Strength Mismatch Effects on Limit Loads for Surface Cracked Tensile Plates

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Abstract. The present work provides mis-match limit loads for tensile plates with constant-depth, part-through surface cracks in the center of the weld metal. Based on systematic three-dimensional FE limit analyses, effects of strength mis-match related variables on limit loads are quantified by the strength mismatch ratio and one geometry-related parameter. Mismatch limit loads for part-through surface cracks are then correlated to those for two-dimensional, through-wall crack problems.

Introduction

As the accuracy of the defect assessment methods modified for strength mis-match effects is directly related to the plastic limit load, accurate estimation of mis-match limit loads is essential for assessing cracked structures with weldments [1-6]. Limit load solutions for mis-matched plates have been presented by several researchers [1,2,7-10]. For instance, via extensive two-dimensional finite element (FE) analyses using elastic-perfectly plastic materials, the authors presented closed-form mis-match limit load solutions for middle cracked tension plates, single-edge-cracked plates in bending and double-edge-cracked plates in tension [7-9]. Note that all these solutions are for through-wall cracks under two-dimensional idealizations (either plane strain or plane stress). In practice, however, part-through surface cracks are of more interest. In contrast to two-dimensional through-wall cracks, part-through surface cracks have one more dimension related to the crack geometry (the crack depth), which makes quantification of mis-match limit loads quite difficult. Accordingly existing work for the strength mis-match effect on plastic limit loads is very limited (see for instance Ref. [11]), and no systematic result has yet been reported.

The present work attempts to provide mis-match limit loads and approximate J estimates for tensile plates with constant-depth, part-through surface cracks in the center of the weld metal. Based on systematic three-dimensional FE limit analyses, effects of mis-match related variables on limit loads are quantified.

Mismatch Limit Loads for M(T) Plates: review

Developing mis-match limit load solutions for constant-depth, surface cracked plates in tension is quite a difficult task, due to its three-dimensional nature and a large number of variables involved. Thus it would be instructive to review solutions for two-dimensional limiting cases before attempting to quantify mis-match limit load solutions for constant-depth, part-through surface cracked plates in tension. One limiting case is the middle-cracked tension (M(T)) plate (Fig. 1a), and the other is the single-edge-cracked plate in (fixed grip) tension, SE(T) (Fig. 1b). Note that solutions for the M(T) plate were given in [7,8] and those for the SE(T) plate in [9]. For the weld configuration, an idealised butt weld configuration (a simple strip model where the weld metal strip has a rectangular cross section) was considered, as shown in Fig. 1. The mis-match in the yield strength between the weld metal, σ_{YW}, and the base plate, σ_{YB}, is quantified by the mis-match factor M:

$$M \equiv \frac{\sigma_{YW}}{\sigma_{YB}} \tag{1}$$

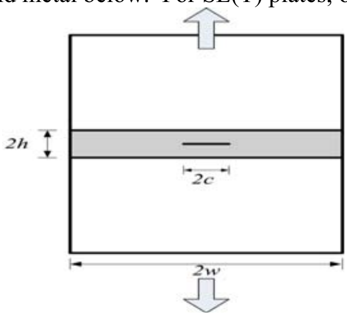
with M<1 referring to under-matching and M>1 referring to over-matching. Another important mis-match related parameter is the slenderness of the weld [4,7]. For the M(T) plate, it is defined as

$$\psi = \frac{(w-c)}{h} \tag{2}$$

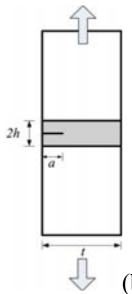
where $2c$, $2w$ and $2h$ denote the crack length, plate width and weld width, respectively (Fig. 1a). For the SE(T) plate, on the other hand, it is defined as

$$\psi = \frac{(t-a)}{h} \tag{3}$$

where a and t denote the crack depth and plate thickness, respectively (Fig. 1b). For the crack location, two locations were considered, in the centre of the weld metal and in the interface of the weld metal and base plate. Both plane strain and plane stress conditions were considered. Below the mis-match limit load solutions will be summarized mainly for M(T) plates with cracks in the center of the weld metal below. For SE(T) plates, only essential features will be briefly given later.



(a)



(b)

Fig. 1. Schematic illustrations of (a) mis-matched middle-cracked tensile (M(T)) plates and (b) mis-matched single-edge-cracked plates in (fixed grip) tension, SE(T).

Middle-Cracked Tension (M(T)) Plates The middle-cracked tension plate is one limiting case of the constant-depth, surface cracked tension plate when the crack depth equals the plate thickness ($a=t$). Mis-match limit load solutions for middle-cracked tension plates with cracks in the center of the weld metal, given in [7], are briefly summarised here. For over-matching, plane strain limit loads, F_{LM}, are given by

$$\frac{F_{LM}}{F_{LB}^{PE}} = \begin{cases} \min\left(M,\ \dfrac{1}{1-c/w}\right) & \text{for } 0 \le \psi \le \psi_1 \\ \min\left(\dfrac{24(M-1)}{25}\left(\dfrac{\psi_1}{\psi}\right) + \dfrac{(M+24)}{25},\ \dfrac{1}{1-c/w}\right) & \text{for } \psi_1 \le \psi \end{cases} \tag{4}$$

$$\psi_1 = \exp\left[-\frac{(M-1)}{5}\right] \quad ; \quad F_{LB}^{PE} = \left(\frac{4}{\sqrt{3}}(w-c)t\sigma_{YB}\right)$$

In Eq. (4), $F_{LB}{}^{PE}$ is the plane strain limit load for the M(T) plate made wholly of the base material. For under-matching, plane strain limit load solutions are given by the smaller one of Eq. (5) and Eq. (6):

$$\frac{F_{LM}}{F_{LB}^{PE}} = \begin{cases} M & \text{for } \quad 0 \le \psi \le 1.0 \\ 1 - \dfrac{(1-M)}{\psi} & \text{for } \quad 1.0 \le \psi \end{cases} \tag{5}$$

$$\frac{F_{LM}}{F_{LB}^{PE}} = \begin{cases} M & \text{for } \quad 0 \le \psi \le 1.0 \\ M \cdot \left[1.0 + 0.462(\psi-1)^2/\psi - 0.044(\psi-1)^3/\psi\right] & \text{for } \quad 1.0 \le \psi \le 3.6 \\ M \cdot [2.571 - 3.254/\psi] & \text{for } \quad 3.6 \le \psi \le 5.0 \\ M \cdot [1.291 + 0.125 \cdot \psi + 0.019/\psi] & \text{for } \quad 5.6 \le \psi \end{cases} \tag{6}$$

For the plane stress condition, mis-match limit loads are given as follows. For over-matching,

$$\frac{F_{LM}}{F_{LB}^{PS}} = \begin{cases} \min\left(M,\ \dfrac{1}{1-c/w}\right) & \text{for } 0 \le \psi \le \psi_1 \\ \min\left(\dfrac{24(M-1)}{25}\left(\dfrac{\psi_1}{\psi}\right) + \dfrac{(M+24)}{25},\ \dfrac{1}{1-c/w}\right) & \text{for } \psi_1 \le \psi \end{cases} \tag{7}$$

$$\psi_1 = \left(1 + 0.43\exp^{-5(M-1)}\right)\exp^{-(M-1)/5} \quad ; \quad F_{LB}^{PS} = \left[2(w-c)t\sigma_{YB}\right]$$

where $F_{LB}{}^{PS}$ is the plane stress limit load for the M(T) plate made wholly of the base material. For under-matching, plane stress limit load solutions are given by the smaller one of Eq. (8) and Eq. (9):

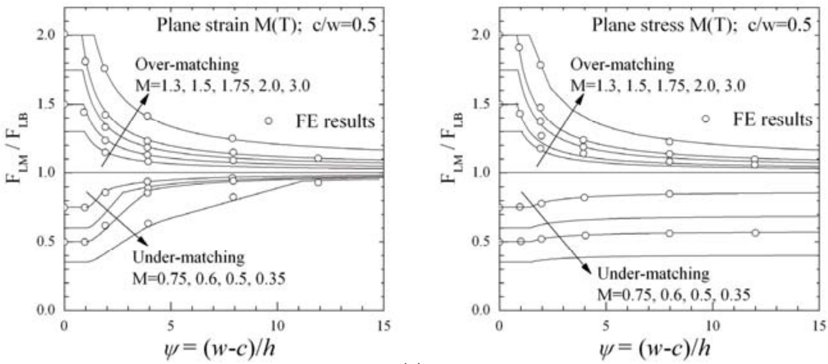
$$\frac{F_{LM}}{F_{LB}^{PS}} = \begin{cases} M & \text{for } \quad 0 \le \psi \le 1.43 \\ 1 - 1.43\dfrac{(1-M)}{\psi} & \text{for } \quad 1.43 \le \psi \end{cases} \tag{8}$$

$$\frac{F_{LM}}{F_{LB}^{PS}} = \begin{cases} M & \text{for } \quad 0 \le \psi \le 1.43 \\ M \cdot \left[1.155 - \dfrac{0.2212}{\psi}\right] & \text{for } \quad 1.43 \le \psi \end{cases} \tag{9}$$

It should be noted that the parameter ψ in above equations, Eqs. (4)-(9), is defined by Eq. (2).

Figure 2 compares above solutions with FE results for mis-matched M(T) plates with c/w=0.5 for a wide range of M and $\psi=(w-c)/h$. The results for plane strain are shown in Fig. 2a and those for plane stress in Fig. 2b. The results in Fig. 2 provide several notable features. The first one is that mis-match limit loads can be characterized by two parameters, M and $\psi=(w-c)/h$. The geometry-related parameter ψ is very important, incorporating effects of c/w and w/h. The second point is that mis-match limit loads are affected not only by M but also by $\psi=(w-c)/h$. In particular, the effect of ψ could be quite significant. The third and final point is that, when a general trend of F_{LM}/F_{LB} are compared for plane strain and plane stress conditions, they are similar for over-matching but are quite different for under-matching. For under-matching cases with sufficiently large values of ψ, mis-match limit loads for plane strain conditions are similar to those made of the base plate, but mis-match limit loads for plane stress conditions are close to those made of the weld metal.

Mis-Match Limit Loads for Single-Edge-Cracked Plates in Tension, SE(T) The single-edge-cracked plate in tension is the other limiting case of the constant-depth, surface cracked tension plate when the crack length equals the plate thickness ($c=w$). Detailed equations for mis-match limit loads of SE(T) plates with cracks in the center of the weld metal can be found in [9], which will not be repeated here. It is important to note that expressions of normalized mis-match limit loads are quite similar to those for M(T) plates, given in Eqs. (4)-(9), as long as mis-match limit loads are normalized with respect to appropriate limit loads for SE(T) plates made wholly of the base material. Another important point is the definition of ψ. For the M(T) plates, it is defined by Eq. (2), whereas, for the SE(T) plate, by Eq. (3).



(a) (b)

Fig. 2. Variations of normalized mis-match limit loads, F_{LM}/F_{LB}, with $\psi=(w-c)/h$ for mis-matched M(T) plates with c/w=0.5; (a) plane strain and (b) plane stress.

Implications Review of existing mis-match limit load solutions for M(T) and SE(T) plates in plane strain and plane stress provides two important implications to three-dimensional surface crack problems. The first one is that, for idealized two-dimensional cases, mis-match limit loads are characterized by two parameters, M and ψ. The parameter ψ, related to the slenderness of the weld, is defined by the ratio of the remaining ligament to the half weld-width. For two-dimensional problems, it can be easily defined. However, for three-dimensional problems such as for surface cracks, its definition is not straightforward. Another point is the plate thickness effect. As shown in Fig. 2, for over-matching, shapes of F_{LM}/F_{LB} for the plane strain condition are similar to those for the plane stress condition. This implies that the plate thickness effect on mis-match limit loads for surface cracked tensile plates would not be so significant for over-matching cases. For under-matching, however, they are very different, suggesting that the plate thickness effect would be significant. This also suggests that systematic analyses should include variations not only in in-plane dimensions but also in the out-of-plane dimension.

Finite Element Limit Analysis

Geometry and Analysis Matrix Figure 3 depicts mis-matched tensile plates with constant-depth, part-through surface cracks in the center of the weld metal, considered in the present work. As depicted in Fig. 3, the analysis was concentrated on an idealized butt weld configuration (a simple strip model where the weld metal strip has a rectangular cross section), and a crack is assumed to locate in the centre of the weld metal. The plate is characterized by its half-width, w, plate thickness, t, and the half length, L. The value of L was set to be sufficiently long (L=5w) to avoid the end effect. The idealized weld has the height of 2h. The constant-depth, part-through surface crack is characterized by its length, 2c, and depth, a. It should be noted that, although semi-elliptical surface cracks would be more realistic than constant-depth surface cracks in reality, the reason to choose constant-depth surface cracks, rather than semi-elliptical surface cracks, in the present work is its clarity and simplicity. Firstly limiting cases of constant-depth surface cracks are clearer. In the limiting case of $a/t \rightarrow 1$, constant-depth surface cracks become through-wall cracks, whereas, in the other limiting case of $c/w \rightarrow 1$, they become edge cracks. The second advantage is that extraction of elastic-plastic J for constant-depth surface cracks is simpler than that for semi-elliptical surface cracks. The mis-match in the yield strength between the weld metal, σ_{YW}, and the base plate, σ_{YB}, is quantified by the mis-match factor M, see Eq. (1).

For systematic investigations, above variables were systematically varied. Four different values for w/t, w/t=1, 5, 10 and 20, were considered to investigate the thickness effect. Values of c/w and a/t were systematically varied from c/w=0.25 to 0.8, and from a/t=0.2 to a/t=1.0, respectively. Note that the case of a/t=1.0 corresponds to the through-wall crack. For the weld width, values of h/t were systematically varied from h/t=0.25 to h/t=2.0. Finally five different values of M were considered, M=0.5, 0.75, 1.0, 1.5 and 2.0. Note that the case of M=1.0 corresponds to homogeneous plates (without welds).

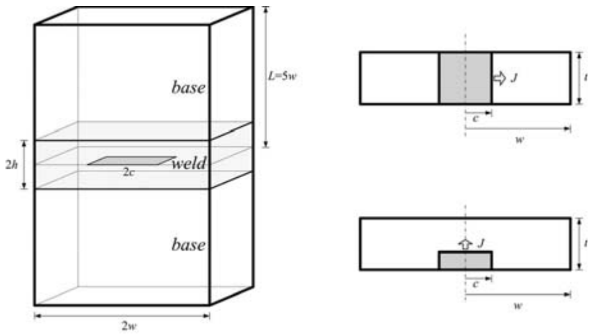


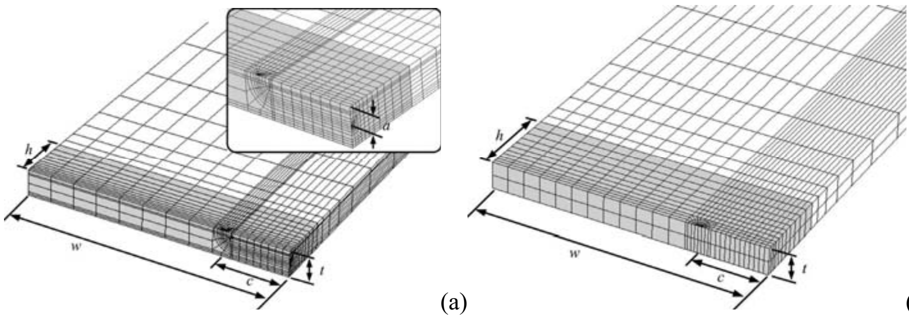
Fig. 3. Schematic illustration of tensile plates with constant-depth, part-through surface cracks in the center of the weld metal, and geometries of through-wall and part-through surface cracks.

Figure 4 depicts typical FE meshes for plates with through-wall cracks and constant-depth part-through surface cracks, employed in the present work. Symmetry conditions were fully utilised in the FE models to reduce the computing time, and thus quarter models were used. Furthermore, twenty-node iso-parametric quadratic brick elements with reduced integration (C3D20R within ABAQUS [12]) were used. For through-wall cracks, typical FE meshes have about 294 elements and 965 nodes. For constant-depth surface cracks, typical FE meshes have about 2,240 elements and 10,869 nodes. The crack tip was designed with focused elements, as shown in Fig. 4. As can be seen in Fig. 4, the present FE model is believed to be sufficiently fine to provide accurate limit loads and elastic-plastic fracture mechanics analyses.

FE Limit Analysis Elastic-perfectly plastic analyses of FE models were performed using ABAQUS [12]. Materials were assumed to be elastic-perfectly plastic, and non-hardening J_2 flow theory was used using a small geometry change continuum FE model. The FE models were subject to pure tension. All nodes in the upper end plane were constrained using the MPC (Multi-Point Constraint) option in ABAQUS, and sufficiently large displacement was applied. Corresponding plastic limit loads can be easily obtained directly from the nodal force.

Mismatch Limit Loads

Through-Wall Cracks In Fig. 5, FE limit loads for mis-matched tensile plates with through-wall cracks (a/t=1.0) for four different values of w/t(=1, 5, 10 and 20) are compared with mis-match limit load solutions for plane stress, Eq. (7) and Eq. (8) (or Eq. (9)) and in plane strain, Eq. (4) and Eq. (5) (or Eq. (6)). The solutions for plane stress are shown in dotted lines and those for plane strain in solid lines. In the figure, the normalized limit load, F_{LM}/F_{LB}, are presented in terms of ψ given by Eq. (2), $\psi=(w-c)/h$, where F_{LB} denotes the FE limit load for homogeneous plates made of the base metal (M=1). It should be noted that for over-matching, plane strain solutions for F_{LM}/F_{LB} are very close to plane stress solutions, and thus dotted lines do not appear in the figures. For under-matching, however, they are very different and thus both solid and dotted lines are shown. The FE limit loads cover three different values of c/w and h/t, leading to wide ranges of ψ values. For over-matching, present FE mis-match limit loads agree well with both (plane strain and plane stress) predictions. For under-matching, present FE mis-match limit loads for w/t=10 and w/t=20 agree well with the plane stress solution. However, for w/t=1 and 5, FE limit loads are in between plane stress and plane strain solutions. For w/t=5, FE results are closer to the plane stress solution, whereas for w/t=1, they are closer to the plane strain one.



(a) (b)

Fig. 4. Typical FE meshes for (a) constant-depth part-through surface cracks and (b) through-wall cracks.

Constant-Depth Surface Cracks For constant-depth part-through surface cracks, one important question to be addressed is how to define the parameter related to the slenderness of the weld, ψ. For two-dimensional through-wall cases, only one variable (ψ) is sufficient, as given in Eq. (2) or in Eq. (3). The part-through surface crack has one more variable related to the crack geometry, and thus one more variable associated with the slenderness of the weld might be needed, which makes quantification of mis-match limit loads quite complicated. It is important to note that careful examination of present FE results suggests that, even for part-through surface cracks, one variable would be sufficient. This is a significant result from an engineering point of view. Such a single parameter must include effects not only of *w/c* but also of *a/t*. An attempt was made simply by linear combination of Eqs. (2) and (3):

$$\psi_{eff} = \frac{1}{f}\left[\frac{(t-a)}{h} + \frac{(w-c)}{h}\right] \tag{10}$$

where ψ_{eff} is used for surface cracks to give contrast to ψ for two-dimensional problems. The non-dimensional factor f is introduced to correlate FE limit loads for mis-matched plates with through-wall cracks and those with part-through surface cracks. Based on many trials, we propose the following empirical form:

$$f = \begin{cases} 2\left(g\left(\frac{w}{t}\right)-1\right)\left(\frac{a}{t}-0.5\right)+g\left(\frac{w}{t}\right) & 0 \le \frac{a}{t} \le 0.5 \\ -2\left(g\left(\frac{w}{t}\right)-1\right)\left(\frac{a}{t}-0.5\right)+g\left(\frac{w}{t}\right) & 0.5 \le \frac{a}{t} \le 1.0 \end{cases} \; ; \; g\left(\frac{w}{t}\right) = 0.13\left(\frac{w}{t}\right)+2.37 \tag{11}$$

The dependence of f on w/t and a/t is assumed to be bi-linear. The value of f increases with increasing w/t. It increases linearly with increasing a/t for $a/t \le 0.5$, and decreases linearly for $a/t \ge 0.5$. The value of f is unity for $a/t=0$ and $a/t=1.0$. Thus, in the limiting case of through-wall cracks ($a/t \to 1$), the value of f becomes unity, and Eq. (11) recovers Eq. (2). The value of f is maximum at $a/t=0.5$, ranging from $f=\sim 2$ for $w/t=1$ to $f=\sim 5$ for $w/t=20$. The FE limit loads for mis-matched plates with part-through surface cracks are shown for four different values of w/t(=1, 5, 10 and 20) in Fig. 6 for $a/t=0.8$ and in Fig. 7 for $a/t=0.5$. In figures, values of F_{LM}/F_{LB} are presented in terms of ψ_{eff} given by Eq. (10) with Eq. (11). The FE results are also compared with predictions based on plane strain (Eq. (4) and Eq. (5) or Eq. (6)) in solid lines and plane stress (Eq. (7) and Eq. (8) or Eq. (9)) in dotted lines. It can be seen that introduction of ψ_{eff} gives much better correlations between FE results and predictions. It is shown in Figs. 6 and 7 that, using Eq. (10) with Eq. (11), the strength mismatch effect on plastic limit loads for part-through surface cracks can be quantified based on corresponding results for through-wall cracks. It should be noted that the above expressions, Eqs. (10) and (11), are empirical in the sense that they are proposed based on present FE results, and thus are valid only for the geometric cases considered in the present work.

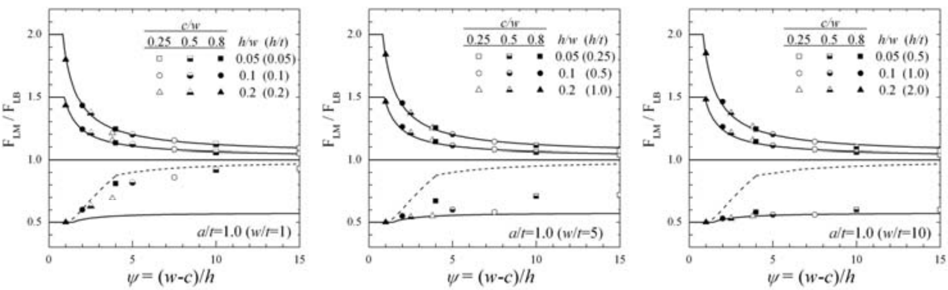


Fig. 5. Variations of normalized mis-match limit loads for through-wall cracks with $\psi=(w-c)/h$.

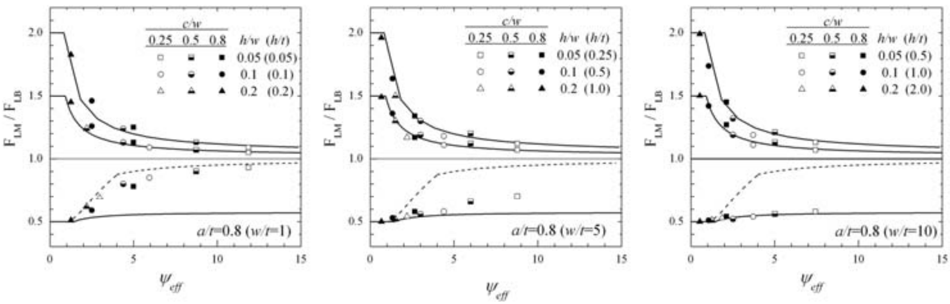


Fig. 6. Variations of normalized mis-match limit loads for part-through surface cracks (a/t=0.8) with the proposed parameter $\psi_{eff}=(w\text{-}c)/h_{eff}$.

Conclusions

The present work provides mis-match limit loads and approximate J estimates for tensile plates with constant-depth, part-through surface cracks in the center of the weld metal. Based on systematic three-dimensional FE limit analyses, effects of mis-match related variables on limit loads are quantified. Note that, for through-wall cracks, mismatch limit loads can be characterized by the strength mismatch factor and one geometry-related parameter. An interesting finding is that, even for part-through surface cracks, one geometry-related parameter is sufficient to characterize mismatch limit loads, not requiring additional geometry-related parameter. Based on extensive FE results, a form of the geometry-related parameter for tensile plates with constant-depth, part-through surface cracks in the center of the weld metal is proposed. This implies that limit loads for welded plates with part-through surface cracks can be estimated simply from mismatch limit load solutions for two-dimensional, through-wall crack problems, which in turn provides significant benefit to assess surface cracks in welded plates in practice. It could be noted that similar analyses are being performed for welded pipes with constant-depth, part-through surface cracks in the center of the weld metal, subject to tension and bending. Preliminary results suggest that a similar conclusion can be drawn that one geometry-related parameter is sufficient to characterize mismatch limit loads, implying that present results are not limited to plates but are rather general.

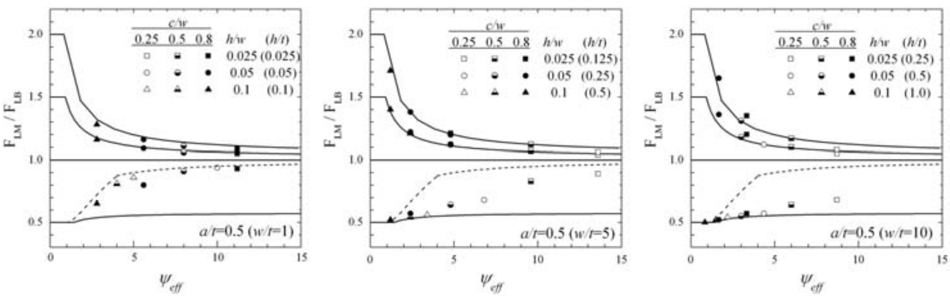


Fig. 7. Variations of normalized mis-match limit loads for part-through surface cracks (a/t=0.5) with the proposed parameter $\psi_{eff}=(w\text{-}c)/h_{eff}$.

References

[1] Hao, S., Cornec, A. and Schwalbe, K-H. (1997). International Journal of Solids and Structures 34, 297-326.

[2] Joch, J., Ainsworth, RA. and Hyde, TH. (1993). Fatigue and Fracture of Engineering Materials and Structures 16, 1061-1079.

[3] Kim, Y-J., Kocak, M., Ainsworth, RA. and Zerbst. U. (2000). Engineering Fracture Mechanics 67, 529-546.

[4] Schwalbe, K-H., Kim, Y-J., Hao, S., Cornec, A. and Koçak, M. (1996). EFAM ETM-MM 96: The ETM method for assessing the significance of crack-like defects in joints with mechanical heterogeneity (strength mis-match). GKSS Research Centre, GKSS/97/E/9, Germany.

[5] Lei, Y. and Ainsworth, RA. (1997). International Journal of Pressure Vessels and Piping 70, 237-245.

[6] Kotousove, A. and Mohmed, M. F. M. (2006). Engineering Failure Analysis 13, 1065-1075.

[7] Kim, Y-J. and Schwalbe, K-H. (2001). Engineering Fracture Mechanics 68, 163-182.

[8] Kim, Y-J. and Schwalbe, K-H. (2001). Engineering Fracture Mechanics 68, 183-199.

[9] Kim, Y-J. and Schwalbe, K-H. (2001). Engineering Fracture Mechanics 68, 1137-1151.

[10] Schwalbe, K-H., Ainsworth, R A., Eripret, C., Gilles, P., Koçak, M., Pisarski, H. and Wang, Y Y. (1997). In: Schwalbe K-H and Koçak M, editors. Mis-Matching of Interfaces and Welds, GKSS Research Centre Publications, Germany, 99-134.

[11] Boothman, D.P., Lee, M.M.K. and Luxmoore, A.R. (1999). Engineering Fracture Mechanics 64, 433-458.

[12] ABAQUS (2005). ABAQUS Standard/User's Manual, Version 6.4, Hibbit Karlsson & Sorensen, Inc., Pawtucket, RI, USA.