Weak Fiber/Matrix Interface Degradation and the Crack Tip Energy Dissipation.

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Keywords: crack tip, weak fiber coating, composite's fracture.

Introduction.

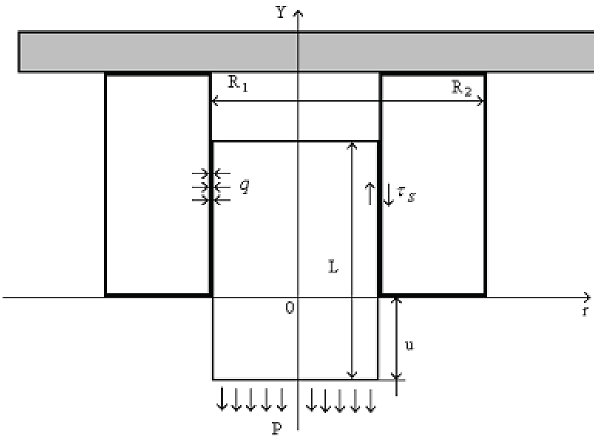
The mechanism of pull-out of brittle fibers in composites with brittle matrix and non-ideal interface is analyzed. The influence of effect on the composite's fracture toughness characteristics is studied. Model of composite fiber/matrix interface degradation in conditions of the steady state creep under cyclic loading is proposed.

Extent:

Composite materials such as oxide/oxide composites with brittle fiber and brittle matrix are perspective modern materials, which can be used in high temperature regimes. Low steady-state creep rates and high crack propagation durability are two sides of their manufacture problem. Weak fiber/matrix interface stimulates high energy dissipation near the crack tip due the pull-out mechanism but very weak interface provides an interesting and dramatic effect of its rapid degradation during the 5-10 cycle loadings. So, we would like to propose a model of cyclic interface degradation and to analyze optimal conditions for fracture energy dissipation in brittle/brittle composites due the fibers pull-out.

Abrasive deterioration model.

Let us consider a cylindrical piece of the fiber extended with tension P from a cylindrical site of a matrix, fixed from above. Fiber and a matrix forces by Coulomb's law of friction due the normal tensions in fiber/matrix boundary. The primary normal tension appears due the manufacture process and is considered set.



During the pull-out process normal tensions are redistributing and the fiber/matrix contact surface is deteriorating simultaneously, that leads to change of boundary conditions and further friction weakening.

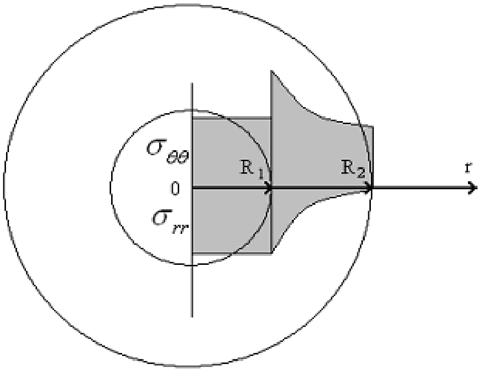
Coulomb's law of friction. $\tau_S = \mu \cdot q$

Depth of deterioration of a matrix in a surface of contact corresponds to model of abrasive deterioration: $u_W = l_i \int_0^v \tau_S dv$

Distribution of normal pressure in a direction of an axis 0y goes from:

$$\frac{d\sigma_m}{dy} = \frac{2\gamma}{R_1}\tau_S$$

$$\frac{d\sigma_f}{dy} = -\frac{2}{R_1}\tau_S$$



For any cross-section we have classical problem:

$$\sigma_{rr} = \frac{B}{r^2} + C$$

$$\sigma_{\theta\theta} = -\frac{B}{r^2} + C$$

$$\varepsilon_{\theta\theta} = \frac{1}{E_m}\left(\sigma_{\theta\theta} - \upsilon_m(\sigma_{rr} + \sigma_m)\right)$$

$$u_r = r\varepsilon_{\theta\theta} = -\frac{1+\upsilon_m}{E_m r}B + \left(\frac{1-\upsilon_m}{E_m}C - \frac{\upsilon_m}{E_m}\sigma_m\right)r$$

$$\sigma_{rr} = \sigma_{\theta\theta} = A$$

$$u_r = \left(\frac{1-\upsilon_f}{E_f}A - \frac{\upsilon_f}{E_f}\sigma_f\right)r$$

with boundary conditions depending on depth of deterioration:

$$u_{r+} - u_{r-} = u_0 - u_w$$

$$\sigma_{rr+} = \sigma_{rr-}$$

Therefore we obtain normal tension dependence of normal stresses and depth of deterioration:

$$q(y,u) = q(u_W, \sigma_f, \sigma_m) = \frac{E_m u_0 / R_1}{\alpha\left(1-\upsilon_f\right)+1+2\gamma+\upsilon_m} - \frac{E_m u_W / R_1}{\alpha\left(1-\upsilon_f\right)+1+2\gamma+\upsilon_m} +$$

$$\frac{\upsilon_m \sigma_m}{\alpha\left(1-\upsilon_f\right)+1+2\gamma+\upsilon_m} - \frac{\alpha\upsilon_f\sigma_f}{\alpha\left(1-\upsilon_f\right)+1+2\gamma+\upsilon_m}$$

Using following

$$u_W = l_i \mu \int_0^u q(y,t)dt$$

$$\sigma_m(y) = \int_0^y \frac{2\gamma}{R_1}\tau_S dy = \frac{2\mu\gamma}{R_1}\int_0^y q(t,u)dt$$

$$\sigma_f(y) = \frac{P}{\pi R_1^2} - \frac{\sigma_m(y)}{\gamma}$$

finally be get the defining equation

$$q(y,u) = q_0 - A_1\int_0^u q(y,t)dt - A_2\int_0^{L-u} q(t,u)dt + A_3\int_0^y q(t,u)dt$$

$$u \in [0;L], y \in [0;L-u]; q(y,u) \geq 0$$

We can solve this equation numerically or can involve simplifications such as: Linearization for tension *q:*

$$q(y,u) = K(u)y + M(u)$$

Thus we have:

$$\begin{cases} K(u) = -A_1\int_0^u K(t)dt + B_3 \\ M(u) = q_0 - A_1\int_0^u M(t)dt - A_2K(u)\dfrac{(L-u)^2}{2} - A_2M(u)(L-u) \end{cases}$$

Solution of this system can be obtained analytically:

$$K(u) = B_3e^{-A_1u}$$

$$M(u) = C\left(1 + A_2L - A_2u\right)^{\frac{A_3-A_2}{A_2}} +$$
$$+\frac{A_2B_3}{2}\left(1 + A_2L - A_2u\right)^{\frac{A_3-A_2}{A_2}}\int_0^u \frac{(L-t)(A_1L + 2 - A_1t)}{\left(1 + A_2L - A_2t\right)^{\frac{A_3}{A_2}}}e^{-A_1t}dt$$

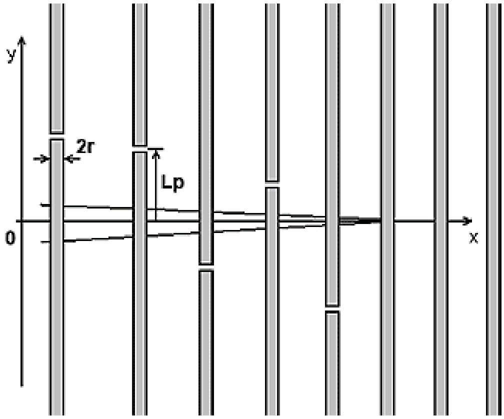
$$C = \frac{2q_0 - A_2B_3L^2}{2(1 + A_2L)^{\frac{A_3}{A_2}}}$$

The second simplification is averaging of normal tensions *q* by *y,* after that we have

$$q_1(u) = q_0 - A\int_0^v q_1(u)du$$

therefore $q_1(u) = q_0 e^{-Au}$

Pull-out energy dissipation. By means of the two cylinders shift problem solution received in the first part we can obtain energy dissipating definition due the fiber pull-out mechanism. Let us consider a unidirectional brittle/brittle composite and and classic cross-plane crack, r - is fiber radius, Lp – is average length of broken fiber due the crack approach. Fiber strength is random value and depends on defects randomly distributed along a fiber and presented by Weibull distribution. τ s - is initial stress along the pull-out fiber.



$$P(y) = 1 - \exp\left[-\frac{1}{L_0} \int_0^y \left(\frac{\sigma(t)}{\sigma_0} \right)^\beta dt \right]$$

Stresses in the fiber we take like in [1]

$$\sigma(y) = \begin{cases} \sigma(0) - \dfrac{2y\tau}{R_1}, y \in (0; L_D) \\ \sigma(0) - \dfrac{2\tau L_D}{R_1} - \left(\sigma(0) - \dfrac{2\tau L_D}{R_1} - \sigma_\infty \right) (L_C - L_D)^{-1} (y - L_D), y \in (L_D; L_C) \end{cases}$$

Therefore

$$\tilde{\sigma} = \int_{\sigma_\infty}^{\infty} \sigma_C \frac{d\tilde{P}}{d\sigma_C} d\sigma_C$$

$$L_p = \int_0^{L_C} y \frac{dP(y)\big|_{\sigma_C = \tilde{\sigma}}}{dy} dy$$

After that we analyze probability of fiber breakage due the pull-out out of the matrix

$$P(\sigma_C)=P(L_P)=1-\exp\left[-\frac{1}{L_0}\int_0^{L_D}\left(\frac{\sigma(t)}{\sigma_0}\right)^{\beta}dt\right]\exp\left[-\frac{1}{L_0}\int_{L_D}^{L_P}\left(\frac{\sigma(t)}{\sigma_0}\right)^{\beta}dt\right]=$$

$$1-\exp\left[\frac{L_D\left(\left(\sigma_C-\dfrac{2\tau L_D}{R_1}\right)^{\beta+1}-\sigma_C^{\beta+1}\right)}{L_0\left(\dfrac{2\tau L_D}{R_1}\right)\sigma_0^{\beta}(\beta+1)}\right]\exp\left[\frac{(L_D-L_P)\left(\sigma_C-\dfrac{2\tau L_D}{R_1}\right)^{\beta}}{L_0\sigma_0^{\beta}(\beta+1)}\right]$$

$$L_{p1}=\int_0^{L_P} y\frac{dP(y)|_{\sigma_C=\tilde{\sigma}_2}}{dy}dy$$

Finally, we can calculate an pull-out energy dissipation range and determine the optimal conditions for its maximum approaching:

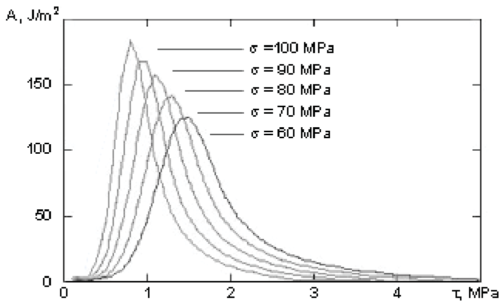
$$A(\sigma_1,\tau)=\frac{2V_f\tau}{R_1}\left[\frac{e^{-Ku_\alpha}}{K}\left(u_\alpha-L+\frac{1}{K}\right)+\frac{L}{K}-\frac{1}{K^2}\right]=$$

$$=\frac{2V_f\tau}{R_1}\left[\frac{e^{-K\frac{L_C\sigma_1}{2E}}}{K}\left(\frac{L_C\sigma_1}{2E}-L_{P1}(\sigma_1,\tau)+\frac{1}{K}\right)+\frac{L_{P1}(\sigma_1,\tau)}{K}-\frac{1}{K^2}\right]$$

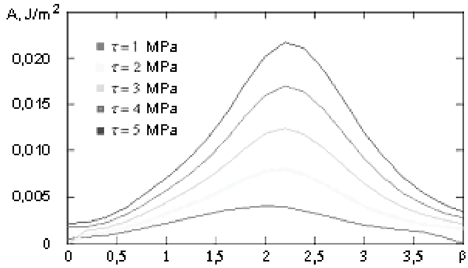
An example calculation for concrete data

Fiber		Matrix		Specimen	
r	$0.5\cdot10^{-4}m$	$10^8\,\Pi a$	$10^8\,Pa$	L_C	$10^{-1}m$
σ_0	$5\cdot10^8\,Pa$			τ	$5\cdot10^6\,Pa$
L_0	$10^{-2}m$			V_f	$1/3$
β	3,6			V_m	$2/3$
E_f	$10^9\,Pa$				

Average length of broken fiber and, correspondingly, pull-out lenth in unidirectional composite with cross-plane crack is estimated. The brunch of optimal Weibull parametr β is established as 2<β<2,5.



Area of effective maximum energy dissipation dependig on the weakness of interface is estimated. But the certain shift in interface properties can completely reduce this effect.



So, we can compare maximum value of pull-out energy dissipation and energy of ceramic matrix destruction.

Ceramic matrix destruction energy		Fiber pull-out energy	
SiC	$10\,J/m^2$	$\sigma_1 = 50\,MPa, \tau = 2\,MPa$	$100J/m^2$
CAS	$25\,J/m^2$	$\sigma_1 = 100\,MPa, \tau = 1\,MPa$	$150\,J/m^2$

Summary.

Thus, fiber pull-out process at the given stage seems very perspective for manufacturing new generation composite materials. However course of this process strongly depends on properties of a material, in particular fiber coating and distribution of fiber strength properties. For example, as shown, fibers without defects are less perspective than fibers with certain defects distribution.

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