Prediction of Crack Growth in Higher Strength Steels Subjected to Constant Amplitude Loading with Overloads

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Abstract. The paper presents the study of fatigue crack growth under constant amplitude loading with single overload cycles for two grade low alloy higher strength steels:18G2A and 18G2AV. The tests was carried out on CCT specimen under tensile fatigue loading at stress ratio R=0,2. Using experimental data of fatigue crack growth rate a deterministic approach has been proposed to predict the crack growth after single overloads. For calculating the fatigue crack growth rate in the tested steel samples the Willenborg crack retardation model was applied.

Introduction

A fatigue crack growth rate under constant amplitude loading (CAL) is retarded after single or multiple tensile overloads [1,2]. This occurs due to the presence of a larger zone of plastic deformation and compressive residual stresses ahead of the crack tip produced by the overloads. The fatigue crack growth rate after overloads is described usually by Wheeler retardation model. This model is bases on known formulas of fatigue crack growth rate, for example Paris formula, with retardation parameter. This paper presents a Willenborg residual stress model for remaining life prediction of the cracked flat steel samples after overload under constant amplitude loading. This model is a modification of Wheeler fatigue crack retardation model.

Description of a fatigue crack growth retardation

For steels and other metals, it has been observed that an initial acceleration of fatigue crack growth rate occurs immediately after an overload (Fig. 1). The fatigue crack growth rate then decelerates to a minimum value dl/dN_{min} [2,3] at some point of the overload retardation (at corresponding crack length l_{min}), upon which the crack growth rate gradually increases and returns to its right value at constant amplitude loading dl/dN_{ca}.

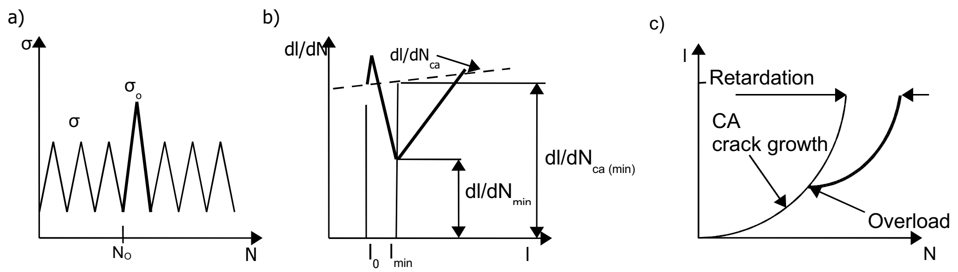

Fig. 1. Fatigue loading with overload(a), crack growth rate (b) and retardation after overload (c)

Delay of the crack growth is results from existence of zone of residual compressive tensions generated by plastic deformation on the crack tip after relief [2] (Fig. 2a). After overload crack grows up more slowly because the development of current plastic zone be holds in the overload plastic zone created near length crack l_o [2,3], (Fig.2b).

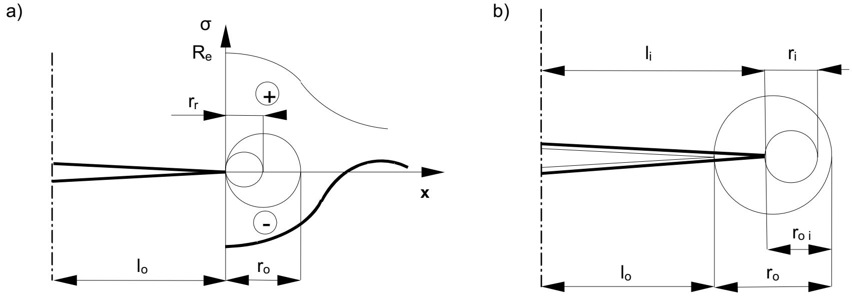

Fig. 2. Plastic zones and stresses on crack tip (a), model of crack growth in overload plastic zone

The Wheeler model. This model is a simple model that calculates the fatigue crack growth retardation following a single tensile overload by introducing a retardation parameter C_{pi}, which can be multiplied to any constant amplitude fatigue model of crack growth rate [2,3]. In here, it is applied to the Paris equation (Eq. 1). It in the Wheeler model was put was, that the crack growth rate after overload steps out only. The Retardation parameter which defines the relation of the plastic zone radius $r_i/r_{o\,i}$ is determine on basis of experimental results of crack growth rate: after overload dl/dN_i and at the constant amplitude load dl/dN_{ca} (Eq. 2).

$$\frac{dl}{dN} = C \cdot \left(\frac{dl}{dN}\right)_{ca} = C_{p_i} \cdot C \cdot \Delta K^m \tag{1}$$

where C, m is parameters of Paris formula and C_{p_i} is the retardation parameter:

$$\left.\begin{array}{ll} C_{p_i} = \left(\dfrac{r_i}{r_{o_i}}\right)^p = \left(\dfrac{r_i}{l_o + r_o - l_i}\right)^p = \dfrac{(dl/dN)_i}{(dl/dN)_{ca_i}} & \text{for } l_i + r_i < l_o + r_o \\ C_{p_i} = 1 \quad \text{for } l_i + r_i \geq l_o + r_o. & \end{array}\right\} \tag{2}$$

The plastic zone radius usually was determined from equations [2,3]:

$$r = \frac{C_d}{2\pi}\left(\frac{K_{\max}}{R_e}\right)^2 \text{ for } C_d\text{=1 – plane stress,} \quad r = \frac{C_d}{6\pi}\left(\frac{K_{\max}}{R_e}\right)^2 \text{ for } C_d\text{=1 – plane strain.} \tag{3}$$

It whole plastic zone on a crack tip [3,4] (describing by diameter d=2r) was applied in proposed by authors the Willenborg model (Eq. 4). Coefficient C_d in Eq. (4) defines on basis test results.

$$d = 2r = \frac{C_d}{\pi}\left(\frac{K_{\max}}{R_e}\right)^2 \text{ for plane stress,} \qquad d = 2r = \frac{C_d}{3\pi}\left(\frac{K_{\max}}{R_e}\right)^2 \text{ for plane strain} \tag{4}$$

The samples with central crack - CCT (Fig. 3a) do not fulfill conditions of plane strain state (Eq.5) with regard on small thickness t [3,4]. Initially the fatigue crack tip is in conditions of plane stress on surface of sample as well as the plane strain in interior of sample. The conditions of plane strain in interior of sample changes in plane stress with development the crack together with. Therefore in this work is proposed the model of plastic zone describing this change (Fig. 3b).

$$\left.\begin{array}{l} t \geq 2{,}5 \cdot \left(\dfrac{K_{\max}}{R_e}\right)^2 \text{ - plane strain,} \qquad w - 2l \geq 2{,}5 \cdot \left(\dfrac{K_{\max}}{R_e}\right)^2 \text{ - plane strain} \\[2ex] t \leq \dfrac{1}{3\pi}\left(\dfrac{K_{\max}}{R_e}\right)^2 \text{ for plane stress} \end{array}\right\} \tag{5}$$

$$d = \frac{C_d}{\pi}\left(\frac{K_{\max}}{R_e}\right)^2 f_d = \frac{C_d}{\pi}\left(\frac{K_{\max}}{R_e}\right)^2\left(C_s + \frac{(1 - C_s)^3}{3}\right), \qquad C_s = \frac{d_{plane-stress}}{t} \tag{6}$$

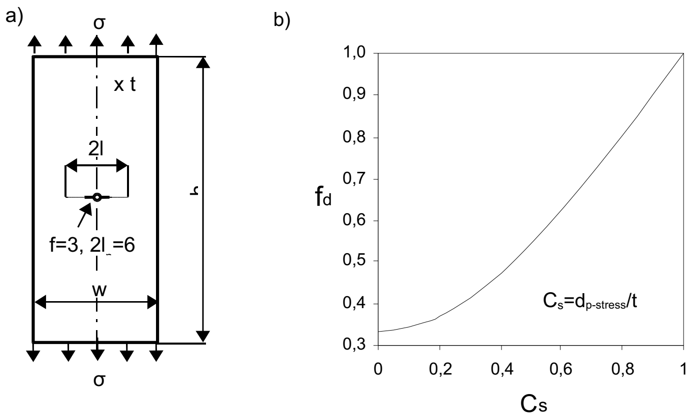

Fig. 3. The sample with central crack (a) and change of the function of strain/stress state

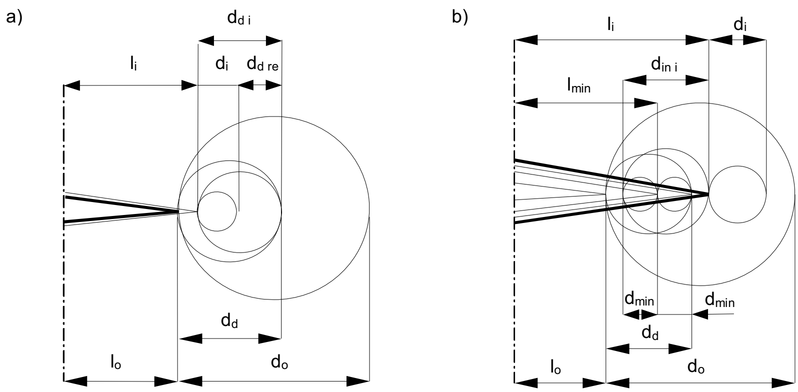

Fig. 4. The Willenborg model in zone of decreasing (a) and increasing (b) of crack growth rate

The Willenborg model. The authors in this paper presented a Willenborg residual stress model for prediction of crack growth in the flat samples after overloading. The proposed model takes into considerate both the zone of decreasing how and increasing the fatigue crack growth rate after overload, similarly how different the modifications of the Wheeler model [5,6]. In the first zone of Willenborg model (Fig. 4a) the reduced stress σ_{red} was introduced was necessary to conquest of the plastic zone d_{di} called out with tensile overload cycle by the crack l_i together with from his the current plastic zone d_i in *i*-cycle of loading. The crack growth to the length l_{min} which answering of lowest crack rate dl/dN_{min} and $d_{di}=d_{min}$ is end of first zone [7] and begin of second zone (Fig. 4b) with conquest of the plastic zone d_{min} [8,9]. The crack growth in this zone be connected with growth of defeated plastic zone $d_{in\,i}$ at the reduced stress σ_{red} then.
According from the Willenborg model in zone of decreasing of crack growth is in force following dependence [9,10]:

$$\text{for } l_i + d_i \le l_o + d_d \quad \text{is} \quad l_o + d_d = l_i + d_{di} \tag{7}$$

$$l_o + d_d = l_i + \frac{C_d}{\pi}\left(\frac{K_{red_i}}{R_e}\right)^2 = l_i + \frac{C_d}{\pi}\left(\frac{\sigma_{red_i}\sqrt{\pi \cdot l_i}\,M_{c_i}}{R_e}\right)^2 \tag{8}$$

$$\sigma_{red} = \frac{R_e}{M_{c_i}\sqrt{C_d}}\sqrt{\frac{l_o + d_d - l_i}{l_i}} \tag{9}$$

$$C_{pd_i} = \left(\frac{d_{d_i}}{d_i}\right)^{pd} = \left(\frac{l_o + d_d - l_i}{d_i}\right)^{pd} = \frac{(dl/dN)_i}{(dl/dN)_{\min}}, \quad \text{for } l_i + d_i = l_o + d_d \; (l_i = l_d) \quad C_{pd_i} = 1. \tag{10}$$

However for zone of increasing of crack growth is in force following dependence [7]:

$$\text{for } l_{\min} + d_{\min} < l_i + d_i < l_o + d_o \quad \text{is} \quad l_{\min} - d_{\min} = l_i - d_{in_i} \tag{11}$$

$$l_{\min} - d_{\min} = l_i - \frac{C_d}{\pi}\left(\frac{K_{red_i}}{R_e}\right)^2 = l_i - \frac{C_d}{\pi}\left(\frac{\sigma_{red_i}\sqrt{\pi \cdot l_i}\,M_{c_i}}{R_e}\right)^2 \tag{12}$$

$$\sigma_{red} = \frac{R_e}{M_{c_i}\sqrt{C_d}}\sqrt{\frac{l_i - l_{\min} + d_{\min}}{l_i}} \tag{13}$$

$$C_{pin_i} = \left(\frac{d_{\min}}{d_{in_i}}\right)^{pin} = \left(\frac{d_{\min}}{l_i - l_{\min} + d_{\min}}\right)^{p_{in}} = \frac{(dl/dN)_i}{(dl/dN)_{\min}}, \quad \text{for } l_i = l_{min} \; C_{pin_i} = 1. \tag{14}$$

The residual stress σ_{res}, effective stress σ_{eff} and stress intensity factor K_{eff} on the crack tip have following form [10]:

$$\left.\begin{array}{l} \sigma_{res_i} = \sigma_{red_i} - \sigma_{\max} \\ \sigma_{eff_{\max i}} = \sigma_{\max} - \sigma_{res_i} = 2\sigma_{\max} - \sigma_{red_i}, \quad \sigma_{eff_{\min i}} = \sigma_{\min} - \sigma_{res_i} = \sigma_{\min} + \sigma_{\max} - \sigma_{red_i} \end{array}\right\} \tag{15}$$

$$K_{eff_{\max_i}} = \sigma_{eff_{\max_i}} \sqrt{\pi \cdot l_i} M_{c_i}, \qquad K_{eff_{\max_i}} = \sigma_{eff_{\max_i}} \sqrt{\pi \cdot l_i} M_{c_i}. \tag{16}$$

According from the Willenborg model the crack growth rate and crack growth increase after overload are defined by force following dependence:

$$\left(\frac{dl}{dN}\right)_i = C \cdot C_{p_i} \left(\Delta K_{eff_i}\right)^m = C \cdot C_{p_i} \left(K_{eff\max_i} - K_{eff\min_i}\right)^m \tag{17}$$

$$\Delta l = \sum_{N_o}^{N_e} \Delta l_i = \sum_{N_o}^{N_e} C \cdot C_{p_i} (\Delta K_{eff\,i})^m \cdot N_i. \tag{18}$$

Experimental investigation

The investigation of fatigue crack growth were carried out on flat CCT samples (Fig. 3a) under constant amplitude loading (σ_{max}=170 MPa, R=0,2) with single overloading (OVL). The samples were made from two grade low alloy higher strength ferritic-perlitic steels: 18G2A-stel (w=120, t=10 mm, R_e= 365 MPa, R_m= 584 MPa,) and 18G2AV (w=120 mm, t=15 mm R_e= 491 MPa, R_m=660 MPa). It was applied three overload ratio OR=1,2, 1,4 and 1,6. To approximation the test results of crack growth (crack length l and number of cycle N) after any overload were applied polynomial function which derivate determinates the crack growth rate (Eq. 19) For example approximating of crack growth test results for sample made of 18G2AV steel after second overload showed in Fig. 5.

$$l = \sum_{j=0}^{n} A_j N^j, \qquad \frac{dl}{dN} = k \sum_{k=1}^{n} k A_k N^{k-1} \tag{19}$$

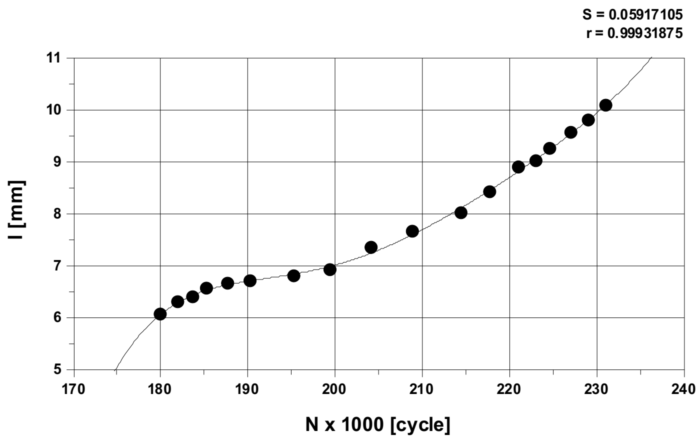

Fig. 5. Polynomial approximating of test results in 18G2AV steel: OR=1,2, 2l/w=0,102

The test results for 18G2AV steel (Fig. 6) showed that after the first overload appears retardation of crack growth with significant decreasing of crack growth rate and after next overloads the falls of crack growth rate were smaller. However after last overloading the fall of crack growth rate was inessential.

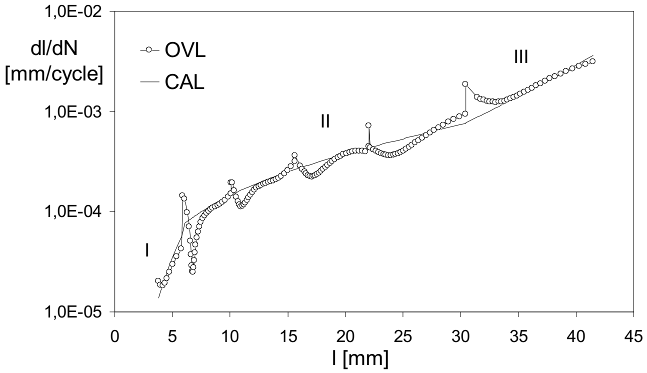

Fig. 6. Fatigue crack growth rate in 18G2AV steel under CAL with overloading (OR=1,2)

Similarly the test results for 18G2A steel showed that after overloads appear retardations of crack growth with significant decreasing of crack growth rate and at first overload was order of magnitude (Fig. 7). However the decreasing of crack growth rate at longer crack was smaller. The lowering the crack growth rate lasted longer at higher ratio overload.

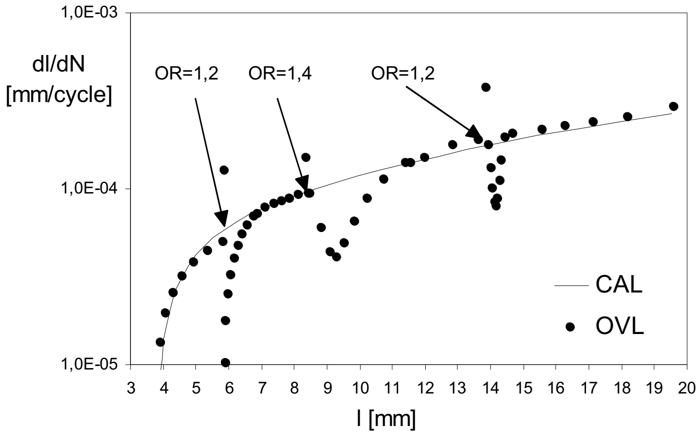

Fig. 7. Fatigue crack growth rate in 18G2A steel under CAL with overloading (OVL)

The influence overloads in this steel for higher overload ratios OR =1,4 and OR=1,6 is presented in Fig. 8a and Fig. 8b respectively. The Paris equation for samples made of this steel was determined by constant amplitude fatigue test as description the crack growth rate in three regions: in the end phase of I region (C=1,47E-13, m=6,78), in II region (C=5,29E-8, m=2,48) and in begin the phase of III region (C=2,68E-12, m=5,03). Generally at higher overload ratio the falls of crack growth rate after overload were greater, than at overload ratio OR =1,2 and after overload were little. After first overload for OR=1,4 and after first, second and third overload for OR=1,6 the falls of crack growth rate were very large (about order of magnitude).

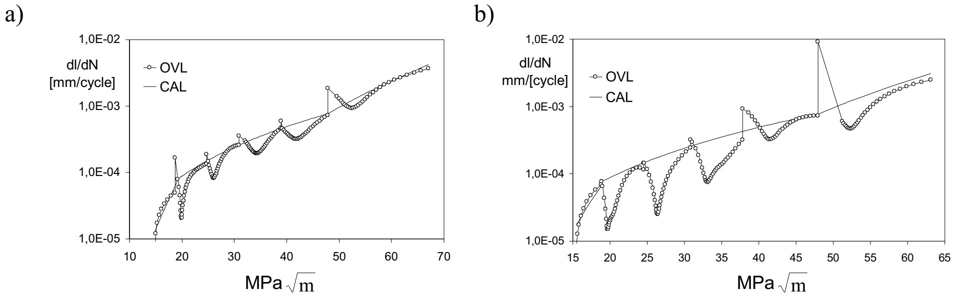

Fig. 8. Fatigue crack growth rate in 18G2AV steel after overloading: a) OR=1,4, b) OR=1,6

The reduced stresses and the residual stresses on investigation basis were calculated. For example the results of these calculation for 18G2AV steel at overload ratio OR=1,4 were introduced on drawings (Fig. 9a and Fig. 9b). The results of calculation showed, that after any overload the stresses σ_{red} and σ_{res} diminished to a minimum values which answered crack growth rate dl/dN_{min}.

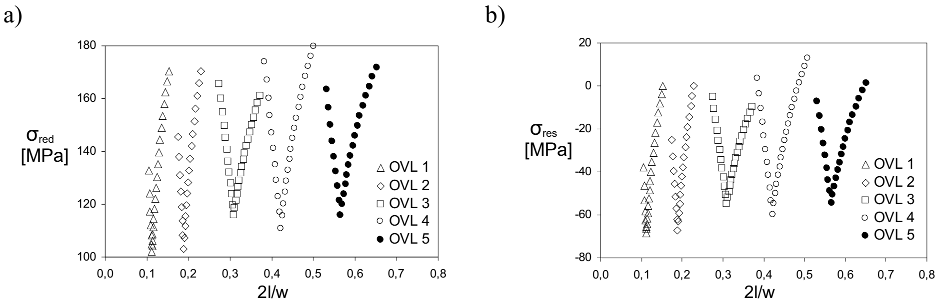

Fig. 9. Reduced stress (a) and residual stress (b) in 18G2AV steel after overloading OR=1,4

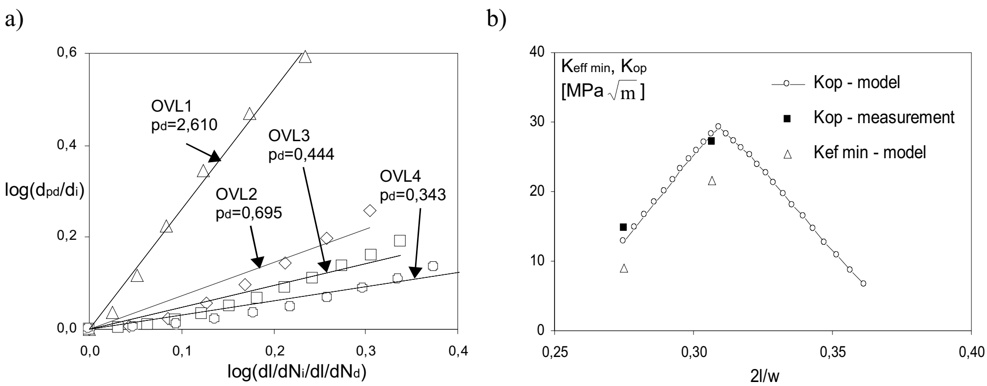

Fig. 10. Exponent of retardation ratio in Eq. 10 (a) and comparison of $K_{eff\,min}$ with K_{op} (b) for 18G2AV steel at OR=1,4

In Fig. 10a the experimental diagrams are presented necessary to obtaining values of exponent of retardation ratio. The stress intensity factor $K_{eff\,min}$ calculated on investigation basis were lower than K_{op} obtained on basis optical measurements of the crack opening by microscope at large magnification (Fig.10b). In this cause the calculation of crack growth increase after overload was one should accept value K_{op} because the crack possible growth is only when crack is open. In Fig.

10b is presented curve obtained values of $K_{op\text{-}model}$; which was calculated on basis real data crack growth rate after overload from transformed Eq.17. The values of stress intensity factor $K_{op\text{-}model}$ were comparable with the test results obtained on basis optical measurements $K_{op\ measurement}$.

Summary and conclusions

1. In This paper is a present the Willenborg residual stress model for prediction of the crack growth in structural higher strength steels under constant amplitude tensile loading with overloads. This proposed model is modification of the Wheeler model retardation. The crack growth is described for two ranges: decrease and increase of crack growth rate at existing reduced stress in zone residual stress called plastic deformation on crack tip.
2. The proposed prediction crack growth model has been carried out by employing the linear elastic fracture mechanics principles. However in this work is proposed the equivalent model of plastic zone in sample, which takes into consideration conditions on crack tip (change from plane strain to plane stress in interior of sample during crack growth).
3. Reduced stress and residual stress in this zone were estimate on basis experimental data of crack growth rate in steel samples at various values of the overload ratio. If value of stress intensity factor $K_{eff\ min}$ is lower than K_{op} this calculation of the crack growth increase after overload was one should accept value K_{op} because the crack possible growth is only when crack is open.

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