Crack-Interface Crack Interaction in FGM/Homogeneous Bimaterials under Thermo-Mechanical Loading: Analysis by Fracture Criteria

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Abstract. Fracture criteria for predicting of extension of an interface crack and of the crack growth direction in a bimaterial consisting of a homogeneous and a functionally graded material (FGM) with systems of internal defects are studied. The bimaterial is subjected to a heat flux and a tensile load applied at infinity. It is assumed that the thermal properties of the FGM have exponential form. Young's modulus and Poisson's ratio are assumed to be constant. In the previous papers [1-4] asymptotic analytical formulas for the stress intensity factors (SIFs) at the interface crack tips were obtained as a series of a small parameter (the ratio between sizes of the internal and interface cracks). These SIFs are used in fracture criteria to obtain the possible direction of crack propagation and critical loads. The following fracture criteria are used: the maximum circumferential stress criterion and the minimum strain energy density. The influence of the geometry of the problem (location and orientation of cracks) and the parameters of non-homogeneity of FGMs on the main fracture characteristics is investigated.

Introduction

The paper is devoted to the problem of the thermal fracture of a bimaterial consisting of a homogeneous and a functionally graded material (FGM) with an interface crack and internal defects subjected to a heat flux and a tensile load applied at infinity. It is assumed that the thermal properties of the FGM have exponential form. Young's modulus and Poisson's ratio are assumed to be constant. Thus, the material is elastically homogeneous, but thermally non-homogeneous. This kind of FGMs includes some ceramic/ceramic and ceramic/metal FGMs. The problem is studied for the case when an interface crack is much larger than internal cracks in the FGM. In the previous papers [1-4] asymptotic analytical formulas for the stress intensity factors (SIFs) at the interface crack tips were obtained as a series of a small parameter (the small parameter is equal to the ratio between sizes of the internal and interface cracks). These SIF functions contain parameters of the geometry of the problem and parameters of the non-homogeneity of the FGM.

The present work is devoted to the analysis of fracture criteria for the prediction of extension of the interface crack and of the crack growth direction in FGM/homogeneous bimaterial. From experimental and theoretical investigations it is known that the crack deflection from initial crack propagation occurs under mixed mode loading and this deflection depends on the details of Modes I and II loadings. For FGMs the near-tip mixity can arise by virtue of the property variation in the material. Besides, the interaction of cracks, defects and interfaces adds additional near tip mixity. In this connection the following fracture criteria are used: the maximum circumferential stress criterion [5,6,7] and the minimum strain energy density [8,9]. The parametric analysis shows the dependence of the initial interface crack propagation on the location and orientation of the crack systems. It is also shown that the non-homogeneity parameter of thermo-conductivity and thermal expansion coefficient notably affect the interface crack deflection angle. The influence of these parameters on the main fracture characteristics is investigated. A comparison of these results for the considered criteria is performed.

Formulation of the problem and solution

Geometry of the problem and assumptions. Fig. 1a shows the geometry of the problem. A bimaterial is composed of a FGM (denoted by number 1) and a homogeneous material (denoted by number 2). The bimaterial is perfectly bonded with the exception of an interface crack of length $2a_0$. It is assumed that the

FGM contains N cracks of length $2a_k$. Cartesian coordinates (x, y) are centered at the midpoint of the interface crack; the x-axis lies along the interface line. Local coordinate systems (x_k, y_k) are attached to each internal crack. The crack position is determined by the defect midpoint coordinate and an inclination angle to the interface, i.e. to the x-axis (Fig. 1a). The bimaterial is subjected to a heat flux of intensity q and a tensile stress P applied at infinity. The cracks are thermally isolated and traction free.

It is assumed that the thermal conductivity coefficient and the thermal expansion coefficient are

$$k_1(y) = k_0 e^{\delta y}, \qquad \alpha_1 = \alpha_0 e^{\omega y}, \tag{1}$$

where the constant k_0 is the thermal conductivity, α_0 is the thermal expansion coefficient of the interface and of material 2, δ and ω are the non-homogeneity parameters for the FGM. The Young's modulus and Poisson's ratio are assumed to be constant, $E_j = const$, $\nu_j = const$ (j = 1, 2).

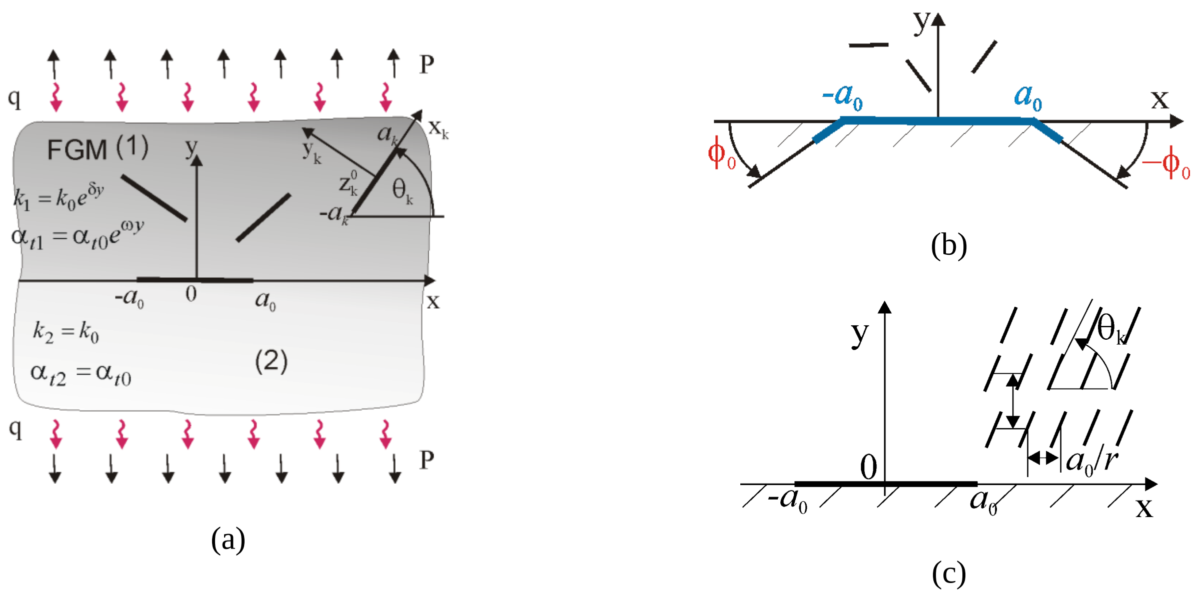


Fig. 1. (a) The geometry of the problem; (b) The angle ϕ_0 of crack deflection; (c) The scheme of locations of the interface crack and microcracks.

The uncoupled, quasi-static thermoelastic theory is applicable to this problem that is the temperature distribution is independent of the mechanical field, and the solution consists of the determination of the temperature field and the determination of the thermal stresses.

Thermal and thermoelastic problem formulations. It is supposed that the cracks are thermally isolated and continuity conditions for thermal fluxes and temperatures are fulfilled at the interface. Using the superposition principle the temperature field in the bimaterial with cracks is presented as a sum of two terms: the temperature distribution in a bimaterial in the absence of cracks and the temperature perturbation caused by the cracks. For the crack problem the temperature perturbation should be determined.

The mechanical boundary conditions are: the cracks are traction-free and the continuity conditions at the interface are assumed, i.e. the stresses are equal and displacements are equal. Using the superposition principle the problem is transformed to the problem with boundary conditions on the crack lines. Because we suppose that the material is elastically homogeneous we can use directly the method presented in [7,10]. The detailed formulation and solution of these problems can be found in [1-4].

Solution by small parameter method. The solution is derived for a special case where an interface crack is significantly larger in size than internal cracks in the FGM. The asymptotic analytical solution of the problem is obtained as a series of a small parameter which is equal to the ratio between sizes of the internal and interface cracks. The method was first suggested by Romalis and Tamuzs at 1984 and then was used for different macro-microcrack interaction problems for homogeneous materials [7,10].

It is assumed that all internal cracks in the FGM have the same size $2a_k = 2a$ (k = 1, 2, ... , N), for example, they have the characteristic size of a grain size of the material. Suppose also that $2a << 2a_0$. In this case the small parameter is $\lambda = a/a_0$ and $\lambda << 1$. The solution is obtained in closed form up to λ^2 (see [1] for details)

$$f_0(\chi) = f_{00}(\chi) + f_{02}(\chi)\lambda^2. \tag{2}$$

Here the function f_0 is the solution of thermal and thermoelastic problems. The zero-th approximation f_{00} in Eq. 2 corresponds to an isolated interface crack and the second one f_{02} is taking into account the influence of each microcrack on the interface crack. Using this solution the SIFs are obtained.

Stress intensity factors. In this work we consider elastically homogeneous materials so that we can use the classical definition of the stress intensity factor. It should be noted, that the crack tip singular field in FGMs has the same form as in homogeneous media [11] and the concept of the stress intensity factors can be also applied directly to cracks in FGMs. Besides, the interface crack between the FGM and the homogeneous material with smooth transition between these materials is also a classical crack with square-root singularities at the crack tips.

For a uniform heat flux and a tension load P acting at infinity the stress intensity factors at the interface crack tips are obtained up to λ^2 as [1,3,4]

$$k_{I0}^{\pm} - ik_{II0}^{\pm} = \delta_t^h q k_0 a_0 \sqrt{a_0}\left[\mp i + \lambda^2 \sum_{k=1}^{N} F_1(w_k, \theta_k, \delta, \omega)\right] + P\sqrt{a_0}\left[1 + \lambda^2 \sum_{k=1}^{N} F_2(w_k, \theta_k)\right] \tag{3}$$

or writing full expressions we have

$$\begin{aligned} k_{I0}^{q\pm} &= \pm\lambda^2 \delta_t^h q k_0 a_0 \sqrt{a_0}\, \frac{1}{2}\sum_{k=1}^{N} \{\mathrm{Re}(I_{k0}^T)\,\mathrm{Im}[m_{k1} - n_{k1}] \\ &\quad + \mathrm{Im}(I_{k0}^T)\,\mathrm{Re}[m_{k1} + n_{k1}] + 2\exp(\omega a_0\, \mathrm{Im}(w_k)) J_k^T(\delta)\,\mathrm{Im}[m_{k0} - n_{k0}]\}, \end{aligned} \tag{4}$$

$$\begin{aligned} k_{II0}^{q\pm} &= \pm\delta_t^h q k_0 a_0 \sqrt{a_0}\{1 + \frac{\lambda^2}{2}\sum_{k=1}^{N}\{2J_k^T(\delta)\,\mathrm{Re}[e^{i\theta_k}(w_k/(w_k^2-1)^{1/2} - 1)] \\ &+ \mathrm{Re}[I_{k0}^T]\,\mathrm{Re}[m_{k1} - n_{k1}] + \mathrm{Im}(I_{k0}^T)\,\mathrm{Im}[m_{k1} - n_{k1}] - 2\exp(\omega a_0\, \mathrm{Im}(w_k)) J_k^T(\delta)\,\mathrm{Re}[m_{k0} - n_{k0}]\}\}, \end{aligned} \tag{5}$$

and the mechanical part of SIFs

$$k_{I0}^{P\pm} = P\sqrt{a_0}\left(1 + \lambda^2 \frac{1}{2}\sum_{k=1}^{N} \mathrm{Re}(J_k m_{k1} + \bar{J}_k n_{k1})\right), \tag{6}$$

$$k_{II0}^{P\pm} = P\sqrt{a_0}\,\lambda^2 \frac{1}{2}\sum_{k=1}^{N} \mathrm{Im}(J_k m_{k1} + \bar{J}_k n_{k1}), \tag{7}$$

where $I_{k0}{}^T$, $J_k{}^T$ and J_k are

$$I_{k0}^T = \frac{1}{2}\left[\frac{e^{-2i\theta_k} - 1}{\sqrt{\bar{w}_k^2 - 1}} + \frac{1}{\sqrt{w_k^2 - 1}} - \frac{e^{-2i\theta_k}(1 - w_k\bar{w}_k)}{(\bar{w}_k^2 - 1)^{3/2}}\right], \tag{8}$$

$$J_k^T(\delta) = \exp(-\delta a_0\, \mathrm{Im}\, w_k)\cos\theta_k - \mathrm{Re}\left[e^{i\theta_k}\left(1 - \frac{w_k}{\sqrt{w_k^2 - 1}}\right)\right], \tag{9}$$

$$J_k = \frac{1}{2}\left[e^{-2i\theta_k} - 1 + e^{-2i\theta_k}\,\frac{\bar{w}_k - w_k}{(\bar{w}_k^2 - 1)^{3/2}} + \frac{w_k}{\sqrt{w_k^2 - 1}} + \frac{\bar{w}_k}{\sqrt{\bar{w}_k^2 - 1}}\right], \tag{10}$$

and m_{k1}, n_{k1}, m_{k0} and n_{k0} are

$$m_{k1} = e^{i\theta_k}\,\mathrm{Re}\left[\frac{e^{i\theta_k}}{(\overline{w}_k \mp 1)\sqrt{w_k^2 - 1}}\right],\; n_{k1} = \frac{e^{-2i\theta_k}}{2(\overline{w}_k \mp 1)\sqrt{\overline{w}_k^2 - 1}}\left[(w_k - \overline{w}_k)\frac{2\overline{w}_k \pm 1}{\overline{w}_k^2 - 1} + e^{2i\theta_k} - 1\right],$$

$$m_{k0} = e^{i\theta_k}\left[1 - \mathrm{Re}\left(\frac{\sqrt{w_k^2 - 1}}{\overline{w}_k \mp 1}\right)\right],\; n_{k0} = \frac{e^{-i\theta_k}}{2}\frac{w_k - \overline{w}_k}{(\overline{w}_k \mp 1)\sqrt{\overline{w}_k^2 - 1}}. \tag{11}$$

Besides $w_k = z_k^0 / a_0$ is the non-dimensional complex coordinate of the midpoint of the cracks. In Eqs. 15–22 "Re" and "Im" denote the real and imaginary parts of complex numbers correspondingly. In Eqs. 3-7 the upper part of the "$\pm$" or "$\mp$" signs refers to the right tip and the lower part to the left tip of the crack.

For a single crack (without microcracks) subjected to a heat flux normal to the crack surfaces the thermal stress intensity factors are given as [5]

$$k_I^{\pm} = 0,\quad k_{II}^{\pm} = \pm\delta_t^h q k_0 a_0 \sqrt{a_0} \tag{12}$$

and for the crack under the tensile load P remotely applied normal to the crack the stress intensity factors are

$$k_I^{\pm} = P\sqrt{a_0},\quad k_{II}^{\pm} = 0. \tag{13}$$

The interaction of cracks leads to mixed mode conditions in the interface crack surfaces, i.e. $k_I \neq 0$ in the first case and $k_{II} \neq 0$ in the second one. The influence of both thermal and mechanical loading results in mixed-mode conditions near the interface crack.

Fracture criteria and direction of crack propagation

From experimental and theoretical investigations of cracks under mixed-mode loading, it is known that the cracks deviate from their initial propagation direction. For prediction of the crack growth and direction of this growth a fracture criterion should be applied. Two criteria will be considered: the maximum circumferential stress criterion [5,6,7] and the criterion based on the strain energy density function [8,9].

Maximum circumferential stress criterion. Using the maximum circumferential stress criterion (see for references [5,6,7]) the direction of the initial crack propagation (Fig. 1b) is evaluated as

$$\phi = 2\arctan\left[\left(k_I - \sqrt{k_I^2 + 8k_{II}^2}\right)/4k_{II}\right] \tag{14}$$

and the critical stresses can be calculated from the expression

$$\cos^3(\phi_0/2)\left(k_I - 3k_{II}\tan(\phi_0/2)\right) = K_{Ic}/\sqrt{\pi}. \tag{15}$$

Here K_{Ic} is the fracture toughness of the material.

For a single interface crack under heat flux the SIF factor k_I is equal to zero and Eq. 14 gives the fracture angle $\phi_0 \approx \mp 70.5^\circ$ (the upper sign is for the right crack tip, the lower – for the left one). The initial direction of crack propagation in the general case is determined from Eq. 14 by substitution of the SIFs $k_I^{\pm}$ and $k_{II}^{\pm}$ from Eqs. 4-7. For the case of a heat flux some results for the fracture angle at the interface crack tips in FGM/homogeneous bimaterials were presented in [2].

Strain energy density criterion. Now the strain energy density criterion is used for fracture interpretation of the results. In [8,9] the minimum strain energy density factor criterion was introduced.

The local strain energy density is given by $dW/dV = S/r$. Based on the stress intensity factor solutions k_I and k_{II}, the strain energy density (SED) factor S(k_I, k_{II}) is defined as

$$S(k_I, k_{II}) = a_{11}k_I^2 + 2a_{12}k_I k_{II} + a_{22}k_{II}^2, \tag{16}$$

where

$$\begin{aligned} a_{11} &= \frac{1}{16\mu}(1+\cos\phi)(\kappa-\cos\phi), \quad a_{12} = \frac{1}{16\mu}\sin\phi[2\cos\phi-(\kappa-1)], \\ a_{22} &= \frac{1}{16\mu}[(\kappa+1)(1-\cos\phi)+(1+\cos\phi)(3\cos\phi-1)] \end{aligned} \tag{17}$$

and $\kappa = 3-4\nu$ is for plain strain, μ is the shear modulus and ν is Poisson's coefficient. In Eq. 17 ϕ is the polar angle of the polar coordinate system (r, ϕ) with the origin at the crack tip. SED factor determines the mixed mode effects, i.e., the direction of crack initiation as well as the critical condition under which the crack would initiate.

The criterion can be expressed mathematically as

$$\frac{\partial S}{\partial \phi} = 0, \quad \frac{\partial^2 S}{\partial \phi^2} > 0. \tag{18}$$

The crack growth occurs when the SED factor reaches critical value, i.e. $S = S_{cr}$ for $\phi = \phi_0$. Here S is Eqs. 16 and 17.

In the SED criterion the angle ϕ_0 depends on Poisson's ratio. For Mode II cracks the maximum stress criterion predicts a fixed angle $\phi_0 \approx \mp 70.5^0$, which corresponds to a material with zero Poisson's ratio in SED criterion. For $\nu = 0.3$ the angle of crack propagation is $\phi_0 \approx \mp 82.3^0$.

Parameters of materials

The formulae for SIFs Eqs. 4-11 and hence the formulae for other fracture characteristics Eqs. 14-18 contain geometrical parameters of the problem, such as, length of cracks, coordinates of the centers of cracks w_k and inclination angles θ_k of small cracks to the interface, and parameters of materials, the main of them are inhomogeneity parameters of thermal conductivity and of the thermal expansion coefficient. The influence of these parameters on the fracture characteristics at the interface crack tips can be investigated.

The values of the inhomogeneity parameters are estimated based on the following considerations. From exponential form of the thermal conductivity Eq. 1 the inhomogeneity parameter δ is $\delta = (1/y)\ln(k_1/k_2)$. That means, the value depends on the ratio of material properties and the value of y. We consider an infinite domain and it is supposed that the value of δ changes slowly, we take $-1.0 \le \delta \le 1.0$. The same concerns the inhomogeneity parameter of the thermal expansion coefficient ω ($\omega = (1/y)\ln(\alpha_{t1}/\alpha_{t2})$).

Tables 1 and 2 give the thermal properties [12, 13] of some FGM/homogeneous material combinations and corresponding values of the inhomogeneity parameters δ and ω. The Young's moduli of these materials are similar.

Table 1. Ceramic/Ceramic FGMs

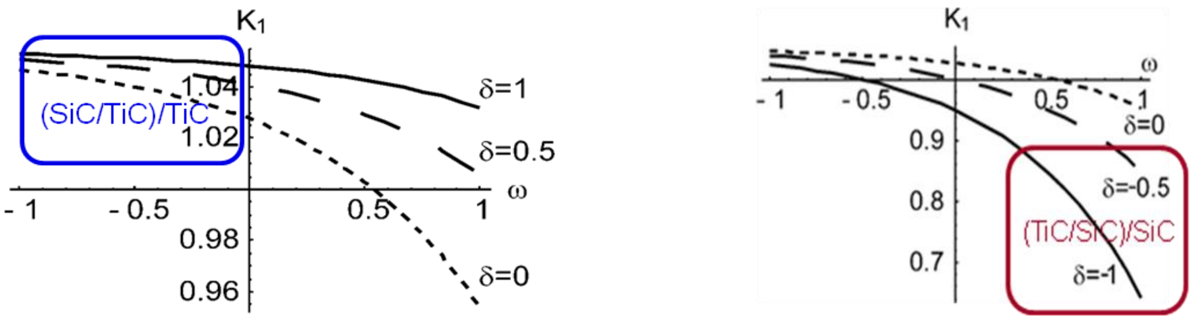
FGM/H ($MoSi_2$/ Al_2O_3)/ Al_2O_3 (Molybdenum disilicide $MoSi_2$, alumina Al_2O_3)						
	Thermal expansion coeff. $*10^{-6}$ K^{-1}			Thermal conductivity $Wm^{-1}K^{-1}$		
$MoSi_2$	α_{t1}	5	$\alpha_{t1}/\alpha_{t2}=1$	k_1	52	$k_1/k_2>1$
Al_2O_3	α_{t2}	5	$\omega=0$	k_2	25	$\delta>0$
FGM/H (Al_2O_3/ $MoSi_2$)/ $MoSi_2$			$\omega=0$			$\delta<0$
FGM/H (SiC/ $MoSi_2$)/ $MoSi_2$ (Silicon carbide SiC, molybdenum disilicide $MoSi_2$)						
	Thermal expansion coeff. $*10^{-6}$ K^{-1}			Thermal conductivity $Wm^{-1}K^{-1}$		
SiC	α_{t1}	4	$\alpha_{t1}/\alpha_{t2}<1$	k_1	60	$k_1/k_2>1$
$MoSi_2$	α_{t2}	5	$\omega<0$	k_2	52	$\delta>0$
FGM/H ($MoSi_2$/ SiC)/ SiC			$\omega>0$			$\delta<0$
FGM/H (SiC/ TiC)/ TiC (Silicon carbide SiC, titanium carbide TiC)						
	Thermal expansion coeff. $*10^{-6}$ K^{-1}			Thermal conductivity $Wm^{-1}K^{-1}$		
SiC	α_{t1}	4	$\alpha_{t1}/\alpha_{t2}<1$	k_1	60	$k_1/k_2>1$
TiC	α_{t2}	7	$\omega<0$	k_2	20	$\delta>0$
FGM/H (TiC/ SiC)/ SiC			$\omega>0$			$\delta<0$

Table 2. Ceramic/metal FGMs

FGM/H (ZrO_2/ Ni)/ Ni (Zirconia ZrO_2, nickel Ni)						
	Thermal expansion coeff. $*10^{-6}$ K^{-1}			Thermal conductivity $Wm^{-1}K^{-1}$		
ZrO_2	α_{t1}	10	$\alpha_{t1}/\alpha_{t2}<1$	k_1	2	$k_1/k_2<1$
Ni	α_{t2}	18	$\omega<0$	k_2	90	$\delta<0$
FGM/H (Ni/ ZrO_2)/ ZrO_2			$\omega>0$			$\delta>0$
FGM/H (ZrO_2/ Steel)/ Steel						
	Thermal expansion coeff. $*10^{-6}$ K^{-1}			Thermal conductivity $Wm^{-1}K^{-1}$		
ZrO_2	α_{t1}	10	$\alpha_{t1}/\alpha_{t2}<1$	k_1	2	$k_1/k_2<1$
Steel	α_{t2}	12	$\omega<0$	k_2	20	$\delta<0$
FGM/H (Steel/ ZrO_2)/ ZrO_2			$\omega>0$			$\delta>0$

Numerical results

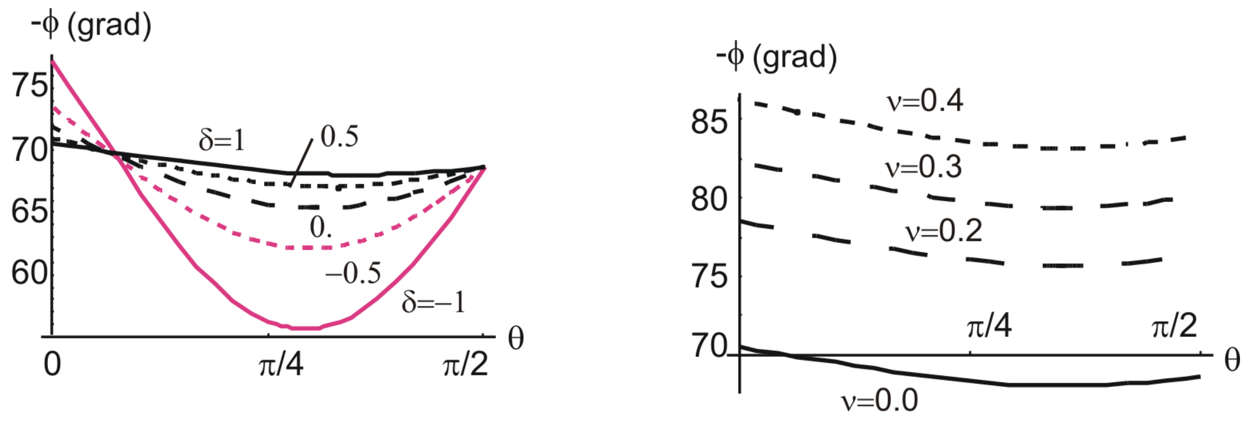
The influence of different arrays of microcracks on the thermal SIFs at the interface crack was investigated in the case of a heat flux in [1-3] and in the case of thermo-mechanical loading some results for SIFs can be found in [4]. It was assumed that all microcracks have the same angle of inclination θ to the x-axis. The microcrack centers were presented by $x_n = a_0 n/r, y_n = a_0 m/s$ (n, m = ±1, ±2,...), with $r = s = 5$, $w_n = (x_n + i\, y_n)/\, a_0$ (Fig. 1c). One of the calculation schemes of the system of microcracks in the FGM is shown in Fig. 1c. SIFs $k_{I,II}$ are normalized by k^0 and denoted by K_1 and K_2 in the figures. k^0 is $|\, k_{II}\,|$ Eq. 12 in the case of only thermal loading and k_I Eq. 13 in the case of tensile loading. It is supposed that $k^0 = |\, k_{II}\,| = k_I$ in the case when both thermal and mechanical loads are applied. The calculations were performed with $\lambda = 0.1$. The non-dimensional inhomogeneity parameters of thermal conductivity and of thermal expansion are δa_0 and ωa_0, but in the figures the designation δ and ω is used.



(a) (b)

Fig. 2. The case of a heat flux and a tensile load. Influence of system of microcracks (Fig. 1c with θ=0) on SIFs K_1 at the right interface crack tip for (a) δ>0 and (b) δ<0.

Fig. 2 shows SIF K_1 at the right interface crack tip as functions of the inhomogeneity parameter ω and for different δ (the angle $\theta = 0$) for both thermal and mechanical loadings. Because the mechanical part of the SIFs does not depend on inhomogeneity parameters, the curves of K_1 are similar as in Fig. 6 in [3], but the values of K_1 are different for these two cases. The difference in values of K_1 due to change of δ is up to 35% (maximum is reached for ω = 1). In Fig. 2 the value of K_1 is positive for all parameters. A part of K_1 is larger than 1 and a part is less than 1, $K_1 = 1$ corresponds to the value for a single crack. The example of an FGM/homogeneous bimaterial is (SiC/TiC)/TiC (Fig. 2a), and in this case a system with cracks increases K_1, but for (TiC/SiC)/SiC (Fig. 2b) the same system of cracks decreases K_1 (at the same loading conditions).



(a) (b)

Fig. 3. The case of a heat flux. The fracture angle ϕ as function of inclination angle θ at the right interface crack tip: (a) the maximum stress criterion, ω=1; (b) SED for different ν, δ=1 and ω=1.

Fig. 3a presents results for the fracture angle ϕ calculated by Eq. 14 for ω=1 and different δ and Fig. 3b – by Eq. 16-18 with ω=1 and δ=1. The case $\nu = 0$ in Fig. 3b corresponds to the maximum stress criterion prediction. Fig. 4 shows non-dimensional critical loads (Eq. 15) for different δ and for ω=0,1.

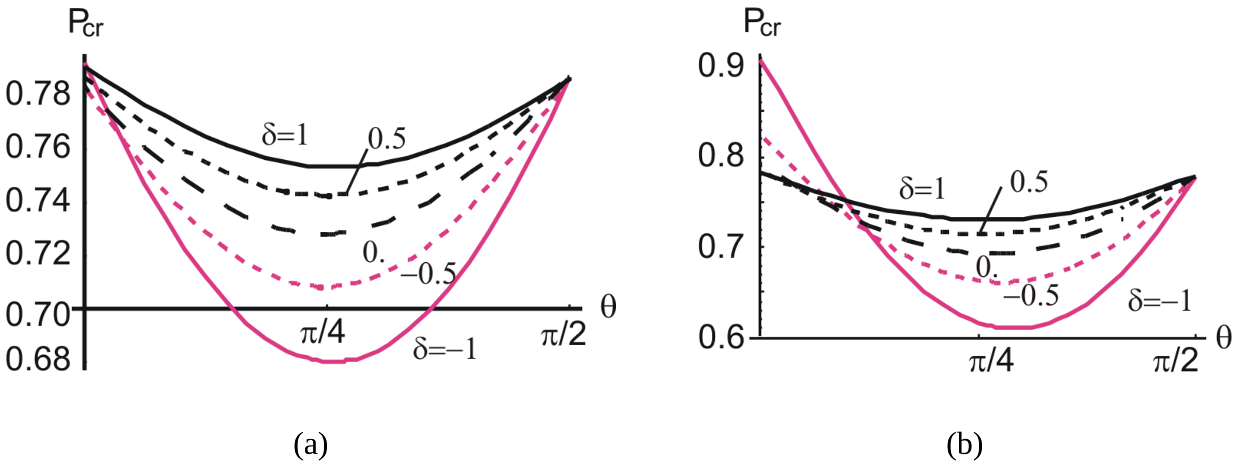


Fig. 4. The case of a heat flux. The critical load P_{cr} as function of inclination angle θ at the right interface crack tip: (a) ω=0 and (b) ω=1 (maximum stress criterion).

Conclusions

Mathematical modeling of the fracture processes in the vicinity of an interface crack in functionally graded/ homogeneous bimaterials with internal defects subjected to tensile loading and a thermal flux remotely applied normal to the interface surface was performed. Asymptotic analytical solution for a special case when an interface crack is significantly larger in size than internal cracks in the FGM was used in two fracture criteria for the determination of the direction of the initial crack propagation and critical loads. The main fracture characteristics were obtained as functions of geometry of the problem and non-homogeneity parameters of FGMs. Examples of actual material combinations were presented, e.g. ceramic/ceramic TiC/SiC, $MoSi_2/Al_2O_3$ and $MoSi_2$/SiC, and also ceramic/metal FGMs, e.g., zirconia/nickel and zirconia/steel. Optimal crack configurations can be determined at which the stress intensity factors at the interface crack tips possess a minimal value or at which the critical loads are maximal and, accordingly, the interface crack failure will be minimal.

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