The influence of vertex singularities on fatigue crack shape

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Abstract. Due to the existence of vertex singularity at the point where the crack intersects the free surface, stress distribution around the crack tip and the type of the singularity is changed. In the interior of the specimen the classical singular behaviour of the crack is dominant and can be described using generally known analytic equations. Contrary to this, at the free surface or in the boundary layer close to the free surface the vertex singularity is significant. The influence of vertex singularity on crack behaviour and crack shape for a three-dimensional structure is described in this paper. To estimate the influence of the free surface on fatigue crack growth rate the model of a general singular stress concentrator was applied. Using this concept of the generalized stress intensity factor it is possible to estimate the behaviour of a crack with general singular stress field. The estimated fatigue crack shape can help to provide a more reliable estimation of the fatigue life of the structures considered.

Introduction

Fatigue crack growth in three dimensional structures is considered in this article. This process can be described in terms of linear elastic fracture mechanics. The crack stress field is usually characterized by a classical stress intensity factor, which is related to square-root stress singularity. But this kind of singularity is generally not present close to the intersection between crack front and free surface. In this area the stress distribution is influenced by vertex singularity [1,2]. Based on experimental investigations [3,4], it can be concluded that the crack front will be curved corresponding to a particular Poisson's ratio (i.e. to the particular vertex singularity).

For a straight crack front and semi-infinite three dimensional body the stress singularity exponent in the vertex point can be found using separation variable technique or variation principle [1,2]. According to that results, the power of the vertex singularity is weaker than 0.5 and for particular Poisson's ratios varies between 0.5 (corresponding to ν= 0) and 0.33 (corresponding to ν= 0.5). Change in the singularity exponent between the interior of the structure and the free surface is continuous and can be estimated using 3D numerical calculation [5].

The aim of the paper is to estimate the influence of the vertex singularity on the shape and rate of the fatigue crack in a three-dimensional structure. Numerically obtained distribution of the stress singularity exponent through the thickness of the specimen provides us with an indication of the crack behaviour close to the free surface.

Stress singularity exponent

To describe the behaviour of the fatigue crack near the vertex point the stress distribution around the crack tip has to be known. Following the literature, the singularity exponent p along the crack front in three dimensional bodies changes from p = 0.5 in the middle of the specimen to a value corresponding to a particular vertex singularity and depending on Poisson's ratio ν. In this case the stress and displacement distribution depends on the distance r from the crack tip as $\sigma_{ij} \approx r^{-p}$ and

$u_i \approx r^{1-p}$. Corresponding to linear elastic fracture mechanics of general singular stress concentrators, the stress distribution around a crack tip in each single plane perpendicular to crack front can be generally expressed as

$$\sigma_{ij} = \frac{H_I}{r^p} \cdot f_{ij}(p,\theta), \tag{1}$$

where H_I is a generalized stress intensity factor, p is a singularity exponent and $f_{ij}(p,\theta)$ are corresponding shape functions. r,θ are local polar coordinates with the origin at the point on the crack front. Based on these assumptions, singularity exponent p along the crack front can be estimated numerically by direct method using log-log regression analysis [5,6]. The used finite element software ANSYS is based on deformation version of finite elements and therefore displacement regression was used for the estimation of the singularity exponent in a three dimensional body, see Fig.1.

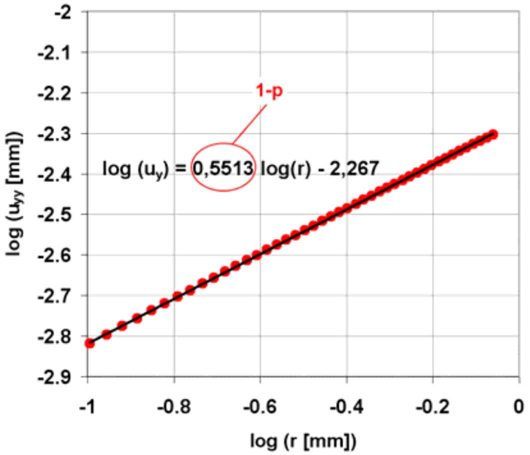

Fig.1. Typical example of displacement regression of the stress singularity exponent in 3D body

Determination of the stress singularity exponent based on regression analysis of the stress field is accurate only for sufficiently fine mesh around the crack front. Therefore, it is necessary to make several calculations for different mesh densities to check that further mesh refinements have no influence on calculated results.

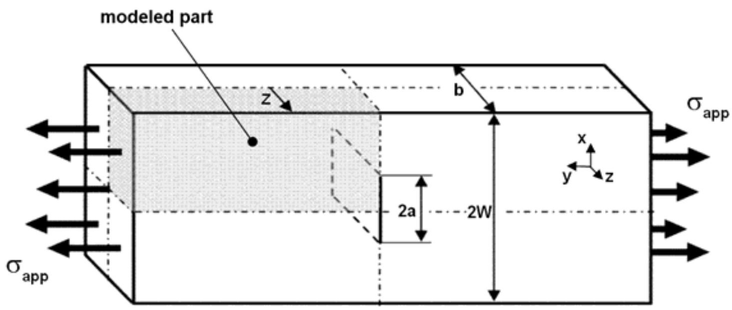

Fig.2. Model used for finite element calculations

Numerical model with straight crack

The effect of the vertex singularity on the stress field around the crack front under small scale yielding conditions in the sense of linear elastic fracture mechanics was assessed using a middle tension (MT) specimen, see Fig.2. The straight crack perpendicular to the free surface is considered. The dimensions of the MT specimen were: crack length $2a$ = 25 mm, a/W = 0.5 and thickness b = 10 mm. Material properties were considered homogenous and isotropic. Uniform applied tensile stress was applied, so only loading mode I was considered. Exploiting the symmetry in the specimen geometry and loading, only one-eighth of the MT-specimen was modeled by finite element analysis (FEM) and the stress and displacement distribution were calculated.
Displacements from a finite element analysis were used to estimate the power of the singularity using log-log regression. Variation of the singularity exponent along the crack front for MT specimen and for Poisson's ratio in the interval 0-0.5 is shown in Fig.3.

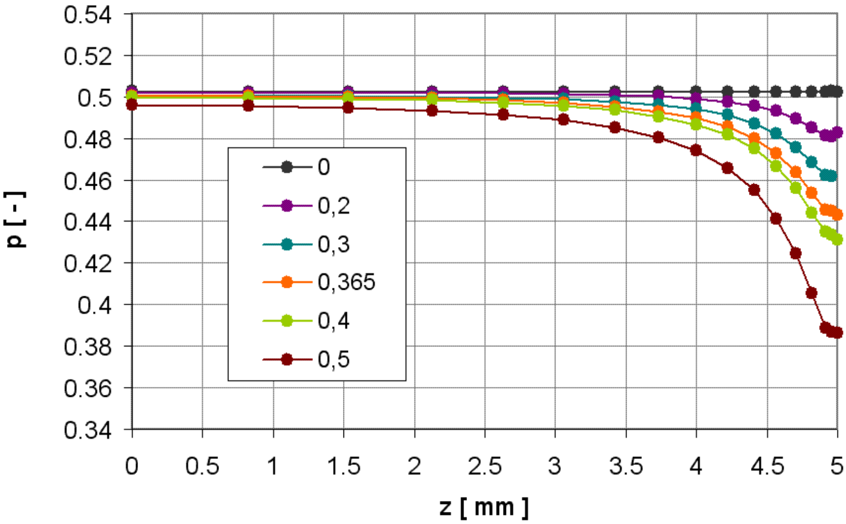

Fig.3. Variation of stress singularity exponent along the crack front for different Poisson's ratios.

As expected, the singularity exponent is close to 0.5 in the center of the specimen (z = 0). At the point where crack front intersects the free surface (z = 5) the numerically obtained values of the singularity exponents correspond to Bažant [1] and Benthem [2]. For ν = 0 the value of the stress singularity exponent is p=0.5 even in vertex point and crack behaviour is similar to the classical 2D solution [7] and can be described by the stress intensity factor. If the value of Poisson´s ratio increases the value of the vertex singularity decreases. Moreover, the thickness of the surface region which is influenced by vertex singularity increases as well. The results of numerical simulations for various thickness of the specimen [5,8] show that the thickness of the surface region influenced by the vertex singularity does not depend on the specimen thickness or geometry, but depends on Poisson´s ratio only.
Fatigue crack growth rate in vertex point, where $p < ½$, can be estimated using the generalized stress intensity factor H_I, see equation (1). To model crack behavior in this case an analogy between the stress distribution around the vertex point and V-notch has been used. The V-notch represents a general singular stress concentrator with stress singularity exponent dependent on notch opening angle α. Note that stress and displacement distribution around the tip of the V-notch are

characterized by a stress singularity exponent from the interval $0 < p \leq 0.50$. To prove the similarity of the stress field, the numerically determined shape functions for vertex singularity and those for V-notch were compared for the same values of p and a reasonable degree of agreement was obtained [8].

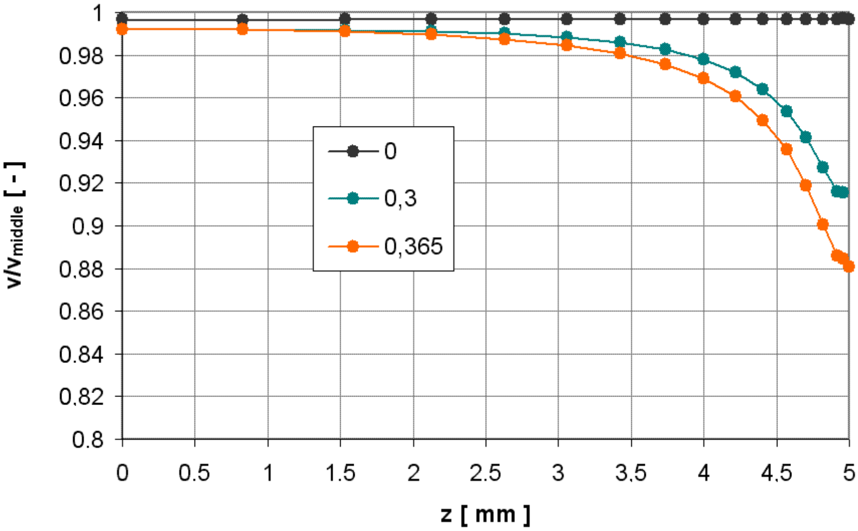

Fig.4. Variation of ratio between real fatigue crack propagation rate v and FCPR in the middle of the specimen v_{middle} along the crack front for three Poisson's ratios.

Therefore, following the quantitative similarity between shape functions in both cases the model of the V-notch in homogenous isotropic material was applied for estimation of fatigue crack propagation rate influenced by the free surface. Fatigue crack growth rate (FCPR) was estimated following the concept published by Knésl et al. in [9]. Assuming that plastic zone size is a value controlling FCPR (e.g. [10]) and based on correlation of the plastic zone parameters for the standard fatigue crack and the V-notch, the relation between the generalized stress intensity factor H_I and the effective value of the stress intensity factor K_{eff} can be expressed in the form [11]:

$$K_{eff} = \left(\frac{H_I . s^{1/2} (2\pi)^{(p-1)}}{(1+4\nu^2 - 4\nu)^p \sigma_0^{(1-2p)}} \right)^{\frac{1}{2p}}, \tag{2}$$

where H_I is a generalized stress intensity factor corresponding to mode I loading, p is a singularity exponent and σ_0 is the cyclic yield stress of the material. Function s depends on the singularity exponent and Poisson's ratio:

$$s = (1-p)^2 (3p^2 + 3p^2q^2 + 6p^2q - 12q^2p - 12qp + 12q^2 + 4 + 16\nu^2 - 16\nu), \tag{3}$$

where q is a function of the singularity exponent and V-notch angle corresponding to particular singularity exponent [11]. The equation (2) makes it possible to recalculate a generalized stress intensity factor H_I with unit MPa m^p to an effective stress intensity factor K_{eff} with standard unit

MPa m$^{1/2}$. Then the effective stress intensity factor can be used as a parameter controlling fatigue crack propagation rate according to the Paris-Erdogan law in the usual form:

$$\mathrm{v} = \frac{da}{dN} = C\left(K_{eff}\right)^{m} \quad (4)$$

where C and m are material parameters characterizing the standard fatigue crack propagation rate in the material studied.

The ratio between real fatigue crack propagation rate v and FCPR in the center of the specimen v_{middle} along the crack front are presented in Fig.4. For a specimen with Poisson's ratio 0.365 the decrease of FCPR in region close to free surface is approximately 12% in comparison with the centre of the specimen (z =0 mm). The FCPR is not constant along the crack front and therefore a hypothetical originally straight crack starts to change its shape during its propagation. The FCPR in regions closer to the free surface is slower than in the centre of the specimen and typical curved crack front observed experimentally for through cracks is created.

Numerical model with curved crack

The influence of vertex singularity on the shape of the crack front, especially in the vicinity of the crack front intersection with the free surface was examined experimentally by Heyder et al. [3,12]. For this experimental work specimens of PMMA were tested and the shape of the fatigue crack was estimated. Corresponding material parameters of PMMA were Poisson's ratio $\nu \approx 0.365$ and the Young modulus $\mathrm{E} \approx 3.6$ GPa.

To simulate this problem numerically a model of the MT specimen with curved crack front was developed. The dimensions of the MT specimen were similar to those in the previous model with straight crack (see Fig.2): crack length $2a$ = 25 mm, a/W = 0.5 and thickness b = 10 mm. Its material properties correspond to PMMA used for the experiment (Heyder et al. [3,12]). Uniform applied tensile stress was applied. Exploiting the symmetry in the specimen geometry and loading, only one-eighth of the MT-specimen was modeled by finite element analysis (see Fig.2).

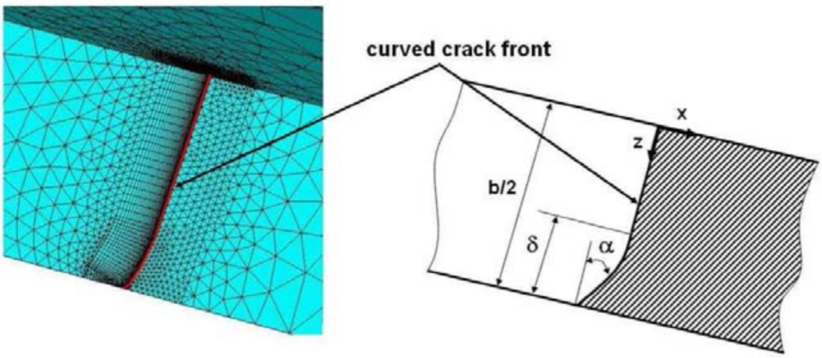

Fig.5. Example of the finite element mesh close to curved crack front and schematic description of the MT specimen cross section area.

The thickness of the boundary layer where the stress field is influenced by the vertex singularity can be estimated from dependence on Fig.3. According to previous numerical simulations [5,8], it was found that boundary layer thickness δ is dependent only on Poisson's ratio and the influence of the specimen geometry is insignificant.

In the case of investigated material (PMMA), variation of the stress singularity exponent along the crack front for different crack front angles is presented in Fig.6. If the crack front angle increases the value of the vertex singularity exponent increases as well. Therefore it can be concluded that the vertex singularity plays an important role in crack shape formation. The basic mechanism consists of a decrease in the fatigue crack propagation rate in the boundary layer influenced by the free surface and slow formation of the curved crack. If the crack front angle increases, the difference between FCPR in the middle of the specimen (p=0.5) and close to the free surface decreases. Hence, the crack front is shaped ensuring a valid square-root stress singularity in the vicinity of the intersection of the crack front and free surface.

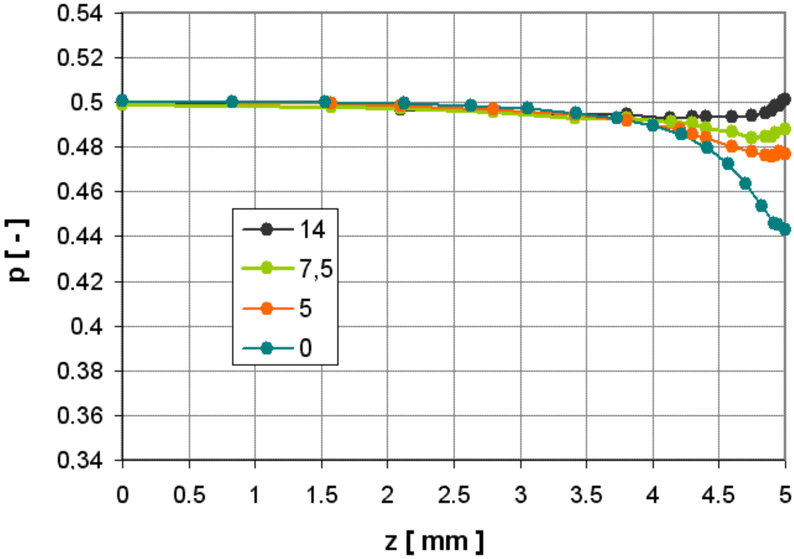

Fig.6. Variation of the stress singularity exponent along crack front for different crack front angles α. Material properties is corresponds to PMMA (Poisson's ratio is 0.365).

Based on numerical calculations for different crack front shapes an angle α for PMMA material, crack front angle 14° corresponding to square-root singularity along the whole crack front (see Fig.6.). Difference between numerically obtained stress singularity exponents along the crack front and 0.5 was smaller than 2 % in this case. Therefore, it can be concluded, that shape of the crack with crack front angle 14° is close to reality. This result is in good agreement with experimental work of Heyder et al. [3,12] and Pook [4].

Conclusions

The influence of vertex singularity in the case of a three-dimensional body with fatigue crack is described in the paper. A middle tension specimen with through crack was used for the numerical investigations. Using a log-log regression of the displacement field close to the crack front, the power of the singularity along crack front was estimated. For a hypothetically straight crack a continuous decrease of the stress singularity exponents close to the free surface was found. To estimate fatigue crack growth rate influenced by a free surface the model of a general singular stress concentrator was applied. Using the model of V-notch following Knesl's concept for estimation of the fatigue crack propagation rate [9], FCPR was estimated along the crack front. The relative change of the fatigue crack propagation rate plays an important role in crack shape formation. The basic mechanism consists in decreasing of the FCPR in the boundary layer influenced by the free surface, and slow formation of the final curved crack. When the crack front angle increases the difference between FCPR in the middle of the specimen (p=0.5) and close to the free surface decreases. The crack front is then curved so as to ensure a valid square-root stress singularity in the vicinity of the intersection of the crack front and free surface. The performed numerical simulations showed a characteristic shape of the crack front close to the free surface which leads to square-root singularity along the whole crack front. Thus, it can be concluded that in this case the classical stress intensity factor concept can be used for suitable fatigue crack propagation criterion close to the free surface.

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