Relationship between J-integral and CTOD for a C(T) specimen: a 3D FE analysis

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Abstract. In this investigation the relationship between *J*-integral and CTOD is studied considering a Compact Tensile (CT) specimen using 3 dimensional finite element analyses. The magnitude of CTOD is estimated by 90°-intercept method on the surface and at the centre of the specimen, and also by plastic hinge model. The results indicate that the magnitudes of CTOD estimated by 90°-intercept method at the centre of the specimen are found to be higher than that on the surface. The results reveal an inconsistency in estimation of CTOD by 90°-intercept method and by plastic hinge model. The CTOD values obtained by both the methods are found to be linearly proportional to *J*-integral. The linear proportionality constant d_n between CTOD and *J* is found to be strongly depend on the method of estimation of CTOD and specimen thickness.

Introduction

Elastic plastic fracture mechanics (EPFM) is the domain of fracture analysis, which considers extensive plastic deformation ahead of crack-tips prior to fracture. It is well known that *J*-integral (J) and crack-tip opening displacement, CTOD (δ) can be used as fracture parameters for analysis of fracture problems under EPFM. In EPFM it is required that J and δ should be interchangeable to each other. Thus, it is essential to examine the relation between J and δ. A well-known general relation between J and CTOD [1] is:

$$J = m\sigma_y\delta \tag{1}$$

where, σ_y- is the yield stress of the material, m- constant, and δ- CTOD. Earlier literature [2-4] indicates that the load intensity measured in terms of *J*-integral as a single parameter alone does not describe the stress/strain field ahead of the crack-tip uniquely and accurately. Hence, there is a necessity of introducing a second parameter with J, which is required to characterize the crack-tip fields. This discrepancy in characterizing the crack-tip fields is due to varied *constraint effects* in fracture. Interestingly, the constant factor m in relation between J and δ given in Eq.(1) is known to be constraint dependent [4]. Thus m can serve as a parameter to characterize constraints [4]. Therefore study of m is important in EPFM analysis.

Shih [5] has shown that the relationship between J and δ can be obtained theoretically by HRR stress field equations [6, 7] as:

$$\delta = d_n \frac{J}{\sigma_y} \tag{2}$$

where, d_n is a constant, which depends on Ramberg-Osgood (R-O) constant N of the material. From Eqn.(1) and Eqn.(2) the relation between m and d_n is:

$$m = \frac{1}{d_n} \tag{3}$$

Shih [5] has also shown that d_n usually varies between 0.4 to 0.8 for common structural steels and for fully plastic materials (N=∞) d_n=1, which is obtained by extrapolation. Omidvar *et al*. [8] using closed form solutions have confirmed their results on relationship between J and CTOD fully corroborate the results of Shih [5]. The magnitude of d_n is found to be dependent on strain hardening component of the material [5, 8], specimen a/W ratio [9] is known from analytical solutions and 2D finite element analysis. As m or d_n can be used as a constraint parameter [4], it is also required to examine the effect of specimen thickness on the factor d_n, which can address *out-of-plane constraint* effecs. The CTOD can be estimated by plastic hinge model [10] and by 90^o intercept method [5]. Hence, the consistency in measurement of CTOD in 3D analysis by both the methods is to be studied. The objective of the present investigation is to examine the relationship between the J and δ for various specimen thicknesses and to study the effect of specimen thickness on magnitude of d_n computed by plastic hinge model and 90^o intercept method.

Finite element analysis

The general-purpose finite element code ABAQUS [11] is used in this study. Compact Tension (CT) fracture specimen geometry has been considered in the present study. The dimensions of CT specimen has been computed according to ASTM standard E1290 [10] with width of the specimen W=20mm. To study the effect of specimen thickness on relationship between J and CTOD, several specimens with thickness (B) to width (W) ratio, B/W= 0.1, 0.2, 0.3, 0.4 and 0.5 are considered for finite element analysis (FEA). A typical specimen configuration used in this analysis is shown in Fig.1. Only one half of the specimen has been considered for FEA due to the geometrical and loading symmetry. The analysis domain is descritized using 20-noded quadratic brick finite elements using reduced integration. This kind of elements is used in the work of Kim *et al.* [12], Courtin *et al.* [13]. The number of elements used in the FEA varied with the thickness of the specimen. A typical finite element mesh generated for the FEA of CT specimen is shown in the Fig.2. In these finite element calculations, the material behaviour has been considered to be multilinear kinematic hardening type pertaining to an interstitial free (IF) steel possessing yield strength of 155 MPa, elastic modulus of 197 GPa, poison's ratio=0.3, and Ramberg-Osgood constants N=3.358 and α=19.22 [14].

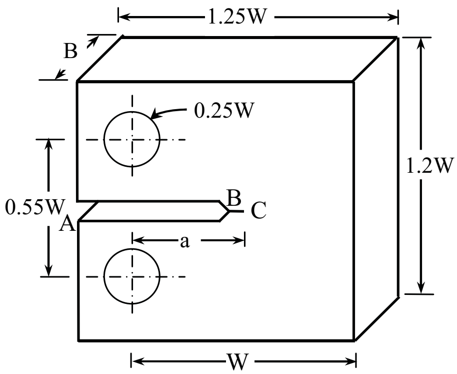

Fig.1 The geometry of CT specimen used in the analysis (W=20 mm).

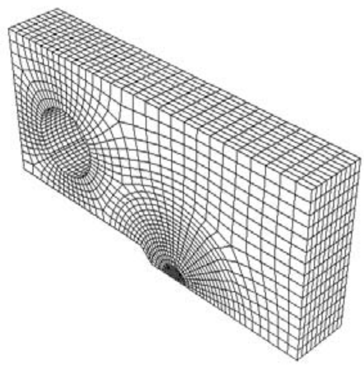

Fig.2 A typical mesh used in the FE analysis for thickness B=4 mm

A series of elastic-plastic stress analyses on CT specimens (Ref. Fig.1) of various thickness *B/W*=0.1, 0.2, 0.3, 0.4 and 0.5 are carried out for different applied load levels. In these analyses, for every load step, elastic-plastic fracture parameters such as 3D *J*-integral and CTOD by 90^o intercept method (on the surface and at the centre of the specimen) and plastic hinge model have been computed.

J-integral

The magnitude of *J*-integral in 3D has been computed with the help of finite element code ABAQUS in the similar manner it is computed in an earlier report [13]. The specimen is divided in to number surfaces equal to the number of elements along the thickness direction. By defining the nodes of the crack front, using domain integral method the software automatically finds the five contours (user defined) in order to carry out *J*-integrals. As it is widely accepted that the first contour does not provide good results because of numerical singularities [13], the magnitude of first contours has been neglected in the analysis. The mean value of rest of four contours on each surface along the thickness direction is computed and typical such magnitudes of *J*-are plotted in Fig.3.

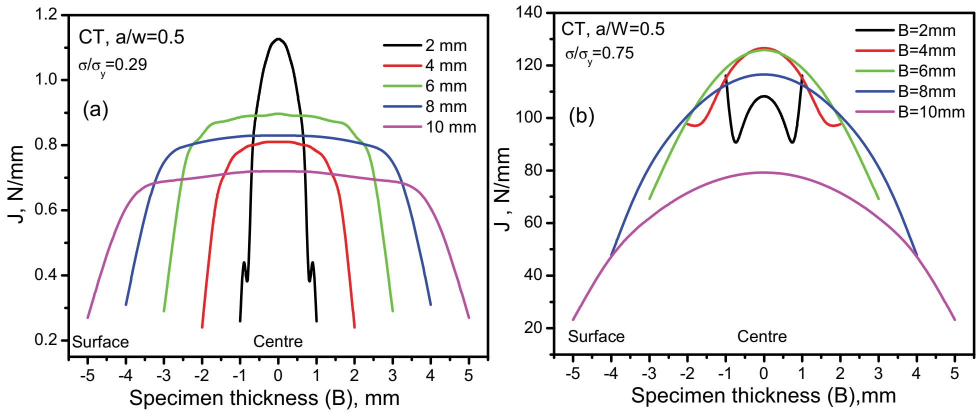

Fig.3 A typical variation of magnitudes of *J*-integral (a) σ/σ_y =0.29 and (b) σ/σ_y = 0.75 for various specimen thickness considered in the analysis.

Figure.3 indicates that the magnitude of *J* varies from surface to the centre of the specimen along the crack front. The magnitude of the *J* is observed to be higher at the centre of the specimen than that of surface. It is also seen from Fig.3 (a) and Fig.3 (b) that the nature variation of *J* along the crack front is dependent on the applied loading. The nature of variation of the magnitude of *J* presented in Fig.3 is in similar agreement with the results presented in the work of Zadeh *et al* [15] and Rajaram *et al* [16]. The average value of *J* (J_{av}) for all specimens having varying thickness is obtained by computing the mean value of *J* using the surface values (Fig.3) on each specimen along the thickness. This mean value of *J* is considered in the analysis. The average magnitudes of *J* for various applied load expressed in terms of normalized applied stress (σ/σ_y) for various thickness of the specimens is depicted in Fig.4. The applied stress (σ) in the analysis is computed with the analytical formulation provided in the work of Priest [17]. Figure.4 demonstrates that the magnitude of *J* average is independent of specimen thickness up to $\sigma/\sigma_y \cong 0.4$. For $\sigma/\sigma_y > 0.4$, it is observed that the specimen having lower thickness experiences a higher magnitude of *J* than the specimen having higher thickness. This nature of dependence of *J* on specimen thickness may be attributed to varied state of stress field ahead of crack front in specimens with different thickness.

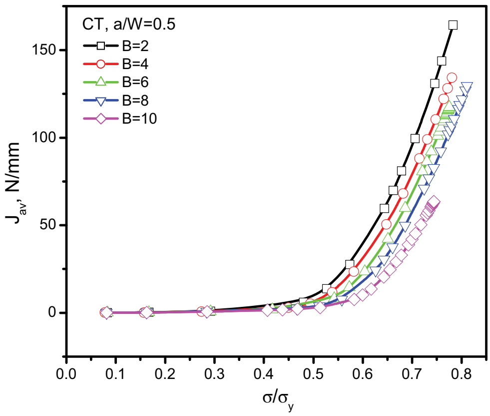

Fig.4 effect of specimen thickness on J_{av} for various applied load

Crack-tip opening displacement (CTOD)

The magnitudes of CTOD for various load steps have been estimated by two methods: (i) by 90^o-intercept method [5] on the surface and at the centre of the specimen and (ii) by conversion of crack mouth opening displacement (CMOD) to CTOD using rotation factor, which is popularly referred as *plastic hinge model* and used in experimental fracture analysis [10]. In 90^o intercept method, for every load step, the *y*-displacement of each node on the crack flank BC (Ref. Fig.1) is plotted. According to this method an intercept of a 45^o line drawn from the crack-tip with the *y*-displacement plot is considered as half part of CTOD.

In the plastic hinge model CMOD is converted to CTOD using rotation factor. At each applied load the magnitude of half CMOD is noted from the *y*-displacement of the node at point A (Ref. Fig.1). The CMOD data obtained from FE results is then used to compute the magnitude of CTOD using a relation given in ASTM E1290 [10]:

$$\delta = \frac{(CMOD).r.b}{a + r.b} \tag{4}$$

where r is rotation factor, the value of r according ASTM E1290 [10] varies with specimen *a/W* ratio and is between 0.44-0.47 for CT specimen, b is the uncracked ligament and a is the crack length of the specimen. The Eqn. (4) estimates only the plastic part of CTOD, as the investigation is elastic-plastic analysis; the elastic part of CTOD is found to be insignificant and is neglected in the present work.

Results and Discussion

Various load steps have been applied on the CT specimen with *a/W*=0.5 and varying thickness to study the stress distribution in the specimen analysis domain. At each load step the magnitude of 3D *J*-integral has been computed as discussed in earlier section. As discussed in earlier section the value of CMOD/2 is noted from the *y* displacement of the point A (Ref. Fig.1). A typical deformed CT specimen compared to un-deformed specimens is shown in Fig.5, which demonstrates the method of obtaining CMOD. It is also observed from the FE analysis that the *y*-displacement along the specimen thickness is similar for a particular loading. This is generally expected, and it shows that the CMOD is same thought the specimen thickness, which indicates that CTOD estimated by this method is independent of specimen thickness. The CMOD data estimated by FEA is used to compute the magnitude of CTOD using Eqn.(4). The magnitude of CTOD is also computed by 90^o intercept method [5]. This method seems to be more suitable in case of 2D fracture analysis. But in 3D fracture analysis, it is known that the stress/strain field is more complex than 2D and it varies along the crack front in thickness direction [18]. This variation of crack-tip stress/strain field can alter the crack-tip opening displacement along the thickness direction. Because of varied crack-tip stress/strain field, it is expected that the CTOD measured by 90^o-intercept method may vary along the crack front. To study the variation of CTOD along the crack front and to save the computation time, it is decided to estimate CTOD only on the surface and at the centre of the specimen. In 90^o-intercept method, for every load step, the *y*-displacement of each node on the crack flank on the surface and at the centre of the specimen (Ref Fig.1) is listed with the help of ABAQUS post processor. The *y*-displacement at the centre of the specimen is obtained by slicing it at the centre and listing the *y*-displacement. The listed *y*-displacements are then plotted using a grapher software. Such a typical plot at the centre of the specimen for various values of *J* is shown in Fig.6. The intercept of *y*-displacement of crack flank with the 45^o lines drawn from the crack-tip is taken as half part of CTOD as indicated in Fig.6.

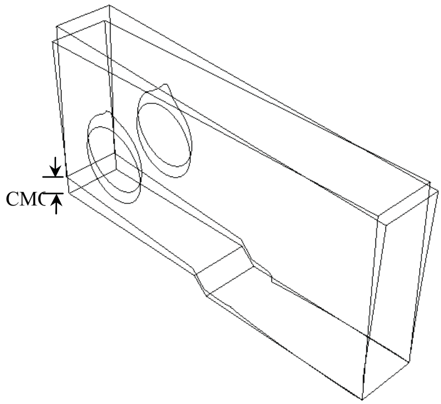

Fig.5 A typical deformed specimen compared to un-deformed specimen showing method of obtaining CTOD

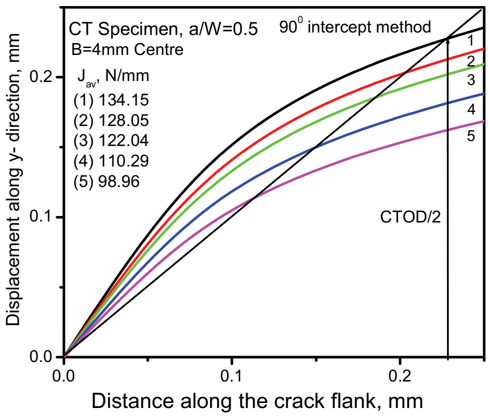

Fig.6 Estimation of CTOD by 90^o intercept method

The magnitudes of δ calculated for various thickness of the specimen using plastic hinge model has been plotted against J/σ_y in Fig.7. This figure shows that slopes of the variation of δ *vs*. J/σ_y apparently look similar, indicating practically there is no effect of thickness on relation between J and δ. Already it is discussed that the magnitudes of δ is expected to vary along the thickness direction. Hence, δ is estimated on the surface and at the centre of the specimen using 90^o intercept method for various specimen thicknesses considered in the analysis. In estimation of δ using 90^o intercept method it is found that the magnitude of δ estimated on the specimen surface is lower than that on the centre. A typical plot of δ on surface and at the centre of specimen is shown in Fig.8. This figure shows that the crack-tip displacements at the surface of the specimen do not open up so as to cut a line drawn at 45^o to crack surface, indicating higher constraint. This nature of CTOD may be attributed the varied stress field along the crack front. In authors one of the recent articles [18] it is shown that size of crack-tip plastic zone at the centre of the specimen is higher than that on the surface. This indicates that centre of the specimen experiences large plastic strains resulting in higher magnitude of CTOD than that on the surface. It is observed form FEA that the magnitudes of δ on the surface are not measurable for specimen having B/W>0.2 mm, and at the centre for normalized specimen thickness, B/W>0.4. The estimated δ at the centre of the specimen thickness has been plotted against J/σ_y in Fig.9. This figure shows that the nature of variation of δ *vs*. J/σ_y is specimen thickness dependent.

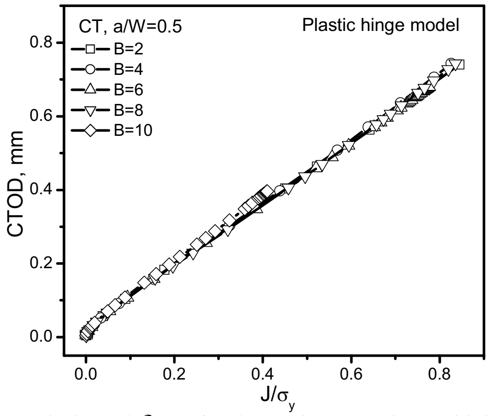

Fig.7 Variation of δ *vs*. J/σ_y for various specimen thicknesses

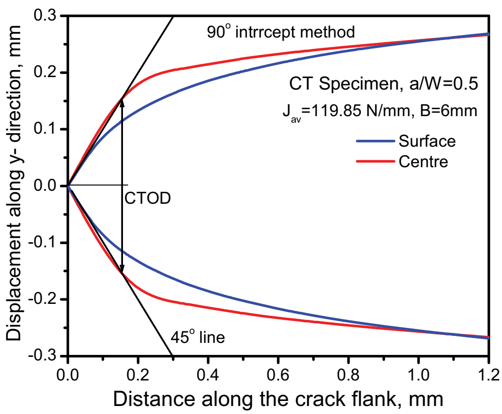

Fig.8 A typical plot of δ on surface and at the centre of specimen

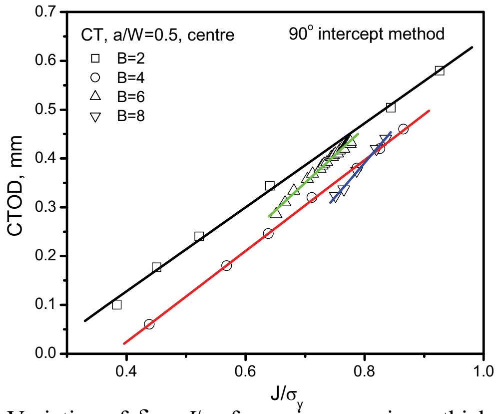

Fig.9 Variation of δ *vs*. J/σ_y for various specimen thicknesses

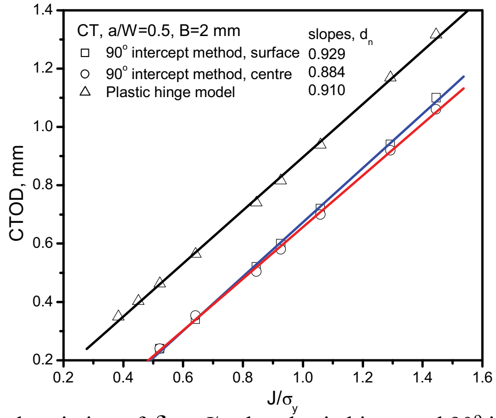

Fig.10 A typical variation of δ *vs*. J/σ_y by plastic hinge and 90° intercept models

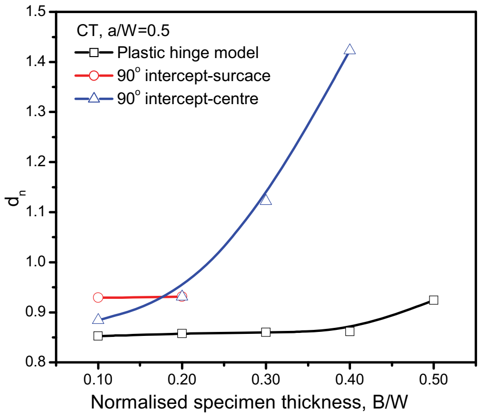

Fig.11 Variation of δ *vs*. J/σ_y for various specimen thicknesses

The magnitudes of CTOD calculated from both the methods have been plotted against J/σ_y in Fig.10. It is interesting to know from this figure that the variation of δ against J/σ_y is linear; this nature of variation of δ against J/σ_y is in good agreement with the results of Panontine *et al*. [9]. It is also clear from Fig.10 that the magnitudes of δ obtained from plastic hinge model and 90^o intercept method differ with respect to J/σ_y. This discrepancy may be attributed to the methods of computing the δ. In reality it is difficult to estimate the exact value of δ. The 90^o-intercept method is a theoretical method of estimating δ by constructing a 90^o triangle at the crack-tip. This method is difficult to use in experimental fracture analysis. An attempt of using this method of estimating δ is carried out by Kulkarni *et al.* [19]. The second method based on plastic hinge model is popularly used in experimental methods of fracture analysis. In this method a clip gauge is used to measure CMOD. The obtained CMOD is then converted to δ by considering the deflection of crack mouth points with respect to a plastic hinge measured as a rotation factor (r) as given in Eqn. (4). It is clearly mentioned in ASTM 1290 [10] that the plastic rotation factor r is not a constant factor. The parameter r is a complex function of specimen configuration and size, applied loading and material. These difficulties in the measuring methods possibly alter the results of δ obtained by both the methods.

In the present investigation the constant d_n in the relationship between δ and J/σ_y (Ref. Eqn. (2)) is obtained by the slopes of the results shown in Fig.7 and Fig.9. The estimated values of constant, d_n by 90^o intercept method and by plastic hinge model (calculated using Eqn. (4)) for CT specimen are plotted in Fig.11. This figure shows that the magnitude of d_n estimated by plastic hinge model is almost independent of specimen thickness. But the d_n value estimated by 90^o intercept method is almost similar to that of one obtained by plastic hinge model for $B/W \leq 0.1$, for $B/W > 0.1$ the d_n increases as B/W increases. This discrepancy may be attributed to varied out-of-plane constraint in the specimen. It is clear from this investigation that the conversion of J to δ or vice versa may be associated with some errors depending on the magnitude of d_n and the method of estimation of CTOD. The analysis also infers that while converting the magnitude of δ to J one need to carefully evaluate the value of d_n depending on the material property, specimen geometry and method of estimation of δ rather than considering it to be 1 in EPFM.

Summary and Conclusions

In this investigation the relationship between J and δ are studied with respect to the method of obtaining δ and specimen thickness using 3D elastic-plastic finite element analysis. Following conclusions are drawn from the present investigation:
(i) CTOD estimated by 90^o-intercept method is higher at the centre of specimen thickness than that on the surface.
(i) There exists a discrepancy in estimation of δ from 90^o-intercept method and by plastic hinge model.
(ii) The relationship between J and δ is linear and the linear proportionality constant, d_n, obtained in this analysis for a CT specimen with a/W=0.5 is found to be specimen thickness dependent.
(iii) The relation between J and δ strongly depends on the method of estimation of δ and specimen thickness.

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