# Modeling of S/N curves for flawed materials in the VHCF regime

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**Abstract.** For years lifetime estimations have been performed by means of the original Miner rule under the assumption of a distinct fatigue limit. To consider damage in the high cycle fatigue regime, empirical modifications of the Miner rule were introduced. For aluminum alloys, these assumptions are no longer sufficient. Leitner/Eichlseder showed by statistical analysis of experimental data that the slope of the S/N curve in the very high cycle (VHCF) fatigue regime is about five times the slope in the high cycle fatigue (HCF) regime.

In order to avoid a time-consuming experimental determination of such S/N curves, a method is proposed to derive S/N curves for different conditions (i.e. stress ratio or age of the material) from crack growth data, with only one measured S/N curve for calibration. The distinct fatigue limit of the original Miner rule is simply calculated from the threshold value of the crack growth curve. With this data it is not possible to obtain a slope in the VHCF area. Based on the rules of crack nucleation and short crack growth, an engineering estimate is given also for the VHCF slope.

## Introduction

In the automotive industry most lifetime estimates are based on S/N curves. These curves are obtained from standardized smooth, defect-free specimens. Another approach to lifetime estimates stems from fracture mechanics. A crack is assumed in the material, its size being estimated from the non destructive testing (NDT) detection limit. Under cyclic loading the crack may grow until fracture occurs.

The advantage of a lifetime estimate from S/N curves is the possibility to calculate a damage parameter for every point of the part with relatively short computation time. On the other hand, at least 12 to 15 specimens are needed for obtaining an S/N curve.

For the fracture mechanics approach the conditions are inverse. It is easily possible to acquire the parameters of the crack growth equation from only two or three specimens (only one if just a first estimate). However, it would be very time consuming to calculate the stress intensity factors for large components because of the very fine mesh needed for finite element crack growth computations.

The aim of this work is to combine both approaches for a quick lifetime estimate at minimum experimental and computational cost. To this purpose a cast aluminium alloy is investigated with respect to crack growth and fatigue.

## Basic principles

**S/N curves.** To calculate the lifetime of a component in the area of low cycle fatigue (LCF) is normally based on strain-life curves, whereas in the high cycle fatigue (HCF) regime stress-life curves are used.

To predict the lifetime of a component a damage parameter was introduced by Palmgren and Miner [6], [7]. The damage induced by one load cycle is defined as follows:

$$\Delta D_i = \frac{1}{N_i} \tag{1}$$

To describe stress-life curves (Fig. 1) three equations are sufficient [8]:

$$\begin{aligned} &N_1 \cdot \sigma_1^{k_1} = N_2 \cdot \sigma_2^{k_1} = const \\ &k = -\left(\frac{d(\log \sigma)}{d(\log N)}\right)^{-1} \\ &k_2 = c \cdot k_1 \end{aligned} \tag{2}$$

The inclination $k$ is the negative reciprocal value of the slope of the S/N curve in a logarithmic scale. The S/N curve is thus approximated by a bi-linear curve in a log-log scale, with the inclination $k_1$ describing low cycle fatigue (LCF) and high cycle fatigue (HCF), and $k_2$ describing very high cycle fatigue (VHCF). The coefficient $c$ defines the ratio between the two slopes and can be estimated between 3 and 5, according to FKM [13] or Leitner [8]. Aluminum alloys do not exhibit a distinct fatigue limit in the region of $10^6$ load cycles; however, for practical design computations, the fatigue limit will be assumed equal to the endurable stress $\sigma_0$ at $10^7$ load cycles. It is not possible to define the fatigue limit $\sigma_f$ by testing due to the extreme increase of testing time for load cycles of $10^8$ or above.

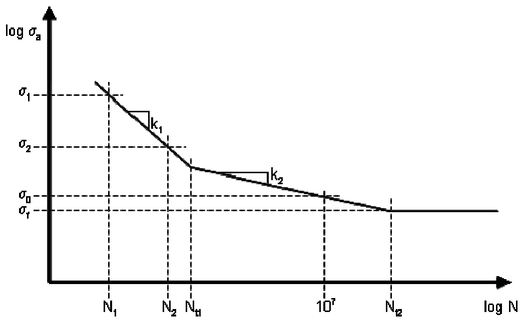

Fig. 1: Stress-life curve

**Crack growth curves.** Different objectives can be assessed using fracture mechanics. If the loading condition of a machine part is known, there are three possible ways of lifetime prediction:

- Calculation of the crack length after a certain number of load cycles, if the initial crack length can be estimated,
- Calculation of the initial crack size, if the critical crack length and the number of load cycles is available,
- Calculation of the number of load cycles, if the initial and critical crack length are known.

To this purpose a link between applied load and crack growth per load cycle has to be found. The first to propose such a crack growth law were Paris and Erdogan [1]; their law captures the region of stable crack growth, nowadays known as Paris region:

$$\frac{da}{dN} = C \cdot \Delta K^m \tag{3}$$

The coefficient and exponent of this equation do not only depend on the properties of the material (age, temperature or environment) but also on mechanical influences like the stress-ratio. The boundaries of this region are (Fig. 2) the near-threshold regime of the stress intensity factor $\Delta K_{\mathrm{th}}$ (region I), where crack initiation occurs, and the region of instable crack growth (region III) near the fracture toughness $\Delta K_{\mathrm{c}}$.

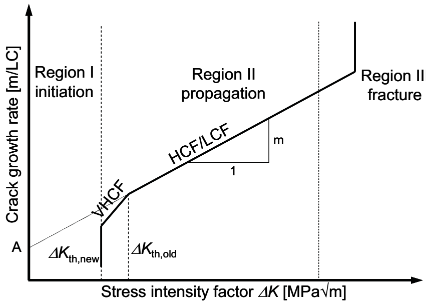

Fig. 2: Regions of the crack growth curve

A lot of alternative equations for crack growth predictions can be found in the literature. Kohout [3] for example proposed a crack growth equation which can approximate all three states of crack growth and also takes the stress-ratio into account. A large number of other approximations are known from the literature, for example Kujawski [4] or Huang and Moan [2].
To define the threshold value of the crack growth curve the ASTM [5] defines a minimum crack growth rate of approximately $10^{-10}$m/cycle. Recent investigations in the threshold region under ultrasonic testing frequencies have shown that crack growth rates of $10^{-14}$m/cycle can occur [12].

**The Kitagawa diagram.** The Kitagawa diagram is a practical tool to describe the relation between the fatigue limit and the crack length. It uses the dependence of the stress intensity factor (SIF) range $\Delta K$ on the stress range and the crack length; the geometry parameter $Y(a)$ is often neglected for engineering considerations:

$$\Delta K = \Delta\sigma \cdot \sqrt{\pi \cdot a} \cdot Y(a) \tag{4}$$

It can be assumed that no crack growth occurs below the threshold value of the SIF $\Delta K_{th}$, which corresponds to the fatigue limit $\Delta\sigma_0$ of the unflawed material. This means that, if the crack is smaller than a certain intrinsic length $a_0$, it has no influence on the fatigue limit. If the initial crack length is larger than $a_0$ the fatigue limit is dominated by the SIF. The length $a_0$ is calculated from Eq. (5):

$$a_0 = \frac{1}{\pi} \cdot \left( \frac{\Delta K_{th}}{\Delta\sigma_0 \cdot Y(a)} \right)^2 \tag{5}$$

$$\Delta\sigma = \frac{\Delta K_{th}}{Y(a) \cdot \sqrt{\pi \cdot (a + a_0)}} \tag{6}$$

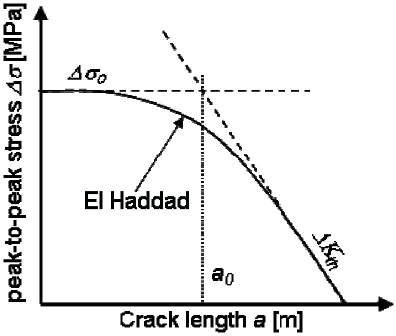

Fig. 3: The Kitagawa diagram [10]

**Calculation of stress-based S/N curves**

To get the lifetime of the material at a certain stress level, the reciprocal of the crack growth equation has to be integrated, Eq. (7).

$$N = \frac{1}{A\left(2\sigma\sqrt{\pi}\,1{,}12\right)^m}\frac{2}{2-m}\left[a_f^{\,1-\frac{m}{2}} - a_0^{\,1-\frac{m}{2}}\right] \tag{7}$$

The limits of the integral are the intrinsic crack length $a_0$, which can be estimated by Eq. (5) based on the Kitagawa diagram, and the fracture crack length $a_f$ that is computed from the actual stress σ and the fracture toughness $K_c$:

$$a_f = \frac{1}{\pi}\cdot\left(\frac{K_c}{\sigma\cdot Y(a)}\right)^2 \tag{8}$$

As discussed before, aluminum alloys have no distinct fatigue limit but a second slope $k_2$ to define the fatigue behavior in the HCF regime. The Paris law is limited to define the HCF/LCF regime of the S/N curve because of the assumption that constant crack growth only occurs between the two limits threshold and fracture toughness. Even though the Kohout equation introduces the exponents $n$ and $p$ to better fit the measurement data, the problem of one single regime of constant crack growth still exists. If the second slope of the S/N curve should be described, a second region of constant crack growth needs to be defined.

As reported in [12] and confirmed by the authors' measurements, crack growth rates below $10^{-10}$m/cycle can be found. For crack growth rates of approximately $1.5\text{x}10^{-12}$m/cycle the new threshold value ($\Delta K_{th,new}$) lies about 25% below the threshold value known from ASTM ($\Delta K_{th,old}$).

For engineering calculations we can assume that the crack growth rates in the regime I can also be described by a Paris-type law. Then it is possible to define a second line of stable crack growth which intersects the first at the threshold $\Delta K_{th,old}$ and ends at $\Delta K_{th,new}$ (Fig. 5)

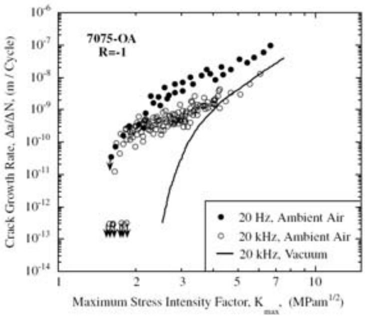


Fig. 4: near threshold crack growth [12]

With respect to the initial Eq. (5) and final crack length Eq. (8) it is possible to use Eq. (7) to calculate the number of cycles to failure. The VHCF and HCF/LCF Paris lines are integrated separately and then the results are added up.

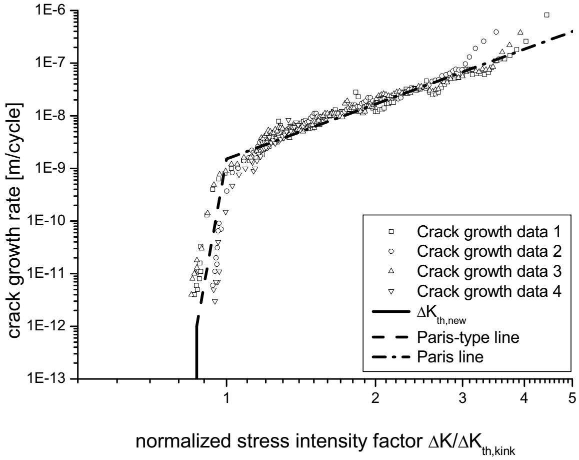


Fig. 5: Adaptation of the Paris law

The measured data in Fig. 5 can only describe the crack growth behavior for long cracks as the testing specimens were common fatigue pre-cracked single edge notch bending (SENB) specimens with an initial crack length at test start of 2.5 mm and a notch of 4 mm according to the ASTM.

If the whole life of a flawed part should be taken into account, short crack growth behavior must also be considered. Chapetti [14] describes a method to approximate the behavior of short cracks by using a crack length dependent threshold $\Delta K_{\mathrm{th}}$:

$$\frac{da}{dN} = C\left(\Delta K^m - \Delta K_{th}{}^m\right)$$
$$\Delta K_{th} = \Delta K_{dR} + \left(\Delta K_{thR} - \Delta K_{dR}\right)\left(1 - e^{-k(a-d)}\right) \tag{9}$$

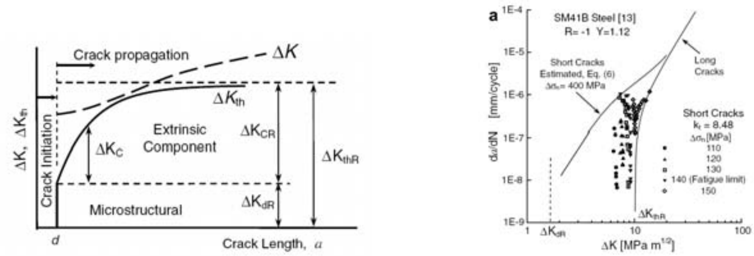


Fig. 6: Crack length dependent threshold and short crack propagation, from [14]

For engineering problems another possibility to estimate this influence is the El Haddad approximation of the Kitagawa diagram. If we know the intrinsic crack length $a_0$ it can be used to shift the measured crack growth curve to smaller cracks lengths by rearranging Eq. (6):

$$\Delta K = \Delta\sigma \cdot \sqrt{\pi \cdot \left(a + a_0\right)} \cdot Y\left(a\right) \tag{10}$$

Fig. 7 shows three stress levels where a small crack starts to grow until final fracture occurs. These curves where derived by calculating the common stress intensity factor Eq. (4) and the adjusted stress intensity factor Eq. (10), which is used to calculate the crack growth rate.

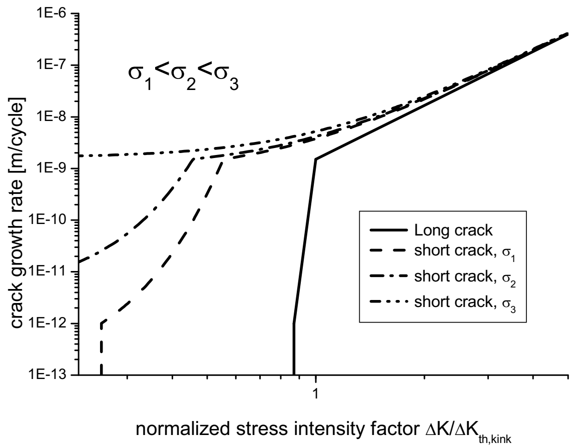


Fig. 7: Estimated short crack growth data

If equation (10) instead of equation (4) is used to integrate the reciprocal crack growth equation we can consider short cracks and describe the influence of the initial crack length on the lifetime curve.

$$N = \frac{1}{A\left(2\sigma\sqrt{\pi}\,1{,}12\right)^m}\frac{2}{2-m}\left[\left(a_f + a_0\right)^{1-\frac{m}{2}} - \left(a_i + a_0\right)^{1-\frac{m}{2}}\right] \tag{11}$$

Fig. 8 compares three different S/N curves. As it can be seen from the two calculated S/N curves, the influence of short crack growth on the slope and position of the S/N curve for this aluminum cast alloy is rather small. Despite the difference in the slope of the crack growth curves that can be found in Fig. 7, the slope of the calculated S/N curve does not change. It was also not possible to extend the S/N curve to load cycles higher than $10^7$. An explanation would be that the short crack passes the region of reduced crack closure effect very fast due to the high propagation rates and therefore has hardly any influence on the slope of the S/N curve and the VHCF behavior.

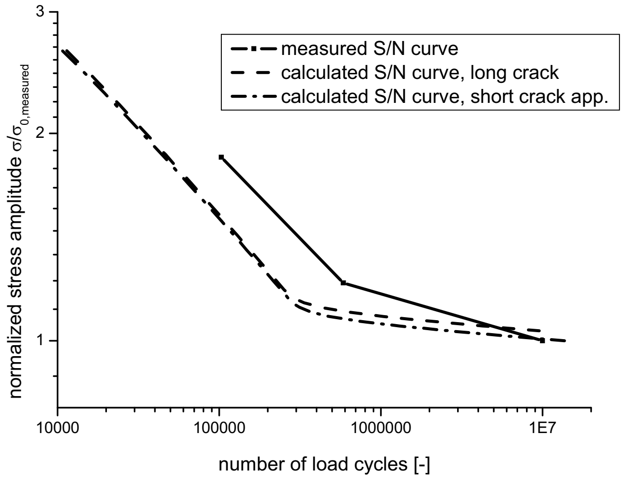


Fig. 8: Comparison of calculated and measured lifetime curves

## Conclusions

A method for the estimation of S/N curves on crack growth curves was presented; the computational predictions are in good agreement with experimental results. The main link between fracture mechanics and S/N curves is formed by the size of intrinsic cracks, $a_0$, the size of the initial crack, $a_i$, and by the allowable crack size at fracture, $a_f$, which are computed from the fatigue limit stress and yield/ultimate stress and the fatigue threshold SIF range and the fracture toughness, respectively.

The influence of short crack growth on the lifetime curve is smaller than expected. However, another advantage is the possibility to consider the minimum and maximum pore or flaw size of the material in fatigue life computations. Investigations of further effects such as ageing are in progress.

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