# Sufficient criterion of quasi-brittle fracture for complicated stress state

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**Abstract.** A general case of proportional loading is considered when not proportional deformation of material in pre-fracture zone takes place. This is peculiar to polycrystalline solids under conditions of plasticity. When investigating, the sufficient fracture criterion for the complicated stress state is used when the material in the pre-fracture zone is subject to not proportional deformation. Inequalities have been derived for describing different mechanisms of material fracture at proportional loading: predominantly the shear fracture mechanism and tendency of material to fracture by cleavage.

## Introduction

A general case of proportional loading is considered when not proportional deformation of material in pre-fracture zone takes place. This is peculiar to polycrystalline solids under conditions of plasticity. When investigating, the sufficient fracture criterion for the complicated stress state is used when the material in the pre-fracture zone is subject to not proportional deformation. Inequalities have been derived for describing different mechanisms of material fracture at proportional loading: predominantly the shear fracture mechanism and tendency of material to fracture by cleavage.

## 1. Fracture criterion at proportional loading.

Suppose the polycrystalline solid consists of fiber-polycrystals located transversely to the front of a sharp macrocrack $2l$ in length. In the Leonov-Panasyuk-Dugdale model, an initial inner straight crack of length $2l_0$ is substituted by an imaginary crack-cut of length $2l = 2l_0 + 2\Delta$, where $\Delta$ is the length of the loaded crack part or the pre-fracture zone length, two pre-fracture zones being on the continuation of the initial crack. In constructing strength criteria, we use the concept on a fiber bundle described in [3, 4]. The bundle of fibers occupies a pre-fracture zone $\Delta$, the length of which is measured in diameters of material grains. The discrete-integral quasi-brittle fracture criterion has the form ($\Delta > 0$), [1, 2]

$$\frac{1}{kr_0}\int_0^{nr_0} \sigma_y(x,0)dx \le \sigma_1, \quad x \ge 0; \qquad \frac{1}{kr_0}\int_0^{nr_0} \tau_{xy}(x,0)dx \le \tau_1, \quad x \ge 0; \tag{1}$$

$$2v(-\Delta) = \frac{\kappa+1}{G}K_{\mathrm{I}}^0\sqrt{\frac{\Delta}{2\pi}} \le 2v^*, \quad x \le 0; \quad 2u(-\Delta) = \frac{\kappa+1}{G}K_{\mathrm{II}}^0\sqrt{\frac{\Delta}{2\pi}} \le 2u^*, \quad x \le 0. \tag{2}$$

Here $\sigma_y(x,0)$ and $\tau_{xy}(x,0)$ are normal and shear stresses on the continuation of a crack having integrable singularity; $Oxy$ is Cartesian coordinate system, the origin of the coordinate system coincides with the right crack tip; $r_0$ is the grain diameter; $n$ and $k$ are integers ($n \ge k$, $k$ is the number of bonds); $nr_0$ is the averaging interval; $2v = 2v(x)$ and $2u = 2u(x)$ are crack opening and displacement of crack flanks, respectively; $2v^*$ and $2u^*$ are critical crack opening and critical displacement of crack flanks, respectively; $\kappa = 3 - 4\nu$ for plane deformation, $\kappa = (3-\nu)/(1+\nu)$ for the plane stress state, where $\nu$ is Poisson coefficient; $G$ is the shear modulus; $K_{\mathrm{I}}$ and $K_{\mathrm{II}}$ are total SIFs in the generalized Leonov-Panasyuk-Dagdale model. Averaged stresses for the continual model $\sigma_y$ and $\tau_{xy}$ are compared with theoretical strengths $\sigma_m$ and $\tau_m$ of polycrystals at given proportional loading way. Interaction between flanks of an imaginary crack takes place only on the loaded cut fragment (Fig. 3 in [1]).

Just as the necessary criterion (1), so sufficient one (1), (2) can be presented in a vector form and be written in projections on the axis of Cartesian coordinate system. Attention should be paid to the symmetry of notation for both criteria since the first or the second relations from (1), (2) will be used in subsequent computations. Ultimate results are bound to be identical if proportional loading and proportional deformation of material in the pre-fracture zone take place.

In order to obtain estimates for crack opening and crack flank displacement, the width of a plastic zone at the tip of a real crack for the generalized stress state is needed. Estimate approximately the form and sizes of the plastic zone in the vicinity of the crack tip at mixed loading. The explicit analytical solution of this problem in elasto-plastic statement encounters substantial difficulties and is not available at present time.

Make use of Mises plasticity criterion that has the following form on the main axes

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2\sigma_{\mathrm{T}}^2. \tag{3}$$

The main stresses $\sigma_1$ and $\sigma_2$ are determined by the relations

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}\,. \tag{4}$$

The third main stress $\sigma_3$ for plane deformation can be given in the form $\sigma_3 = \sigma_z = \nu(\sigma_x + \sigma_y)$. Asymptotic behavior of a stress field in the vicinity of a crack tip deformed by a mixed mode has the form (smooth parts of the solution are omitted)

$$\begin{aligned}
\sigma_x &= \frac{K_{\mathrm{I}\infty}^0}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) - \frac{K_{\mathrm{II}\infty}^0}{\sqrt{2\pi r}}\sin\frac{\theta}{2}\left(2 + \cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right)\\
\sigma_y &= \frac{K_{\mathrm{I}\infty}^0}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) + \frac{K_{\mathrm{II}\infty}^0}{\sqrt{2\pi r}}\sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}\\
\tau_{xy} &= \frac{K_{\mathrm{I}\infty}^0}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\sin\frac{\theta}{2}\cos\frac{3\theta}{2} + \frac{K_{\mathrm{II}\infty}^0}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)
\end{aligned}\,. \tag{5}$$

Here $\theta$ is the polar angle; the plastic zone width depends on SIFs $K_{\mathrm{I}\infty}^0$ and $K_{\mathrm{II}\infty}^0$, where $K_{\mathrm{I}\infty}^0 = \sigma_\infty\sqrt{\pi l}$ and $K_{\mathrm{II}\infty}^0 = \tau_\infty\sqrt{\pi l}$ are SIFs generated by stresses $\sigma_\infty$ and $\tau_\infty$, respectively [1, 2, 5]. Substituting relation (5) into (4), we get the main stresses $\sigma_1$, $\sigma_2$ and $\sigma_3$. After substitution of the main stresses into original relation (3), we obtain estimate for the plastic zone size. This estimate is defined by the radius-vector $r_p$ that depends on the polar angle $\theta$ for conditions of plane deformation

$$r_p(\theta) = \frac{K_{\mathrm{I}\infty}^{0\;2}}{4\pi\sigma_{\mathrm{m}}^2}\cos^2\frac{\theta}{2}\left[6\sin^2\frac{\theta}{2} + 2(1-2\nu)^2\right] + \frac{2K_{\mathrm{I}\infty}^0 K_{\mathrm{II}\infty}^0}{4\pi\sigma_{\mathrm{m}}^2}\left[3\cos\theta - (1-2\nu)^2\right] + \\ + \frac{K_{\mathrm{II}\infty}^{0\;2}}{4\pi\sigma_{\mathrm{m}}^2}\left[\frac{15}{4} + 2(1-2\nu)^2\sin^2\frac{\theta}{2} + \frac{9}{4}\cos 2\theta\right]. \tag{6}$$

The estimate for the plastic zone size for the case of the plane stress state can be easily obtained from estimate (6), putting $\nu = 0$.

In Fig. 1a, 1b, and 1c, forms of plastic zones determined by expression (6) are given as solid curves for the relations $K_{\mathrm{I}\infty}^0 / K_{\mathrm{II}\infty}^0 = 5$, 1, 0,5, respectively; in Fig. 1a and 1c, dashed curves of similar zones for modes I and II fracture, respectively, are shown. Construction was performed in polar coordinates using the dimensionless radius-vector $\rho = r_p / (K_{\mathrm{I}\infty}^0 K_{\mathrm{II}\infty}^0 / 4\pi\sigma_{\mathrm{m}}^2)$.

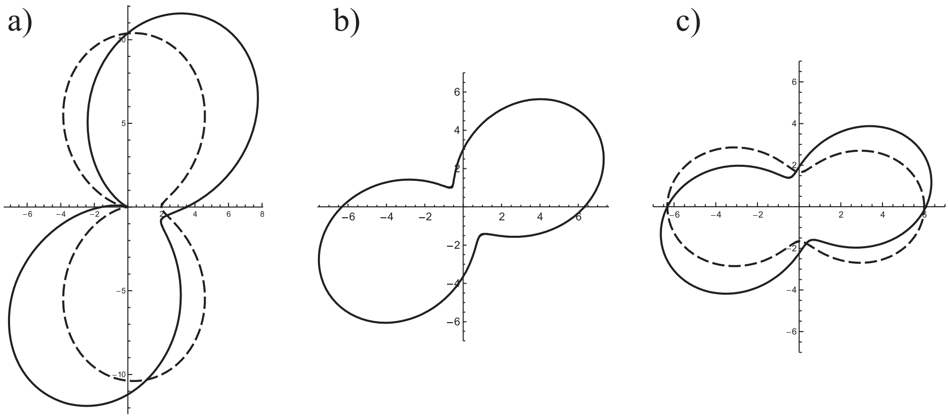


Fig. 1. Forms of pre-fracture zones.

In the general case, it is difficult to propose estimate for the plastic zone width for an arbitrary ratio $K_{\mathrm{I}\infty}^0 / K_{\mathrm{II}\infty}^0$, therefore, we have restricted ourselves to relations for two mixed modes in two cases: 1. $K_{\mathrm{I}\infty}^0 \gg K_{\mathrm{II}\infty}^0$, more exactly $5 < K_{\mathrm{I}\infty}^0 / K_{\mathrm{II}\infty}^0 < \infty$ and 2. $K_{\mathrm{II}\infty}^0 \gg K_{\mathrm{I}\infty}^0$, more exactly $2 < K_{\mathrm{II}\infty}^0 / K_{\mathrm{I}\infty}^0 < \infty$. This corresponds to domination of modes I and II fracture in either of two cases, respectively, that is peculiar for materials that are characterized by brittle and quasi-brittle fracture types for the first case and ductile and quasi-ductile fracture types for the second case. Let the pre-fracture zone width only for these two cases $b$ be coincident with the plastic zone width $2r_p\big|_{\theta=0}(\theta)$ at the tip of a real crack

$$b = 2r_p(0). \tag{7}$$

In order to make use of relation (7) for estimations of the critical crack opening $2v^*$ and critical crack flank displacement $2u^*$, we need for criterion (2) information about plastic properties of the material at proportional loading $\mathbf{s} = (\sigma, \tau)$ for $\tau = a\sigma$, where $a = const$. Take these data from the experiment [6]. Further the general case is considered when non proportional deformation of material in the pre-fracture zone under plastic flow at proportional loading takes place. Experiments are assumed to be carried out using thin-walled pipe specimens. Fig. 2a, b displays the simplest approximations of $\sigma-\varepsilon$ and $\tau-\gamma$ diagrams at proportional loading. Here by $\varepsilon_1$ and $\gamma_1$ denote relative limit lengthening and limit shear angle for the elastic way of deformation, $\varepsilon_m$ and $\gamma_m$ are relative limit lengthening and limit angle for plastic flow. Taking into consideration that pre-fracture zone occupies a rectangle with the width $b$, we get simple expressions for estimation of critical crack opening $2v^*$ and critical crack flank displacement $2u^*$ for mentioned two cases in the form (Fig. 2)

$$2v^* = 2r_p(0)\left(\varepsilon_m - \varepsilon_1\right), 2u^* = 2r_p(0)\left(\gamma_m - \gamma_1\right). \tag{8}$$

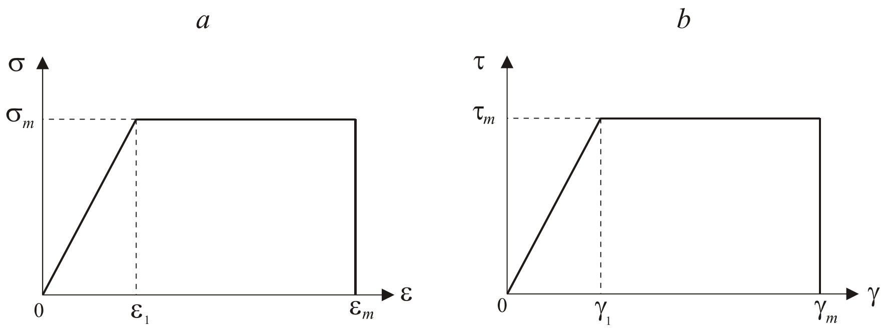


Fig. 2. Simple approximations of $\sigma-\varepsilon$ and $\tau-\gamma$ diagrams at proportional loading.

The obtained critical parameters $2v^*$ and $2u^*$ from (8) are used in the first and second relations for criterion (2). Further, all the necessary transformations [1, 2] are performed. Finally, the critical parameter $\sigma_\infty^{0*}$ is obtained from the first relations for criterion from (1) and (2), and the critical parameter $\tau_\infty^{0*}$ is obtained from the second relations from (1) and (2) when non proportional deformation of material in the pre-fracture zone takes place. Compare the obtained magnitudes. Here three cases can be

$$\sigma_\infty^{0*} = \tau_\infty^{0*} / a, \tag{9}$$

$$\sigma_\infty^{0*} > \tau_\infty^{0*} / a, \tag{10}$$

$$\sigma_\infty^{0*} < \tau_\infty^{0*} / a. \tag{11}$$

The first case (9) corresponds to deformation of material in the pre-fracture zone at proportional loading $\tau_\infty = a\sigma_\infty$ and proportional deformation of material in the pre-fracture zone. Two other cases (10) and (11) correspond to non proportional deformation of material in the pre-fracture zone at proportional loading $\tau_\infty = a\sigma_\infty$. In these cases, for inequality (10), material in the pre-fracture zone has a tendency to fracture by shear, and for inequality (11), material in the pre-fracture zone has tendency to fracture by cleavage.

Thus, in the general case, investigation of fracture mechanism for material in the pre-fracture zone when proportional loading takes place has been completed.

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