Pseudo-Shell Model for Crack Analysis of Tunnel Linings

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Abstract. As an application of the extended fictitious crack model, a pseudo-shell model for crack analysis of the tunnel lining is introduced in this paper, which enables the CMODs of individual cracks to be calculated, corresponding deformations of the tunnel lining to be obtained, and pressure loads to be determined by a quasi loosening zone model. For verification, a soil mechanics model is selected to analyze an aging waterway tunnel, which sustained multiple cracking in its lining and underwent a major renovation work more than thirty years after its construction. The same problem is then studied using the pseudo-shell model to reproduce the recorded cracking patterns and the CMODs at the time of the renovation work. Comparisons between the two numerical results on the lining deformation at the time of renovation are made, and the results are found quite satisfactory. Based on the obtained lining deformation, the size of a quasi loosening zone as well as the pressure loads is calculated.

Introduction

From a structural point of view, a tunnel lining containing through-thickness cracks as shown in Photo 1 (cracks of this scale usually penetrate the whole depth of the wall), can hardly be considered as structurally stable without the interactive support from the surrounding rock mass. In the following numerical analysis, this shielding effect from the surrounding geological materials is replaced by a thin layer of pseudo-shell, which is rigidly connected to the lining and assumes material properties equivalent to those of steel, as shown in Figs. 1 and 2. As a result, the lining is stiffened. This simplified approach

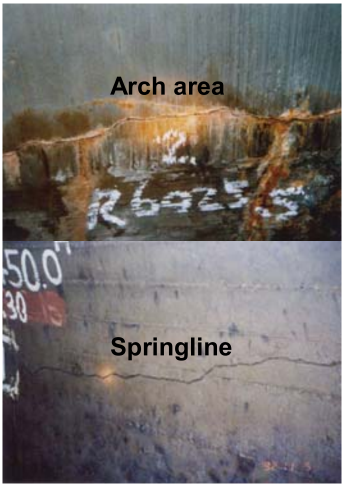

Photo 1 Longitudinal cracks observed in an aging waterway tunnel

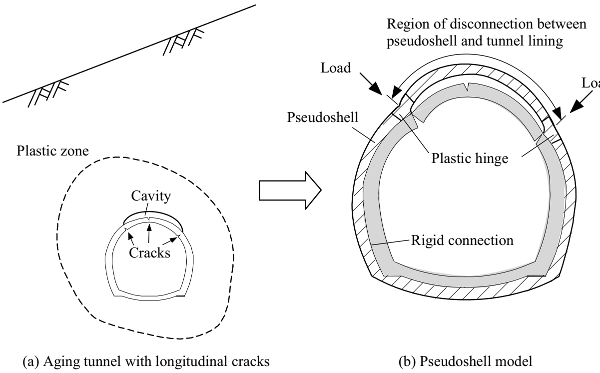

Fig. 1 Concept of the pseudoshell model

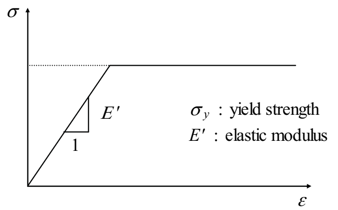

Fig. 2 Elasto-plastic stress-strain relationship of the pseudoshell

is based on the beam theory that ensures the uniqueness of the solution on deformation when the beam is subjected to the same ratio of load to flexural rigidity. Therefore, by stiffening the lining in the pseudo-shell model to increase its flexural rigidity, theoretically it is still possible to obtain the true lining deformation under certain conditions. To achieve this, crack analysis is indispensable. By applying relevant loads to the pseudo-shell with the aim of reproducing the observed cracking behavior as shown in Fig. 1, crack analysis is carried out based on the extended fictitious crack model (EFCM) [1]. When the tip of a crack reaches the pseudo-shell a plastic hinge is introduced into the shell, allowing a rigid-body rotation to take place at the crack surfaces while the crack analysis is continued using a COD-controlled algorithm. Hence, the cross-sectional deformation of the tunnel can be obtained at any specified CMODs. Next, a quasi loosening zone model is used to calculate the ground pressure. Based on the assumption that the ground deformation must equal the cross-sectional deformation of the tunnel, the external loads can be obtained by adjusting the extent of the loosening zone through iterative computations.

Pseudo-Shell Model

Modeling Concept. As shown in Fig. 3, for a beam element of arbitrary cross section, the governing equation is derived as

$$\frac{d^2}{dx^2}(EI\frac{d^2\nu}{dx^2}) = q_z . \qquad (1)$$

where EI is the flexural rigidity, ν is the displacement in the z-direction, and q_z is the distributed load acting perpendicularly to the beam axis. Assuming a constant flexural rigidity EI along the beam axis, Eq. 1 becomes:

$$EI\frac{d^4\nu}{dx^4} = q_z \text{ or } \frac{d^4\nu}{dx^4} = \frac{q_z}{EI} . \qquad (2)$$

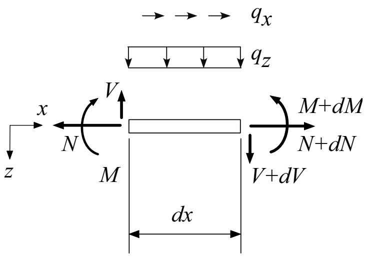

Fig. 3 A beam element

Hence, the beam deformation ν becomes a function solely of the ratio of the external load q_z to the flexural rigidity EI. As long as this ratio is kept unchanged, the beam deformation is uniquely defined, i.e., unaffected by the proportional variations of the load and the flexural rigidity of the beam. As shown below, this unique feature of the beam theory can be exploited to greatly simplify crack analysis problems in tunnels in which the interaction between the lining and the surrounding rock mass is too complex to be modeled realistically. Since the aim of the analysis is to obtain the pressure loads from the CMODs, it seems that a two-step solution can be used. Focusing on the bending action of the tunnel lining under the ground pressure, it is hoped that the cross-sectional deformation of the tunnel can be calculated using a simpler structural model, with which detailed crack analysis can be carried out using the EFCM. Assuming that the tunnel deformation is thus obtained, the external loads can then be calculated using some simplified models in soil mechanics, which will be discussed later.

The pseudo-shell model is thus proposed, which is composed of the lining concrete and a thin layer of pseudo-shell. As is illustrated in Fig. 1, artificial loads are applied to the pseudo-shell to propagate cracks in the lining, which should match the prescribed cracking behavior. Though the dummy loads represent the interactive forces between the lining and the rock mass, the magnitudes of these loads have no direct bearing on the actual loads. As a crack reaches the pseudo-shell, yielding is enforced in a localized zone in the shell next to the crack, resembling the formation of a plastic hinge to allow rigid-body rotations at the crack surfaces. With such a modeling concept, crack analysis can be carried out discretely, thus allowing any prescribed cracking behaviors to be reproduced in detail and the lining deformation to be calculated at the designated CMODs. Obviously, the theoretical justification for the obtained lining deformation lies in the assumption that the same ratio of load to flexural rigidity is obtained, if the actual cracking behavior in a tunnel can be reasonably reproduced through crack analysis using the pseudo-shell model. Of course, the validity of this assumption should be carefully verified in the following numerical studies.

Numerical Formulation. Fig. 4 illustrates a situation with two cracks of the mode-I type, crack A and crack B, in which crack A has reached the pseudo-shell while crack B is still propagating in the lining concrete. In crack analysis, the rotation of the crack surfaces at crack A is achieved by monotonically increasing W_{tip}, which is the COD of the separated dual nodes next to the tip of the crack, as

$$W_{tip} = W_a^N = BK_a^N \cdot P + \sum_{k=1}^{N} AK_a^{Nk} F_a^k + \sum_{j=1}^{M} AK_{ab}^{Nj} F_b^j \,. \tag{3}$$

where BK_a^N is the compliance at the N-th node of crack A due to the external load P. The influence coefficients AK_a^{Nk} and AK_{ab}^{Nj} are the CODs at the N-th node of crack A due to a pair of unit cohesive forces at the k-th node of crack A, and at the j-th node of crack B, respectively. Note that W_{tip} in Eq. 3 is not a variable, but an enforced COD at the N-th node of crack A. Obviously, there is no more need to distinguish between the restrained and the active cracks in the present situation. The remaining CODs along the two fictitious cracks are given by

$$W_a^i = BK_a^i \cdot P + \sum_{k=1}^{N} AK_a^{ik} F_a^k + \sum_{j=1}^{M} AK_{ab}^{ij} F_b^j \,. \tag{4}$$

$$W_b^j = BK_b^j \cdot P + \sum_{i=1}^{N} AK_{ba}^{ji} F_a^i + \sum_{k=1}^{M} AK_b^{jk} F_b^k \,. \tag{5}$$

$$F_a^i = f(W_a^i) \,. \tag{6}$$

$$F_b^j = f(W_b^j) \,. \tag{7}$$

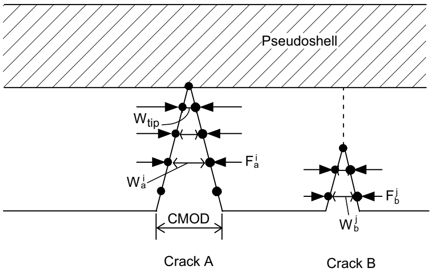

Fig. 4 COD-controlled modeling of multiple cracks

where i = 1, 2,…, N, and j = 1, 2,…, M. The crack equations formed by Eq. 3 to Eq. 7 stipulate the conditions for the rotation of the crack surfaces at the tip of crack A and the growth of crack B in the lining. The problem can be uniquely solved because the number of equations (2N + 2M), matches the number of unknowns, also (2N + 2M). As stated before, numerical results obtained must be checked to eliminate invalid solutions [1]. Obviously, the same procedure can be applied when crack B reaches the pseudo-shell ahead of crack A, and it can be readily generalized to include any number of cracks.

It should be emphasized that, once W_{tip} exceeds the limit crack-opening displacement W_c, the leading crack then becomes a fully open crack, i.e., no more cohesive forces are transmitted through the crack surfaces. Thus Eq. 3 is reduced to

$$W_{tip} = W_a^N = BK_a^N \cdot P \cdot \tag{8}$$

This implies that the COD-controlled solution procedure results in a mere formality, and the problem is now solved under the load control. The outline of the solution procedure is shown in Fig. 5, which is composed of two separate analytical routines, i.e., the crack analysis routine and the stress analysis routine. For each given W_{tip}, the crack equations are solved first to determine the unknown external loads and the cohesive forces acting along each crack. After confirming the validities of the solutions, stress analysis is carried out. Note that as a crack becomes a through-thickness crack, a plastic hinge is set in the pseudo-shell. Following the stress analysis, the solution is checked again. The whole procedure is repeated until the designated CMODs are reached.

Evaluation of Ground Pressure Based on the Quasi Loosening Zone Model

Ground pressure acting on tunnel linings can be defined as the external loads induced when that part of the ground in the vicinity of the tunnel endures large deformation due to tunneling, as a result of the release and redistribution of the stress fields during and after tunnel excavation. Among the many theoretical approaches for estimating these pressure loads which may involve different mechanisms for their occurrence, methods based on Terzaghi's theory on loosening zones have been widely used [2]. In the following, a simple model is defined for calculating the ground pressure, which assumes a simple form so as to facilitate iterative computations required by the solution procedures as described below. Named a quasi loosening zone model, it basically follows Terzaghi's concept of calculating the ground pressure from the depth of the loosening zone, even though other mechanisms might be involved. As shown in Fig. 6, the model is defined by three points. The first two points are the locations where the two slip lines meet the horizontal line passing the crown. The third point represents the estimated depth of loosening zone, usually based on field measurements. The three points are then connected by smooth curves to introduce the arch action in the ground. If the ground and the lining are assumed to be in contact with each other (excluding areas where voids exist in between), the ground deformation must equal to the cross-sectional deformation of the tunnel. As the lining deformation can be calculated independently by crack analysis using the pseudo-shell model, in order to find out the ground pressure it is sufficient to modify the quasi loosening zone through iterative computations until

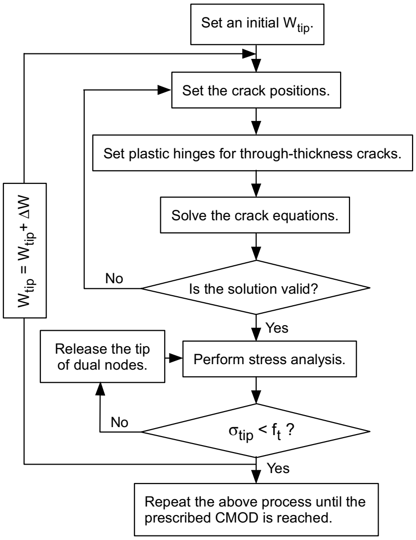

Fig. 5 Solution procedure for crack analysis using the COD-controlled method

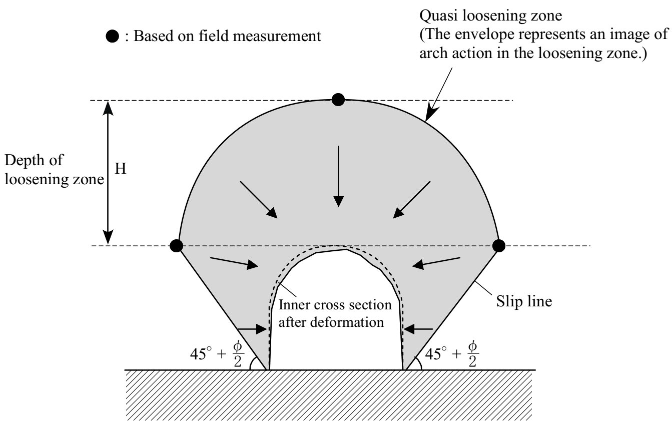

Fig. 6 Conceptual view of the quasi loosening zone model

the differences between the ground deformation and the lining deformation at key points of comparison (usually at inner cracks) can be ignored. In numerical analysis, the ground deformation is calculated under the vertical and lateral pressure loads. The task is to carefully adjust the depth of loosening zone, the coefficient of lateral pressure and other material properties until the ground deformation converges to the cross-sectional deformation of the tunnel.

Numerical Analysis of an Aging Waterway Tunnel in Comparison with a Soil Mechanics Approach

Background. Numerical analyses of an aging waterway tunnel of a hydraulic power facility are carried out, and the numerical results are compared with those obtained previously using the Adachi-Oka model, which is an established constitutive model in geotechnical engineering [3]. The model simulates the time-dependent behavior of geological materials that show strain-softening characteristics.

Fig. 7 shows a cross section of the tunnel, which was constructed in the early 1960s and had been in service for over 30 years before major maintenance work was carried out. The tunnel passes under massive layers of unaltered sedimentary rocks of approximately 400 meters in depth. In its vicinity much more weakly consolidated sandstone and clay, which are extensively disturbed, exist. Based on the results of boring tests and PS logging, the loosening zone, which was formed during tunnel excavation, is estimated to reach a depth of 2 m. During excavation a void was presumably formed above the ceiling area, and the records show that two longitudinal cracks nucleated in the arch areas on the lining surface shortly after the completion of the tunnel. Circumstantial evidence also points to the existence of another crack at the crown, from the outer surface of the ceiling. At the time of the maintenance work, the CMODs of the two surface cracks had reached approximately 2 mm and 3 mm, respectively.

Numerical Analysis by Adachi-Oka Model. A detailed description of the Adachi-Oka model is out of the scope of this paper. Thus, a simple outline of the constitutive model is stated. The shear strength of soft rock and over-consolidated clay consists of the strength due to cementation or bonding and the strength due to friction. With the gradual increase of shear strain, the former diminishes while the latter grows. This process is manifested through strain softening. In Adachi-Oka's elasto-viscoplastic model, the stress history tensor is expressed by introducing a single exponential type of kernel function,

$$\sigma_{ij}^{*} = \frac{1}{\tau}\int_{0}^{z} \exp(-(z-z')/\tau)\sigma_{ij}(z')dz' . \qquad (9)$$

where τ is a material parameter which expresses the retardation of stress with respect to the time measure. The incremental time measure is defined as

$$dz = g(\dot{\varepsilon}_{ij})dt \cdot \qquad (10)$$

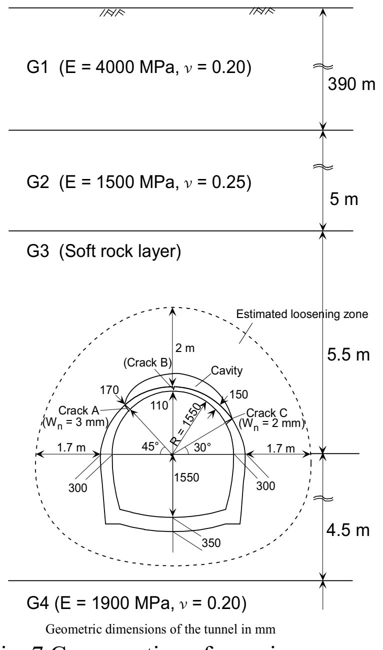

Fig. 7 Cross-section of an aging waterway tunnel and geological conditions

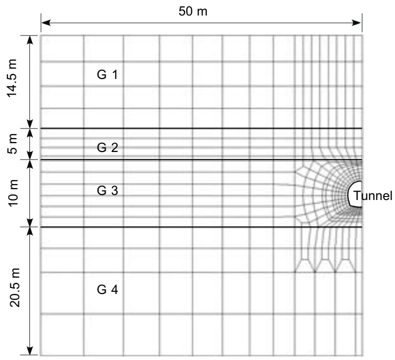

Fig. 8 Finite element meshes in the elasto-viscoplastic modeling of tunnel deformation

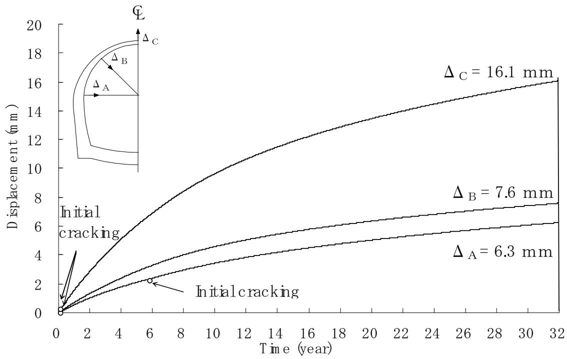

Fig. 9 Time-dependent history of the cross-sectional deformation obtained by the elasto-viscoplastic modeling

where g is an experimentally determined function of strain rate, and t is the time. Introducing a yield function and a non-associated flow rule, a constitutive equation is derived to describe the time-dependent stress-strain relation of geological materials. The constitutive model employs the following material parameters: E of the modulus of elasticity; ν of Poisson's ratio; b and σ_{mb} of plastic potential parameters; G' and M_f^* of strain hardening-softening parameters; τ of a stress-history parameter; M_m of a parameter of over-consolidated boundary; a and C of parameters of time dependency. These parameters are determined from conventional triaxial tests.

The main purpose of the original study using the Adachi-Oka model was to investigate the cross-sectional deformation of the tunnel lining and the time variation of earth pressure on the lining. The numerical analyses were carried out in the plane strain condition, and the tunnel lining was modeled using beam elements. Fig. 8 shows the finite element mesh, and Table 1 lists the parameters of the elasto-viscoplastic model. Table 2 summarizes the main parameters of the beam elements, where E is the modulus of elasticity, I is the moment of inertia, A is the cross-sectional area of the lining, and M_y is the moment of crack initiation. Note that as the moment reaches M_y, a hinge is set to the beam at the location where the critical moment occurs.

The obtained numerical results were examined and verified by geotechnical surveys and in situ tests. Here, the time-dependent, cross-sectional deformation of the tunnel lining is shown in Fig. 9. According to the numerical analyses, approximately two months after the completion of the tunnel initial cracks appeared in the arch area and at the crown simultaneously. Approximately six years later, another crack occurred at the spring line. The final cross-sectional deformation after 32 years in

Table 1 Material parameters of layer G3 in elasto-viscoplastic modeling

E [Mpa]	ν	b [Mpa]	σ_{mb} [Mpa]	G'	M_f^*	τ	M_m	a	C
300.00	0.25	0.87	18.00	45.40	1.15	90000	1.25	0.959	0.565

Table 2 Material properties of the beam elements in elasto-viscoplastic modeling

Member \ Item	E [Gpa]	A [cm^2]	I [cm^4]	M_y [kN·m]
Lining	26.60	1500	28125.00	7.50
Invert	26.60	3500	357291.67	40.83

Note: unit thickness = 1.0 m.

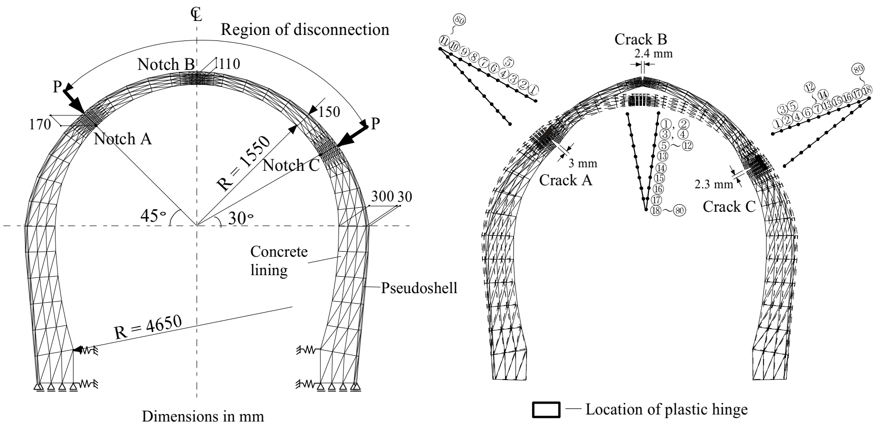

Fig.10 FE model and the results of crack analysis

service was estimated as 16.1 mm at the crown, 7.6 mm in the arch area, and 6.3 mm at the spring line.

Numerical Analysis by the Pseudo-Shell Model. Fig. 10 presents the FE model of the cross section of the tunnel, excluding the invert. Here, the thickness of the pseudoshell is assumed to be 30 mm, just one tenth of the lining thickness at the spring line. Note that the lining is completely separated from the pseudoshell in the range of the void. At the bottom of the wall, hinge and spring supports are assumed in the vertical and horizontal directions, respectively. As illustrated, the numerical case contains three initial notches at the actual crack locations. Judging from the crack condition, two concentrated loads are applied to the pseudo-shell, along the crack paths of the two surface cracks. The material properties used in numerical studies are summarized in Table 3, and a bi-linear tension-softening relation is employed to solve the crack equations.

Also shown in Fig. 10 are the results of crack analysis on crack propagation, with detailed information showing the tip position of each crack at the given computational step. As seen, crack A of the left arch is most active, progressing forward at every computational step, except for the 5th step when it becomes temporarily inactive as crack B becomes a one-step leading crack. As crack A becomes a through-thickness crack at the 11th step, subsequent computations are then carried out using the COD-controlled method. It is worth repeating that, as a crack penetrates through the lining, a plastic hinge is introduced to the pseudo-shell to allow the crack to fully open. At the 18th step, the other two cracks penetrate through the lining simultaneously, and all the three cracks are open at this stage. Finally at the 80th step, the designated CMOD of 3 mm for crack A is reached, while the CMODs of crack B and crack C reach 2.4 mm and 2.3 mm, respectively. Note that the obtained CMOD of crack C represents well its on-site measurement of approximately 2 mm.

Fig. 11 presents the relations between the cross-sectional deformation and the CMOD. As shown, the obtained vertical displacement at the crown is 15.5 mm, and the arch deformations at A and C are 7.3 mm and 6.1 mm, respectively. Compared with the results obtained by the Adachi-Oka model, which predict a deformation of 16.1 mm at the crown and 7.6 mm in the arch area some 32 years after the completion of the tunnel, the agreement between the two approaches is indeed remarkable. These results convincingly prove the validity of the previous assumption that the lining deformation can be uniquely determined if the actual cracking behavior in a tunnel can be reproduced through crack analysis using the pseudo-shell model.

Table 3 Material properties of lining concrete and pseudoshell

Lining concrete					Pseudoshell	
E [Gpa]	ν	f_c [Mpa]	f_t [MPa]	G_F [N/mm]	E' [Gpa]	ν
26.6	0.20	39.0	2.0	0.1	200.0	0.2

In building a powerful

constitutive model to describe the gross behaviors of geological materials, it is inevitable to employ a large number of material parameters to capture every important facet of the mechanisms that cause the overall behavior. Obviously, the accuracy of these material models depends very much on the accuracy of these material parameters. Most of them can only be determined through rigorous tests. The pseudo-shell model, which is a unique structural model, presents a different approach. Exploiting the uniqueness of the solution on deformation in the beam theory, this approach focuses on the individual cracks in the lining and the cross-sectional deformation of the tunnel is obtained as a result of crack analysis, without considering the complicated details of geological materials that are of time-dependent nature.

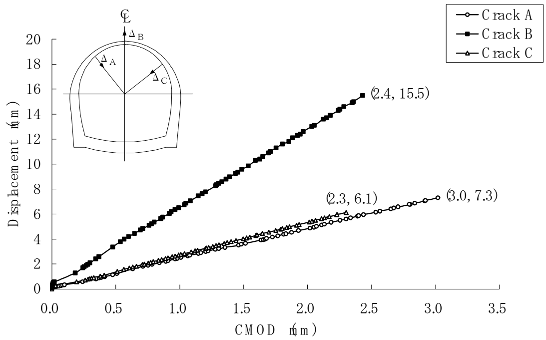

Fig.11 Cross-sectional deformation vs. CMOD relations

Table 4 Material properties of rock mass

γ [g/cm^3]	E [Mpa]	ϕ [°]	ν
2.4	100	20	0.25

Evaluation of Ground Pressure. Material properties of the rock mass are shown in Table 4. Based on the obtained lining deformations at cracks A and C, the depth of loosening zone is adjusted first through iterative computations until a reasonable match is reached. Then, the coefficient of lateral pressure and the loading range are also modified so that the ground deformation in the sidewall sufficiently converges to the lining deformation there. Numerical results are shown in Fig. 12, and the depth of loosening zone is found to be 4 m, twice the initial estimation of 2 m. According to the analysis, the tunnel is under isotropic pressure loads of 0.096 MPa.

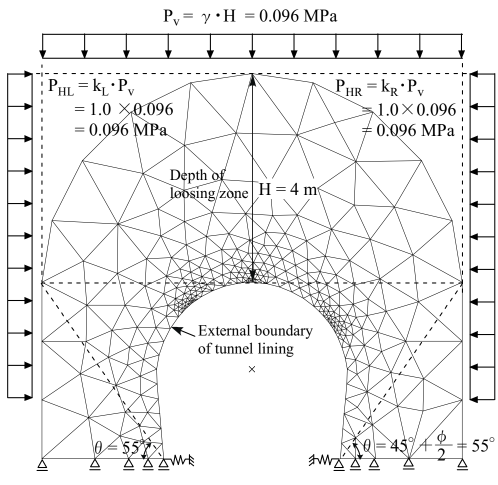

Fig.12 Obtained loosening zone and pressure loads

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