New correlations for the cyclic properties of engineering materials

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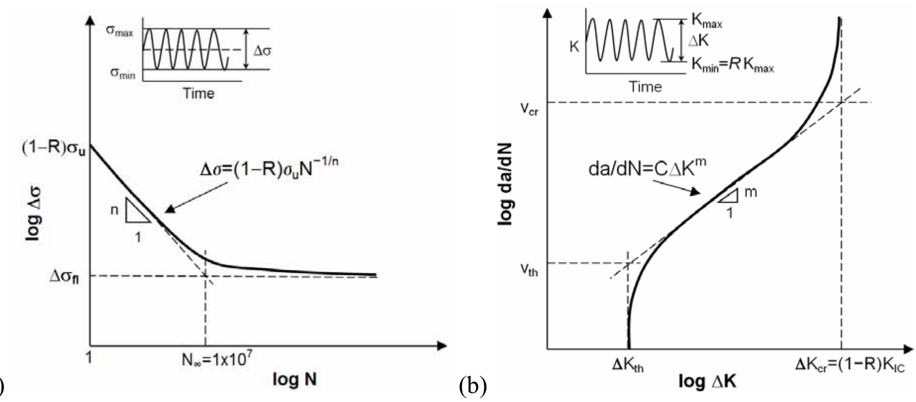
Abstract. A phenomenological description of the fatigue life of engineering components can be given either by plotting the applied stress range as a function of the total number of cycles to failure, i.e., according to the Wöhler's curve, or, after the advent of fracture mechanics, by plotting the crack growth rate in terms of the stress-intensity factor range, i.e., using the Paris' curve. In this work, an analytical approach is proposed for the study of the relationships existing between the Wöhler's and the Paris' representations of fatigue. According to dimensional analysis and the concepts of complete and incomplete self-similarity, generalized Wöhler and Paris equations are determined, which provide a rational interpretation to a majority of empirical power-law criteria used in fatigue. Then, by integration of the generalized Paris' law, the relationship between the aforementioned generalized representations of fatigue is established, providing the link between the cumulative fatigue damage and the fatigue crack propagation approaches. Moreover, paying attention to the limit points defining the range of validity of the classical Wöhler and Paris power-law relationships, whose co-ordinates are referred to as *cyclic* or *fatigue properties*, alternative expressions for the classical laws of fatigue are proposed. Finally, the correlations between such fatigue properties are determined according to theoretical arguments, giving an interpretation of the empirical trends observed in the material property charts.

Introduction

The existing approaches for the prediction of fatigue life can be distinguished in two main categories: those related to the Cumulative Fatigue Damage (CFD) approach, which is the traditional framework for fatigue strength assessment, and those based on the Fatigue Crack Propagation (FCP) approach, developed since the 1960s after the advent of fracture mechanics.

At present time, the CFD analysis based on the Wöhler or S-N curves [1] still plays a key role in predicting the life of components and structures subjected to field-load histories. In the empirical S-N curve, the fatigue life, N, is related to the applied stress range, $\Delta\sigma$ or S, and a reasonable power-law approximation was discovered since 1910 by Basquin [2]. A schematic representation of the Wöhler's curve is shown in Fig. 1a, where the cyclic stress range, $\Delta\sigma = \sigma_{\max} - \sigma_{\min}$, is plotted as a function of the number of cycles to failure, N. In this diagram, we introduce the range of stress at static failure, $\Delta\sigma_u = (1-R)\sigma_u$, where σ_u is the material *tensile strength*, and we define the *endurance* or *fatigue limit*, $\Delta\sigma_{fl}$, as the stress range that a sample will sustain without fracture for $N_\infty = 1\times 10^7$ cycles, which is a conventional value that can be thought of as "infinite" life.

With the advent of fracture mechanics, a more ambitious task was undertaken, i.e., to predict, or at least understand, the propagation of cracks. Plotting the crack growth rate, da/dN, as a function of the stress-intensity factor range, $\Delta K = K_{\max} - K_{\min}$, most of the experimental data can be interpreted in terms of a power-law relationship, i.e., according to the so-called Paris' law [3,4]. A schematic representation of the Paris' curve is shown in Fig. 1b. Note that the power-law representation presents some deviations for very high values of ΔK approaching $\Delta K_{\text{cr}} = (1-R)K_{\text{IC}}$ [5], where K_{IC} is the material *fracture toughness*, or for very low values of ΔK approaching the *threshold stress-intensity factor range*, ΔK_{th}. Again, in close analogy with the concept of fatigue limit, the fatigue threshold is defined in a conventional way as the value of ΔK below which the crack grows at a rate of less than 1×10^{-9} m/cycle.



(a) (b)

Figure 1: schemes of the (a) Wöhler and (b) Paris' curves with the related fatigue parameters.

For a long time, the CFD and the FCP approaches have been considered as totally independent. In the last few decades, the researchers have attempted to extend the field of application of the FCP approach [6-9]. These advances in understanding the complex phenomenon of fatigue crack growth shed a new light on the possibility to unify the CFD and the FCP approaches, and to solve the challenging task of interpreting the Paris and Wöhler power-law regimes within a unified theoretical framework (see also [10-12]).

In the present paper, we extend the dimensional analysis approach, originally proposed by Barenblatt and Botvina [13-14] for the study of the size-scale effects on the Paris' law, to derive generalized Paris and Wöhler representations of the phenomenon of fatigue. Moreover, by integration of the generalized Paris' equation and comparison with the generalized Wöhler's representation, the relationships existing between these two approaches is obtained. This will permit to interpret both FCP and CFD approaches within a unified theoretical framework. Finally, alternative expressions to the Paris' and Wohler's curves are provided in the corresponding fields of variation, replacing the parameters entering the power-law equations by the so-called static and fatigue properties, such as the *tensile strength*, the *fracture toughness*, the *fatigue limit* and the *threshold stress-intensity factor range*. In doing so, analytical correlations between the fatigue properties of engineering materials are determined and compared with the empirical trends proposed by Fleck *et al.* [15], giving a rational interpretation to the fundamental fatigue property charts.

Generalized mathematical representations of fatigue and their relationships

According to the pioneering work by Barenblatt and Botvina [13], the following functional dependence can be considered for the phenomenon of fatigue crack growth:

$$\frac{\mathrm{d}a}{\mathrm{d}N} = F\left(\sigma_u, K_{\mathrm{IC}}, \omega; \Delta K, \Delta K_{\mathrm{th}}, h, d, a; 1-R\right), \tag{1}$$

where the governing variables are summarized in Table 1, along with their physical dimensions expressed in the Length-Force-Time class (LFT).

Variable definition	Symbol	Dimensions
Ultimate tensile strength	σ_u	FL^{-2}
Fracture toughness	K_{IC}	$\mathrm{FL}^{-3/2}$
Frequency of the loading cycle	ω	T^{-1}
Stress-intensity factor range	ΔK	$\mathrm{FL}^{-3/2}$
Threshold stress-intensity factor range	ΔK_{th}	$\mathrm{FL}^{-3/2}$
Stress range	$\Delta\sigma$	FL^{-2}
Fatigue limit	$\Delta\sigma_{fl}$	FL^{-2}
Characteristic structural size	h	L
Microstructural dimension (grain size)	d	L
Crack length	a	L
Loading ratio	R	–

Table 1. Governing variables of the fatigue crack growth phenomenon

Considering a state with no explicit time dependence, it is possible to apply the Buckingham's Π Theorem [16] to reduce the number of parameters involved in the problem (see also [17-20]). As a result, we have:

$$\frac{\mathrm{d}a}{\mathrm{d}N} = \left(\frac{K_{\mathrm{IC}}}{\sigma_u}\right)^2 \Phi\left(\frac{\Delta K}{K_{\mathrm{IC}}}, \frac{\Delta K_{\mathrm{th}}}{K_{\mathrm{IC}}}, \frac{\sigma_u^2}{K_{\mathrm{IC}}^2}h, \frac{\sigma_u^2}{K_{\mathrm{IC}}^2}d, \frac{\sigma_u^2}{K_{\mathrm{IC}}^2}a; 1-R\right) = \left(\frac{K_{\mathrm{IC}}}{\sigma_u}\right)^2 \Phi\left(\Pi_i\right), \tag{2}$$

where Π_i $(i = 1,\ldots,6)$ are dimensionless numbers. Note that Π_3 corresponds to the square of the dimensionless number Z introduced by Barenblatt and Botvina [13] and to the inverse of the square of the brittleness number s introduced by Carpinteri [17-20]. The number Π_5 was firstly considered by Spagnoli [8] for the analysis of the crack-size dependence of the Paris' law parameters.

At this point, we want to see if the number of quantities involved in the relationship (2) can be reduced further from six. This can occur either in the case of *complete* or *incomplete self-similarities* in the corresponding dimensionless variables. In the former situation, the dependence of the mechanical response on a given dimensionless number, say Π_i, disappears and we can say that Π_i is *non essential* for the representation of the physical phenomenon. In the latter situation, a power-law dependence on Π_i can be proposed, which usually characterizes a physical situation intermediate between two asymptotic behaviours.

Considering incomplete self-similarity in the nondimensional variables Π_1, Π_4, Π_5 and Π_6, we obtain the following generalized representation of fatigue crack growth:

$$\frac{\mathrm{d}a}{\mathrm{d}N}=\left(\frac{K_{\mathrm{IC}}}{\sigma_u}\right)^2\left(\frac{\Delta K}{K_{\mathrm{IC}}}\right)^{\alpha_1}\left(\frac{\sigma_u^2}{K_{\mathrm{IC}}^2}d\right)^{\alpha_2}\left(\frac{\sigma_u^2}{K_{\mathrm{IC}}^2}a\right)^{\alpha_3}(1-R)^{\alpha_4}\Phi_2\left(\Pi_2,\Pi_3\right)=$$
$$=\Delta K^{\alpha_1}d^{\alpha_2}a^{\alpha_3}(1-R)^{\alpha_4}\frac{\Phi_2\left(\Pi_2,\Pi_3\right)}{K_{\mathrm{IC}}^{\alpha_1+2\alpha_2+2\alpha_3-2}\sigma_u^{2(1-\alpha_2-\alpha_3)}}. \tag{3}$$

Equation (3) can be considered as a generalized Paris' law (see the classical expression in Fig. 1b), in which the main functional dependencies of the parameter C have been explicitated. This generalized mathematical representation encompasses several improved versions of the Paris' law proposed in the past to cover specific anomalous deviations from the simplest power-law regime suggested by Paris, such as the grain-size [21,22] and crack-size dependencies of C [6,8,9]. The effect of the loading ratio R is also included through the incomplete self-similarity in Π_6.

So far, the crack growth rate has been chosen as the main output parameter characterizing the phenomenon of fatigue crack growth. However, we can also consider the number of cycles, N, as the parameter representative of fatigue. Following this route, we postulate the following functional dependence:

$$N=F\left(\sigma_u,K_{\mathrm{IC}},\omega;\Delta\sigma,\Delta\sigma_{fl},h,d,a;1-R\right), \tag{4}$$

where the definition of the governing variables is provided in Table 1. Considering a state with no explicit time dependence, it is possible to apply the Buckingham's Π Theorem [16] to reduce the number of parameters involved in the problem:

$$N=\Psi\left(\frac{\Delta\sigma}{\sigma_u},\frac{\Delta\sigma_{fl}}{\sigma_u},\frac{\sigma_u^{\ 2}}{K_{\mathrm{IC}}^{\ 2}}h,\frac{\sigma_u^{\ 2}}{K_{\mathrm{IC}}^{\ 2}}d,\frac{\sigma_u^{\ 2}}{K_{\mathrm{IC}}^{\ 2}}a;1-R\right)=\Psi\left(\Pi_i\right), \tag{5}$$

where Ψ is a nondimensional function. At this point, we want to see if the number of quantities involved in the relationship (5) can be reduced further from six. In close analogy with the procedure discussed for the Paris' law, we assume incomplete self-similarity in Π_1, Π_4, Π_5 and Π_6, obtaining:

$$N=\left(\frac{\Delta\sigma}{\sigma_u}\right)^{\beta_1}\left(\frac{\sigma_u^{\ 2}}{K_{\mathrm{IC}}^{\ 2}}d\right)^{\beta_2}\left(\frac{\sigma_u^{\ 2}}{K_{\mathrm{IC}}^{\ 2}}a\right)^{\beta_3}\left(1-R\right)^{\beta_4}\Psi_2\left(\Pi_2,\Pi_3\right)=$$
$$=\Delta\sigma^{\beta_1}d^{\beta_2}a^{\beta_3}\left(1-R\right)^{\beta_4}\frac{\Psi_2\left(\Pi_2,\Pi_3\right)}{K_{\mathrm{IC}}^{\ 2(\beta_2+\beta_3)}\sigma_u^{\ \beta_1-2\beta_2-2\beta_3}}. \tag{6}$$

Equation (6) provides a generalized Wöhler relationship of fatigue and encompasses the empirical S-N curves approximated by the Basquin power law and by the Coffin-Manson criterion as limit cases when $\beta_1=-n$, $\beta_2=\beta_3=0$ and $\beta_4=n$ (see the Equation reported in Fig. 1a).

The cornerstone for determining the relationships existing between the CFD and the FCP approaches is represented by the integration of the generalized Paris' law in Eq. (3) between an initial defect size, a, and a generic final crack length, a_f, corresponding to a given fatigue life N. Recalling that $\Delta K=\Delta\sigma\sqrt{\pi a}$ for a Griffith crack, then the integration gives the following result:

$$N = \frac{\Delta\sigma^{-\alpha_1} d^{-\alpha_2} (1-R)^{-\alpha_4}}{\left[1-\left(\frac{\alpha_1}{2}+\alpha_3\right)\right]\pi^{\alpha_1/2}\dfrac{\Phi_2(\Pi_2,\Pi_3)}{K_{\mathrm{IC}}^{\alpha_1+2\alpha_2+2\alpha_3-2}\sigma_u^{2(1-\alpha_2-\alpha_3)}}}\left[a_f^{1-\left(\frac{\alpha_1}{2}+\alpha_3\right)} - a^{1-\left(\frac{\alpha_1}{2}+\alpha_3\right)}\right], \tag{7}$$

which can be simplified by noting that $a_f^{1-\left(\frac{\alpha_1}{2}+\alpha_3\right)} \ll a^{1-\left(\frac{\alpha_1}{2}+\alpha_3\right)}$, since the exponent of the crack length is negative valued and $a_f \gg a$. Under such conditions, the fatigue life can be approximated as follows:

$$N \cong \frac{\Delta\sigma^{-\alpha_1} d^{-\alpha_2} (1-R)^{-\alpha_4} a^{1-\left(\frac{\alpha_1}{2}+\alpha_3\right)}}{\left[\left(\frac{\alpha_1}{2}+\alpha_3\right)-1\right]\pi^{\alpha_1/2}\dfrac{\Phi_2(\Pi_2,\Pi_3)}{K_{\mathrm{IC}}^{\alpha_1+2\alpha_2+2\alpha_3-2}\sigma_u^{2(1-\alpha_2-\alpha_3)}}}. \tag{8}$$

A comparison between Eq. (6), obtained according to dimensional analysis arguments, and Eq. (8), obtained through the integration of the generalized Paris' law in Eq. (3), leads to the following relationships between the powers entering the two representations:

$$\beta_1 = -\alpha_1;\ \beta_2 = -\alpha_2;\ \beta_3 = 1-\left(\frac{\alpha_1}{2}+\alpha_3\right);\ \beta_4 = -\alpha_4;\ \Psi_2 = \left[\frac{\pi^{-\alpha_1/2}}{\left(\frac{\alpha_1}{2}+\alpha_3\right)-1}\right]\frac{1}{\Phi_2}. \tag{9}$$

Alternative representations in the classical power-law regimes and analytical correlations between the fatigue properties of engineering materials

Let us consider the limit points in the Wöhler's curve defining the range of validity of the power-law approximation relating the stress range, $\Delta\sigma$, to the cycles to failure, N, i.e. the points corresponding to the *cyclic stress at static failure*, $\Delta\sigma_u$, and to the *fatigue limit*, $\Delta\sigma_{fl}$. In this range, the S-N curve can be approximated by a simple equation fully characterized by its exponent n:

$$\Delta\sigma = \Delta\sigma_u N^{-1/n}. \tag{10}$$

Evaluating the S-N curve in correspondence of the fatigue limit, $\Delta\sigma_{fl} = \Delta\sigma_u N_\infty^{-1/n}$, a one-to-one relationship between the exponent n and the co-ordinates of this special point of the Wöhler's curve can be determined:

$$\frac{1}{n} = -\frac{\log\Delta\sigma_{fl} - \log\Delta\sigma_u}{\log N_\infty}, \tag{11}$$

where, by definition, $N_\infty = 1\times10^7$ cycles corresponds to an "infinite" fatigue life. As a result, an alternative expression for the classical Wöhler's curve can be considered, where the exponent n can be written in terms of the fatigue properties $\Delta\sigma_u$ and $\Delta\sigma_{fl}$.

As far as the Paris' law is concerned, let us consider the limit points defining the range of validity of the power-law approximation. They correspond, respectively, to the points with horizontal co-ordinates equal to the *fatigue threshold*, ΔK_{th}, and to ΔK_{cr}, where the Paris' instability coincides with the Griffith-Irwin crack growth instability when $K_{\max}$ tends to the *fracture toughness*. In this range, the Paris' curve is usually defined in terms of the parameters *C* and *m* (see the Equation reported in Fig. 1b).

Now, let us consider the useful construction added with dashed line to Fig. 1b, as proposed in [15]. If a tangent is drawn at the mid-point of the central linear region of the curve and extrapolated, it is found empirically that it intersects the vertical line $\Delta K = \Delta K_{\text{th}}$ in correspondence to a crack growth rate of approximately $v_{\text{th}} = 1\times 10^{-9}$ m/cycle, and it intersects the line $\Delta K = \Delta K_{\text{cr}} = (1-R)K_{\text{IC}}$ at about $v_{\text{cr}} = 1\times 10^{-5}$ m/cycle. Evaluating the Paris' law in correspondence of the second point, the following correlation between the parameters *C* and *m* of the Paris' curve can be obtained [5,11], which provides an explanation of the empirically-based correlations available in the Literature [23]:

$$C = \frac{v_{\text{cr}}}{\left[(1-R)K_{\text{IC}}\right]^m}. \tag{12}$$

Repeating this reasoning for the point defined by the fatigue threshold, the following relationship holds:

$$C = \frac{v_{\text{th}}}{\left(\Delta K_{\text{th}}\right)^m}, \tag{13}$$

which establishes a link between the Paris' law parameter *C* and the coordinates of the point defining the condition of non-propagating cracks. Equating the second members of Eqs. (12) and (13), we find that the ratio between the fatigue threshold and the fracture toughness is a function of the Paris' law parameter *m*, i.e.:

$$\frac{\Delta K_{\text{th}}}{K_{\text{IC}}} = (1-R)\sqrt[m]{\frac{v_{\text{th}}}{v_{\text{cr}}}} \quad \Rightarrow \quad \log\left(\frac{\Delta K_{\text{th}}}{K_{\text{IC}}}\right) = \log(1-R) + \frac{1}{m}\log\left(\frac{v_{\text{th}}}{v_{\text{cr}}}\right). \tag{14}$$

Equation (14) establishes a one-to-one correspondence between ΔK_{th}, K_{IC} and *m* in the long-crack regime and was experimentally confirmed by Fleck *et al*. [15] for a wide range of materials. Considering the fatigue property chart reported in Fig. 2a, we notice a very good agreement between the experimental trend and the proposed correlation, being *R*=0 and $\log(v_{\text{th}}/v_{\text{cr}}) \cong \log(1\times 10^{-9}/1\times 10^{-5}) = -4$.

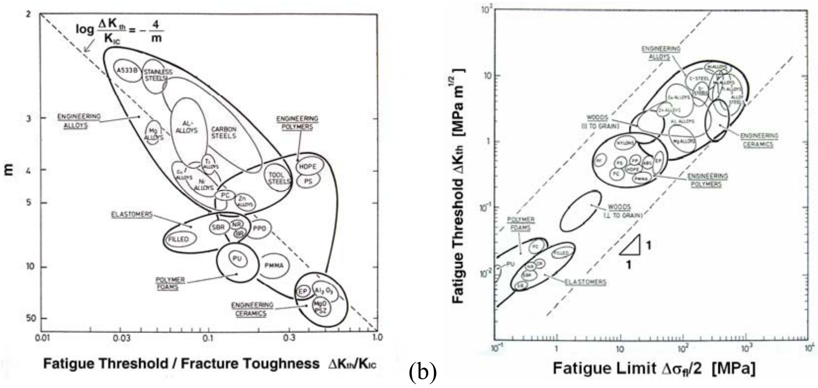
Finally, a relationship between the fatigue stress-intensity factor threshold and the fatigue limit can be derived by considering the propagation of a Griffith crack of length $2a_0$ in an infinite elastic plate subjected to cyclic loading with $\Delta\sigma = \Delta\sigma_{fl}$ acting at infinity and $R = 0$. The initial crack length is chosen as representative of the size of the existing microdefects, i.e., $a_0 = (K_{\text{IC}}/\sigma_u)^2/\pi$. Considering the integrated Paris' law in Eq. (8) from a_0 up to $a_f = (K_{\text{IC}}/\Delta\sigma_{fl})^2/\pi$ corresponding to $N = N_\infty$ (setting also $\alpha_1 = m$, $\alpha_2 = \alpha_3 = \alpha_4 = 0$, and $C = K_{\mathrm{IC}}^{m-2}\sigma_u^2/\Phi_2$ for the sake of simplicity), we find:

$$C\left(\Delta\sigma_{fl}\right)^m N_\infty \cong -\frac{2\pi^{-m/2}}{2-m}a_0^{\frac{2-m}{2}} = \frac{2}{\pi(m-2)}\left(\frac{K_{\mathrm{IC}}}{\sigma_u}\right)^{2-m}, \tag{15}$$

where the definition of a_0 has been suitably introduced. Equation (15) permits to obtain a closed-form relationship between the fatigue limit and the fatigue threshold. In fact, considering Eq. (13), we can relate the parameter C to ΔK_{th}. Moreover, noting that $K_{\mathrm{IC}}/\sigma_u = \Delta K_{\mathrm{th}}/\Delta\sigma_{fl}$, we obtain, after some manipulation, the following approximate correlation between the fatigue threshold and the fatigue limit, strictly holding for R=0:

$$\Delta K_{\mathrm{th}} \cong \sqrt{\frac{\pi(m-2)v_{\mathrm{th}}N_\infty}{2}}\Delta\sigma_{fl} \;\Rightarrow\; \log\Delta K_{\mathrm{th}} \cong \frac{1}{2}\log\left[\frac{\pi(m-2)v_{\mathrm{th}}N_\infty}{2}\right] + \log\Delta\sigma_{fl}. \tag{16}$$

A direct comparison between this proposed correlation and the experimental trend observed for a wide range of materials and collected in the fatigue property chart by Fleck *et al.* [15] is proposed in Fig. 2b. As can be seen, the analytically predicted linear relation between the fatigue threshold and the fatigue limit is correctly reproduced.



(a) (b)

Figure 2: fatigue property charts (adapted from [15]).

Conclusions

In the present contribution, a dimensional analysis approach and the concepts of complete and incomplete self-similarity have been applied to the Paris' curve, extending and generalizing the pioneering work by Barenblatt and Botvina [13], and, for the very first time, to the Wöhler curve. As a main conclusion, it has been shown that the large number of power laws used in fatigue are the result of incomplete self-similarity in the corresponding dimensionless variables. This gives a rational interpretation to such empirically-based fatigue criteria, towards a unified description of fatigue and a possible standardization. Moreover, the integration of the proposed generalized Paris' law and the comparison with the generalized Wöhler curve has permitted to find the relationship between these two representations of fatigue.

Alternative expressions of the classical Wöhler and Paris equations have also been proposed, where the parameters entering the power laws are rewritten in terms of the cyclic properties of engineering materials, that are true material parameters. In doing so, analytical correlations between the cyclic properties have been established, providing an analytical interpretation to the empirical correlations existing in the Literature and to the well-known fatigue property charts.

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