On Constraint-modified Failure Assessment Diagrams for Solids with a Crack/Notch

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Abstract. The local failure criterion in the form of the average stress limitation in the cohesive zone ahead of the crack/notch tip was employed to describe the failure assessment diagram for a solid with a finite crack or notch under mode I loading. The cohesive strength is treated according to von Mises yield criterion as a property of both the yield stress and the *T*-stress which was introduced into the criterion of the average stress to quantify constraint in different configurations of specimens and type of loading (uniaxial and biaxial tension, bending, etc). The failure criterion has been presented for constraint-modified failure assessment diagrams for solids with a crack and U-notch. The results clearly demonstrate that the stress biaxiality ratio as a normalised measure of the structural constraint does not change the character of the failure assessment curves. But, biaxial failure assessment curves move outward from the uniaxial case in conformity with quantitative and qualitative changes of the cohesive stress.

Introduction

The defect assessment method of the damaged component can be based on the failure assessment diagram (FAD). The basic failure curve of the FAD is written as $K_r = f(L_r)$, where $K_r = K_1 / K_{mat}$ is the ratio of the applied stress intensity factor K_1 to the material's fracture toughness K_{mat} and L_r is equal to the ratio of applied load P to plastic collapse load P_Y.

At the present time the different sources of a change in constraint due to type of loading, crack size and notch effect have been treated separately to modify the FAD. For example, the FAD has been modified using the concept of the notch stress intensity factor for a notch-like defect taking into account a finite notch tip radius [1, 2]. In this case, the fracture toughness or so-called the notch fracture toughness, which is applied to the notch FAD, should be measured for a structural component. However, assessment tools for the complex treatment of the loss of constraint are limited in the literature. It should be noted that the global treatment of the loss of constraint recently proposed in Ref. [3] was successfully validated.

The aim of this paper is to give the alternative complex approach for the treatment of a reduction in constraint to modify the FAD. The constraint-modified FAD has been based on the criterion of the average stress ahead of the crack/notch tip taking into account the cohesive strength as a function of the yield stress, the failure applied stress and the biaxiality ratio. In the present research the FAD is considered in terms of K_1 / K_{mat} and σ_C / σ_Y, where σ_Y is the yield strength.

Failure criterion and failure assessment diagrams

The cohesive zone model and criterion of average stress in the cohesive zone ahead of the crack/notch tip have been used to describe failure assessment diagrams for cracked and notched bodies [4, 5].

For an infinite plate with a crack under a remotely applied tensile stress, the stress ahead of the crack tip on the crack extension line was given by the exact elastic solution according to the Westergaard's theory. In this case, the failure criterion for a solid with a finite crack is written by the following criterion equation

$$K_1 = K_{mat}\sqrt{1-\left(\frac{\sigma_C}{\sigma_{coh}}\right)^2}, \tag{1}$$

which allows describing the FAD. Here, σ_{coh} is the cohesive strength ahead of the crack tip, K_{mat} is the fracture toughness, σ_C is the applied (critical) stress at failure.

For a solid with a U-notch, the normal stress distribution at the notch tip, used in the criterion of the average stress, was employed in the form suggested in Ref. [6]. It should be pointed out that such stress distribution was suggested for blunt cracks when the distance ahead of the crack tip is much smaller than the crack length and greater than the crack tip radius. The failure criterion of the average stress in the cohesive zone ahead of the notch tip leads to the FAD as follows [5]

$$K_{1notch} = K_{mat}\sqrt{1-\left(\frac{\sigma_C}{\sigma_{coh}}\right)^2}\left[1-\left(\frac{\sigma_{coh}}{\sigma_C}\right)^2\frac{1}{K_t^2}\right]^{-1/2}, \tag{2}$$

where K_t is the elastic stress concentration factor. The stress intensity factor at the notch tip is denoted as K_{1notch}.

Influence of the *T*-stress on the cohesive strength

The cohesive strength for an infinite plate. The cohesive zone parameters are, in general, not material constants but dependent on the local constraint conditions around the crack tip (e.g., [7]). For example, the *T*-stress can be used as a constraint parameter which has an influence on the cohesive strength ahead of the crack tip [8]. The *T*-stress is a non-singular stress in the general form of the linear elastic crack tip stress fields given by Williams

$$\sigma_{ij} = \frac{K}{\sqrt{2\pi r}} f_{ij}(\theta) + T\delta_{1i}\delta_{1j},$$

(3)

where δ_{ij} is Kronecker's delta, functions $f_{ij}(\theta)$ define the angular variations of in-plane stress components. The sign and magnitude of the *T*-stress can substantially alter crack tip constraint. Positive *T*-stress leads to high crack tip constraint; while negative *T*-stress leads to the loss of the crack tip constraint.

In the present work, the cohesive strength σ_{coh} ahead of the crack tip is treated as the stress which is independent on the separation distance in the cohesive zone and determined by von Mises yielding criterion within the cohesive zone according to flow law

$$\Phi(\sigma_1,\sigma_2,\sigma_3,\sigma_Y) = (\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_1-\sigma_3)^2 - 2\sigma_Y^2 = 0, \tag{4}$$

where σ_Y is the yield strength. The stresses σ_1, σ_2 and σ_3 are the principal normal stresses and in the cohesive zone ahead of the crack tip for certain crack geometries and loading conditions.

Analogously to Williams's solution, the stress σ_2 parallel to the crack plane is suggested to be given by equation $\sigma_2 = \sigma_1 + T$. The stress σ_3 is equal to $\nu(\sigma_1 + \sigma_2)$ and 0 for plane strain and plane stress, respectively. The cohesive strength σ_{coh} is supposed to be σ_1 and can be expressed from von Mises yield condition (4) by the following equations

$$\sigma_{coh} = -\frac{T}{2} + \sigma_Y \sqrt{\frac{1}{4}\left(\frac{T}{\sigma_Y}\right)^2 - \frac{(1+\nu^2-\nu)(T/\sigma_Y)^2 - 1}{(1-2\nu)^2}} \quad (5)$$

for the case of plane strain and

$$\sigma_{coh} = -\frac{T}{2} + \sigma_Y \sqrt{1 - \frac{3}{4}\left(\frac{T}{\sigma_Y}\right)^2} \quad (6)$$

for the case of plane stress.

The cohesive strength for finite geometries. Since the *T*-stress scales linearly with applied load in infinite geometries, most computational studies report *T*-stress values for various crack configurations as follows

$$T = \beta(a/W)\sigma^{\infty}, \quad (7)$$

where $\beta(a/W)$ is a dimensionless parameter (so-called biaxiality ratio) which depends on geometry and loading mode, σ^{∞} is an applied stress, a and W are the crack length and width of a body, respectively. Values of $\beta(a/W)$ can be considered as a normalized measure of the crack tip constraint and have been tabulated (e.g. [9]) for various geometries.

Thus, the cohesive strength for finite geometries can be rewritten as a function of the applied failure stress $\sigma^{\infty} = \sigma_C$ and the crack tip constraint characterized by the value of $\beta(a/W)$

$$\frac{\sigma_{coh}}{\sigma_Y} = -\frac{\beta}{2}\left(\frac{\sigma_C}{\sigma_Y}\right) + \sqrt{\frac{1}{4}\left(\frac{\beta\sigma_C}{\sigma_Y}\right)^2 - \frac{(1+\nu^2-\nu)(\beta\sigma_C/\sigma_Y)^2 - 1}{(1-2\nu)^2}} \quad (8)$$

for the case of plane strain and

$$\frac{\sigma_{coh}}{\sigma_Y} = -\frac{\beta}{2}\left(\frac{\sigma_C}{\sigma_Y}\right) + \sqrt{1 - \frac{3}{4}\left(\frac{\beta\sigma_C}{\sigma_Y}\right)^2} \quad (9)$$

for the case of plane stress. For the classical infinite plate under uniaxial loading β=-1, special case of the cohesive strength occurs [10].

The present results have been used below for describing failure assessment diagrams for finite geometries of a solid with a crack or notch.

Validation

To demonstrate the validity of the FAD described by the failure criterion (1), it is directly applied to a number of experiments reported in the literature.

The validation study is made on through cracked plates under uniaxial loading. The fracture toughness K_{mat} and the cohesive strength in Eq. 1 have been evaluated from the measured fracture data. The FAD has been constructed using these parameters and the stress intensity factor at failure.

The fracture data of through thickness centre crack tensile specimens (having 1.6 mm thickness and 76.2 mm width) made of AA2014-T6 aluminium alloy at 20 K has been employed from Ref. [11]. For the through thickness centre cracked plate subjected to an uniform tensile stress σ_C, the stress intensity factor K_1 at failure can be given by the following well-known equation

$$K_1 = \sigma_C \sqrt{\pi a} Y , \tag{10}$$

where

$$Y = \sqrt{\frac{\pi}{4} \sec\left(\frac{\pi a}{W}\right)} ,$$

a is half the crack length, W is the width, σ_C is the failure stress.

The fracture toughness K_{mat} in Eq. 1 are obtained by correlating the fracture data K_1 of through thickness centre crack tensile specimens. The cohesive strength was calculated from Eq. 9 for plane stress. In this case, the biaxiality ratio β is assumed to be -1 independently on the crack aspect ratio a/W.

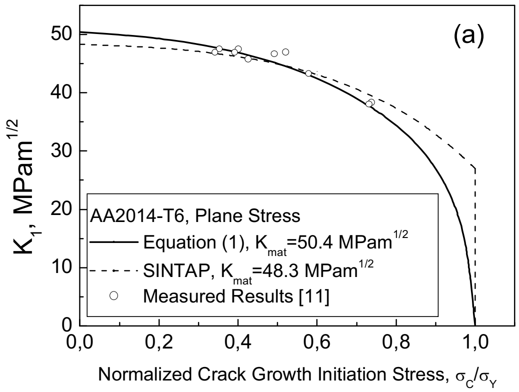
Using the fracture toughness K_{mat}, as the average value for test specimen sets, in the failure criterion (1), fracture analysis has been carried out on these specimens and compared with the test results. Figure 1a shows the failure assessment diagram of the material along with the fracture data (the measured stress intensity factor K_1 at failure versus the measured normalized failure stress σ_C) of through thickness cracks of tensile specimens. The predicted failure assessment diagram is quantitatively consistent with the measured fracture data. Moreover, Eq. 1, 9, 10 and the determined fracture toughness K_{mat} have been employed to generate the failure stress. Fig. 1b shows the comparison of analytical calculation and the experimental data of the failure stress σ_C. The analytical results are found to be within $\pm 5\%$ of the test results.

At the same time, the FAD constructed according to the SINTAP (option 1) gives a good agreement with the presented FAD as well as the measured fracture data (Fig. 1a).

Moreover, the crack growth initiation data of through thickness centre crack tensile specimens made of D16T-1, V-95 and AKCH-1 aluminium alloys (having 0.8 mm thickness and different width) at 293 K was examined and showed similar results.

The effect of biaxial loading on failure assessment diagrams

The following illustration of the FAD-estimation versus the failure stress according to Eq. 1 and 8 has been considered for the centre-cracked plate under uniaxial and biaxial remote loading (with the load biaxiality parameter $k = \sigma_{xx}^{\infty} / \sigma_{yy}^{\infty}$). Generally, the biaxiality ratio $\beta(a/W)$ and the failure stress σ_C are interdependent, since the value of σ_C is also a function of crack length. But, finite-width effects do not drastically modify the value of $\beta(a/W)$ for uniaxially loaded plate ($k = 0$), which remains about -1 over the whole domain of crack lengths a/W [9]. Constantly of this degree in $\beta(a/W)$ is highly advantageous, because the cohesive strength can be calculated as a function of the failure stress from Eq. 8 without employing the data of σ_C versus a/W (Fig. 2).



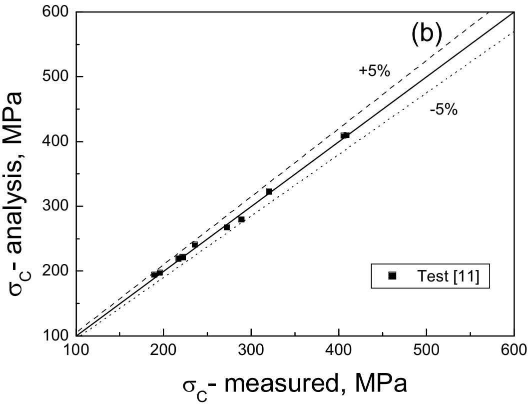


Fig. 1. Comparison of analytical and measured results of fracture analysis for AA2014-T6 aluminium alloy centre crack tensile specimens at 20 K. The failure assessment diagram (a), the failure stress (b).

At the same time, for high values of k crack tip conditions are no longer by any means constant during crack extension. For example, $\beta(a/W)$ varies from -0.5 to -0.75 at crack length $0 < a/W \leq 0.7$ for $k = 0.5$ [9]. Therefore, dependence of the failure stress versus crack length should be employed. However, some conservative estimation has been done to analyze trends in the cohesive strength behavior for biaxially loaded plate. In this case, the biaxiality ratio β is assumed to be -0.5 for the value of k=0.5 independently on a/W .

The failure assessment curve increases with the decrease of the critical applied stress tending to its limiting value, i.e. the fracture criterion $K_1 = K_{mat}$ becomes valid. It can be also seen that the biaxial assessment curves ($k > 0$) move outward from the uniaxial case ($k = 0$) for plane strain (Fig. 2). The main reason of such dependence of the failure assessment diagram on biaxiality is that the tendency of the failure assessment curves for different loading conditions is governed by the cohesive stress. Obtained result is qualitatively consistent with the experimental results published in [12].

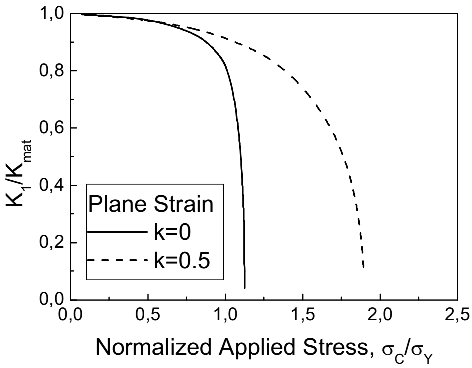


Fig. 2. The effect зa biaxial loading on the FAD of a plate with a crack.

The effect of biaxial loading has been analyzed for a solid with a notch. The notch FAD is based on Eq. 2 and 8. It should be noted that the failure criterion (2), describing the notch FAD, suggests that the loss of constraint due to a notch (characterized by the elastic stress concentration factor K_t) is independent on the loss of constraint due to the T-stress which was introduced into the cohesive strength to quantify constraint in different geometries, crack size and type of loading. It is seen that the notch failure assessment curve becomes progressively raised above the curve for a crack as the notch elastic stress concentration factor K_t decreases. The stress biaxial ratio does not change the character of the failure assessment curves for the notch. But biaxial failure assessment curves move outward from the uniaxial case (Fig. 3) in conformity with quantitative and qualitative changes of the cohesive stress.

Summary

The complex approach and failure criterion have been proposed for the treatment of a reduction in constraint due to type of loading, crack size and notch effect to modify the failure assessment diagram for solids with a crack or U-notch. The approach is based on the criterion of the average stress ahead of the crack/notch tip taking into account the cohesive strength (as a function of the yield stress, failure applied stress and biaxiality ratio) and the elastic stress concentration factor for the notch. This methodology was successfully validated with experimental results of through cracked plates under uniform loading.

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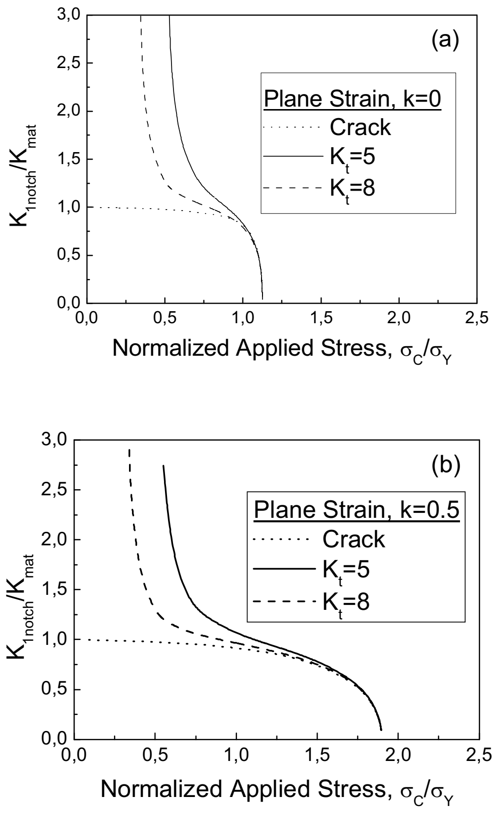

Fig. 3. Failure curves in terms of the stress intensity factor for a plate with a notch under biaxial loading. Stress biaxial ratio $k = 0$ (a) and $k = 0.5$ (b).