (4.14) of the Ref. [2].
On the other hand, by analogy with eqs. (4.15) of the Ref.[2], (18) can be considered as an equation for determining the parameter $R_{cr} = R_0$ for a specified value of $t_{cr} = d_{cr}/\ell$

$$\xi^2 - \xi B(t_{cr})\sqrt{2\eta} + C(t_{cr}) = 0, \quad \xi = \sqrt{R_{cr}} \tag{19}$$

From eq. (19), we obtain

$$R_{cr} = \left(B(t_{cr})\sqrt{\eta} + \sqrt{\eta B^2(t_{cr}) - 2C(t_{cr})}\right)^2 / 2 \tag{20}$$

When $t_{cr} \to 0$, from the solution (20), we have

$$R_{cr} = t_{cr}\left(\sqrt{\eta+1} - \sqrt{\eta}\right)^{-2} \tag{21}$$

An expression for R_{cr} which is identical to (21), follows from expression (4.16) of Ref. [2] when $t_{cr} \to 0$, which is evidence of the equivalence of the fracture criteria being considered in this case.
For a comparative analysis of the energy and force fracture criteria, we will consider relations (4.15) of Ref. [2] and (20) for the parameters $R_{cr} = d_0/\ell_{cr}$ for these fracture criteria as a function of the relative length of the bridged zone of a crack t_{cr} in the limit equilibrium state. For small values of t_{cr}, both criteria give close results (see Fig. 2, $\eta = 1$), and the difference increases as the relative size of the bridged zone increases. Note that, in the case of a fixed relative size of the bridged zone, the energy criterion gives a greater value of the parameter R_{cr} than the force criterion, which, in its turn, corresponds to a shorter crack and a greater critical external stress. The increase in the critical load when the energy criterion is used is explained by taking account of the work done in deforming the bonds. When $\eta \to \infty$, both criteria give similar results for $0 < t_{cr} \le 1$ and $R_{cr} \to 2\eta$ when $t \to 1$.
The results in the case of a matrix with a low fracture toughness ($\eta = 0$) are fundamentally different (see Fig. 3). In the case of a short bridged zone the results are close but, already when $t_{cr} > 0.1$, a considerable divergence is observed. Note that the force criterion is inapplicable in the case of a

crack filled with bonds when $\eta = 0$, since expression (8), which has been written taking account of the finiteness of the stresses ($K_\infty - K_{br} = 0$), gives a zero opening of the crack.

In Fig. 3, when $t_{cr} = 1$, we have $R_{cr} = 0$ in the case of the force fracture criterion, which formally corresponds to a crack of infinite length and, correspondingly, to an bridged zone of infinite size. The maximum value of the parameter $R_{cr}^m \approx 0.368$ is reached when $t_{cr} \approx 0.632$. When $R_{cr} > R_{cr}^m$, cracks, which satisfy the limit equilibrium conditions, do not exist within the framework of the force fracture criterion.

Hence, the energy and force criteria for the development of a crack give close estimates of the fracture parameters in the case of crack with a short bridged zone and, also, in the case of a composite material with a matrix possessing a high fracture toughness.

Acknowledgement This research was supported by the Russian Foundation for Basic Research (11-08-01243).

References

[1] R.V. Goldstein, M.N. Perelmuter, Modeling of bonding at the interface crack, Int. J. Fracture, 99, (1-2) (1999), P. 53-79.

[2] M. Perelmuter, A criterion for the growth of cracks with bonds in the end zone Journal of Applied Mechanics and Mathematics (PMM), 71, (2007), P.137-152. (English translation)

[3] B.N. Cox, D.B. Marshall, Concepts for bridged cracks in fracture and fatigue. Acta metal. mater. V.42. N 2 (1994), P. 341-363.

[4] V.V. Panasyuk, Limit Equilibrium of Brittle Bodies with Cracks. Kiev: Naukova Dumka (1968)

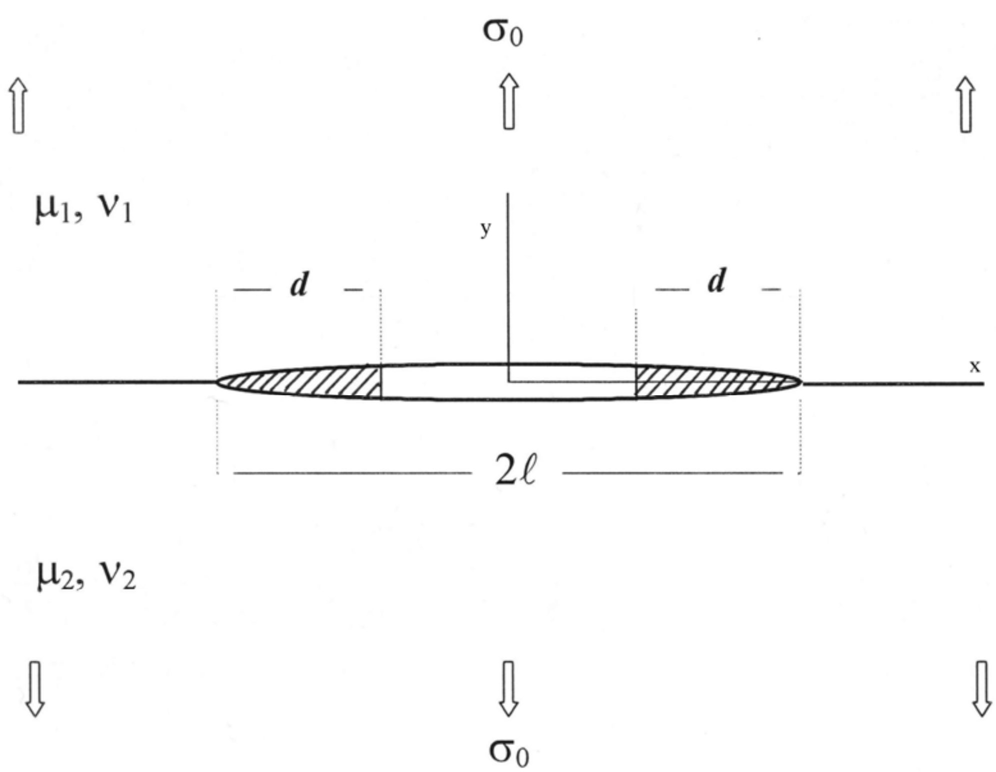

Fig. 1. Bi-material plate with bridged interface crack.

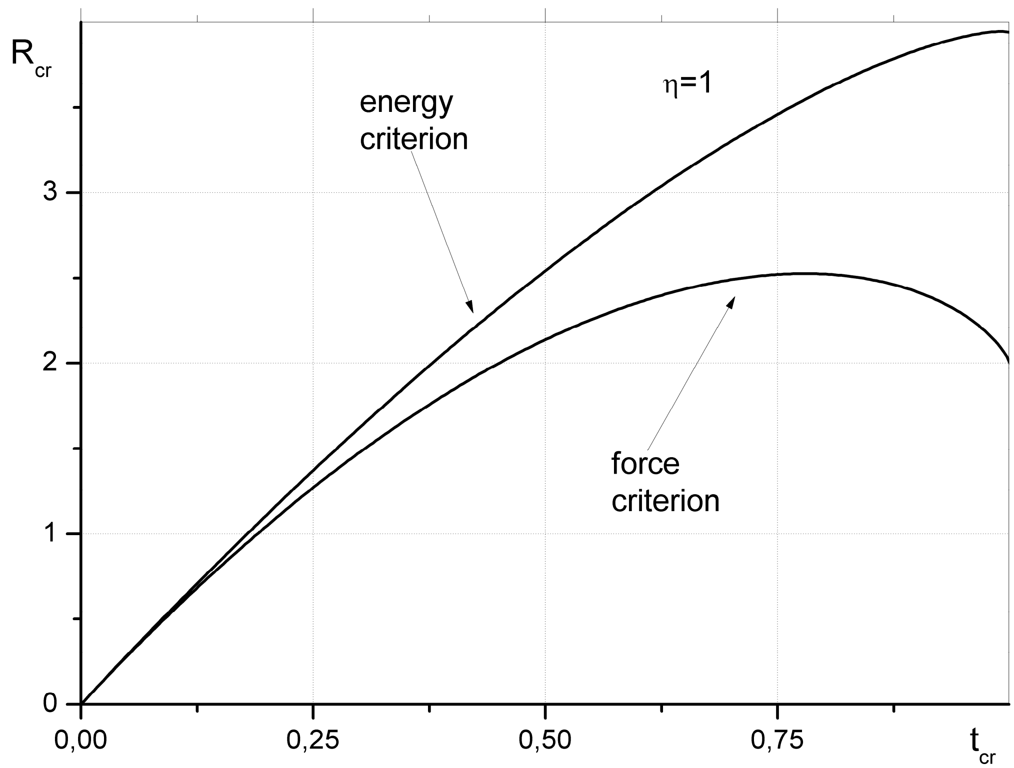

Fig.2. Dependence of R_{cr} on the relative bridged zone size, the criterion comparison, $\eta = 1$.

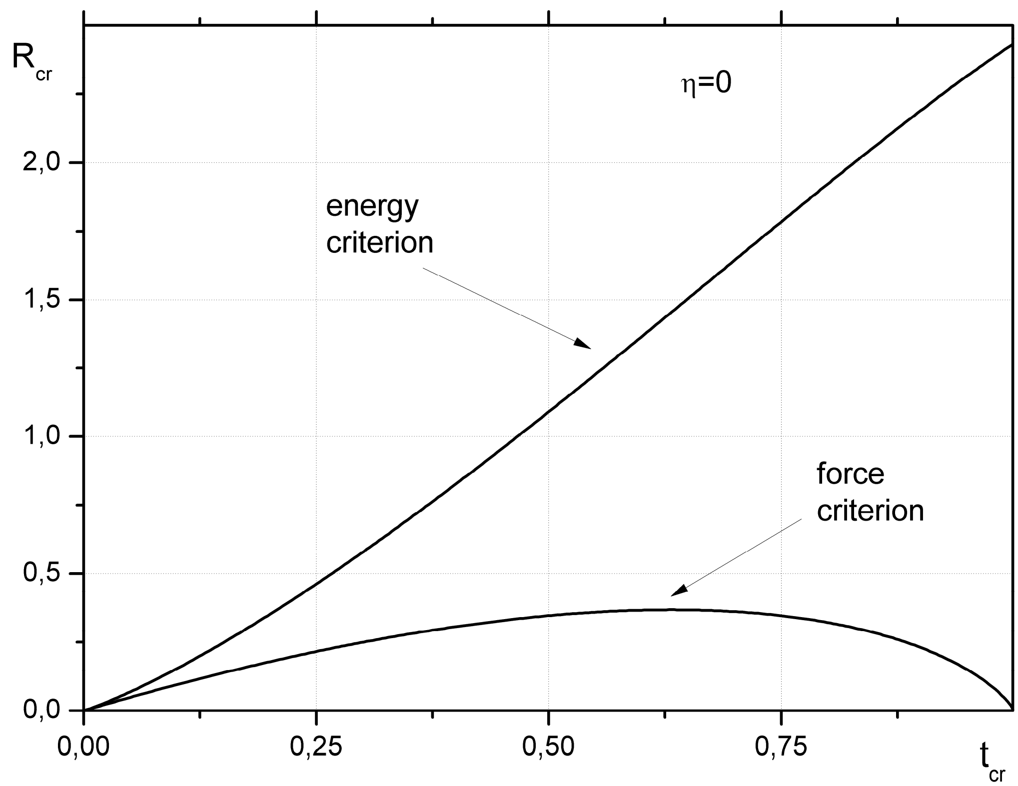

Fig.3. Dependence of R_{cr} on the relative bridged zone size, the criterion comparison, $\eta = 0$.