Jump-like Crack Growth Models or Theory of Critical Distances. Are They Correct?

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Abstract. The theory of critical distances was reexamined from the point of view of classical fracture mechanics. It was demonstrated that it is based on very rough assumptions which are not necessary justified. The new *critical distance* is postulated.

Introduction

The aim of this paper is to critically review some of the basic concepts which support the theory of critical distances in fracture mechanics. It is not an intention of the author to prove that the concept of the critical distances in fracture mechanics is wrong. On the contrary, the author believes that the process of crack growth depends on some critical distances in front of the crack. In the author's opinion these critical distances should depend both on characteristic distances in the material's microstructure and on some distances following from structure of the stress and strain fields in front of the crack.

However, this papers contains several critical remarks concerning the existing hypotheses. Intention of the author was to stimulate discussion which, hopefully, may remove some doubts listed in this paper. At the end of the paper, the present author proposes a slightly different definition of the *critical distance*, which might be applied at least for fracture analysis of metal alloys.

Microscopic observations show that the crack extension is not a continuous process of the atomic bonds breaking just at the crack tip along the whole crack front. It is a very complex process of the micro-cracks or micro-voids or cavities nucleation, growth, coalescence and joining the dominant crack front at different time and places during the process of the structural element loading. Experiments show that the important sites in front of the crack, where the failure takes place, are located at the distance of the order of several to 100-150 μm. These distances are of the order of crack tip opening displacement (CTOD) and the distance of the maximum opening stress location in front of the crack (the maximum is revealed when the finite strains are used in the stress analysis) [1,2]. Taking these observation into account one would expect that the length of the order of the crack tip opening displacement might be so called the critical distance. The crack tip displacement is of the order of $K_I^2 / E\sigma_o$ or J/σ_o. In the classical theory of critical distances [3] it is defined by the formula

$$\text{Critical distance} = \frac{2}{\pi}\left(\frac{K_{IC}}{\sigma_u}\right)^2 \quad \text{or} \quad = \frac{1}{2\pi}\left(\frac{K_{IC}}{\sigma_u}\right)^2 \tag{1}$$

here σ_u has different physical meaning depending on author. If σ_u is an ultimate strength [3] the critical distance is about $(E/\sigma_o)(\sigma_o/\sigma_u)^2$ times greater than the CTOD (it is (80 ÷ 400) times greater than the CTOD for a wide range of steels), where σ_o is the yield strength. If one assumes that the critical stress in front of the crack is equal to $4\sigma_o$ the critical distance is about 50 times greater than the crack tip opening displacement. In the former case the critical distance is closer to the plastic zone than to the process zone length, in the later case this distance is still big and the stress level at this place is very close to the yield stress and nothing "critical", from the failure point of view, can be expected there. Eq. (1) is a rough estimation of the plastic zone length if σ_u is replaced by σ_o. However, it is so for thick, plane strain specimens, with a high in-plane constraint only. It is widely known that for short cracks, Eq. (1) does not provide a good estimation of the plastic zone length . The theory of critical distances is mainly aimed at short cracks.

However, the application of the critical distance, defined by the Eq. (1) led to very interesting results in the fracture analysis of many materials [3]. Why this distance is so important? The theory provides good correlations for a wide range of materials from ceramics, through laminates, polycarbonates, aluminum alloys to steels. It is good for notches and cracks, for fracture under monotonously increasing external loading and for fatigue. The application is so wide that the question arises: why? What is a reason for good correlations between experiment and postulated quasi-theoretical results in such distant cases? Several theories have been proposed to formulate the theoretical basis in order to justify Eq.1. These theories will be critically reviewed in the present article.

The theory of critical distances supports the concepts of discontinuous crack growth. Many authors (e.g. A.Carpintieri [5], N.Pugno 6], D.Leguillon [7], R.Goldstein [8], D.Taylor [9], P.Cornetti [10], A.Yavari [11], M.Wnuk [12], A.Neimitz [13, 14]) introduce the discontinuous crack growth into analysis. The names: "fracture quantum"[15], "finite fracture mechanics" or „quantized fracture mechanics" are well known in the fracture mechanics analysis.

How Eq.1 was derived?

One of the main arguments supporting the theory of critical distances is to remove the unphysical result that the critical stress, to cause the failure, applied to the element containing crack, approaches infinity when the length of the crack approaches zero. It follows directly from the formula:

$$\sigma_f = \sqrt{\frac{G_C E}{\pi a}} = \frac{K_C}{\sqrt{\pi a}} \tag{2}$$

where K_C is the critical stress intensity factor, σ_f is external stress to failure and a is the crack length. Eq. (2) can be derived starting from the well known Griffith formula [16] on a strain energy due to the crack in an infinite plate:

$$W = \frac{\sigma^2 a^2 \pi}{2E}. \tag{3}$$

Then one can use the classical definition of the energy release rate

$$G = \frac{\partial W}{\partial a} = \frac{\sigma^2 a \pi}{E}, \tag{4}$$

and replace σ by σ_f at the critical moment. Finally, introducing the Irwin's [17] relation between energy release rate and stress intensity factor

$$G = \frac{K^2}{E'} \tag{5}$$

where $E' = E$ for plane stress and $E' = E/(1-\nu^2)$ for plane strain, Eq.(2) is obtained. Continuum crack growth model follows directly from the definition, Eq(4).

When the crack „jump" Δa is finite one can write, e.g. [18]:

$$G_\Delta = \frac{\Delta W}{\Delta a} = \frac{\sigma^2 \pi (a+\Delta a)^2 - \sigma^2 \pi a^2}{2E\Delta a} = \frac{\sigma^2 \pi}{E}\left(a + \Delta a / 2\right) \tag{6}$$

and using Eq.(5)

$$\sigma_f = \sqrt{\frac{G_{\Delta C} E}{\pi (a + \Delta a / 2)}} = \frac{K_C}{\sqrt{\pi (a + \Delta a / 2)}} \tag{7}$$

Equation (5) is correct both for infinitesimally small and finite crack jumps. In the former case Eq. (5) is always true. In the later case it is true only if one assumes a'priori that the higher terms in the Williams' series [19] are neglected. As will be shown later, such an assumption is very strong, too strong in many cases.

If it is assumed in (7) that $a \to 0$ the critical stress σ_f approaches:

$$\sigma_f \to \frac{K_C}{\sqrt{\pi \Delta a / 2}} \tag{8}$$

A'priori made assumption that higher terms in the Williams' series are neglected seems to be a strong one. It is well known, that one term approximation of the stress field is sufficiently exact (the error is less than 10%) in a very small domain in front of the crack $r \le 0.01a$. In the theory of critical distances the jump length Δa is of the order of the crack length or even greater. Thus G should be computed using more than one term in the Wiliams' or Yang, Chao, Sutton [20] series. The general formula for G for several terms was derived in [15]. If the more general expression for G is used, the finite value of critical stress is still preserved but the formula for the critical length is different.

Eq. (7) can also be derived using another approach, e.g. [3]. The strain energy change during the crack jump over the distance Δa is equal to

$$\Delta W = \int_a^{a+\Delta a} \left(\frac{\partial W}{\partial a}\right) da = \frac{\pi \sigma^2}{E}\left(a\Delta a + \Delta a^2 / 2\right) \tag{9}$$

where Eq.3 was used. If this value is compared with the product $G_C \Delta a$, Eq. 7 is obtained [3].

Here again the two different approaches were mixed in one derivation: continuous model through Eq.2 and finite jump approach used in Eq.9. Notice, that again the Griffith crack was used to derive Eq. 7.

The similar to Eq. 7 formula leading to the conclusion that the strength of the specimen is finite when crack length approaches zero was derived by Cornetti et al [10]. They used the Novozhilov's [15] "fracture quantum" idea. According to Novoshilov the onset of crack growth is observed when the average, over the distance Δa, opening stress in front of the crack reaches the critical value.

$$\int_{a}^{a+\Delta a} \sigma_{22}(x)dx = \sigma_m \Delta a \tag{10}$$

σ_m is the critical stress in front of the crack, considered often as a material constant, e.g. [1], [2],[21],[23]. Corneti et al [10] replaced the σ_{22} stress in Eq.[10] by the well known, e.g. [22], formula for a Griffith crack (the notation is shown in Fig.6).

$$\sigma_{22} = \frac{x\sigma}{\sqrt{x^2 - a^2}} \tag{11}$$

After integration Eq. 10 assumes the form:

$$\frac{\sigma_f}{\sigma_m} = \frac{1}{\sqrt{\frac{2a}{\Delta a} + 1}} \tag{12}$$

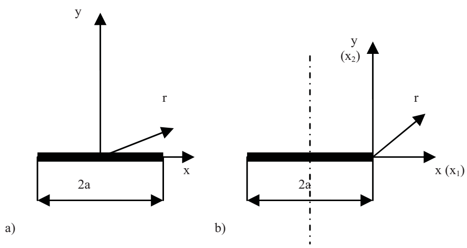

Fig.6. Symbols used in Eq. 11.

where σ_f is the external stress at the onset of crack growth and σ_m is the critical stress in front of the crack. Authors of [10] claim that using Eq. (12) the critical value of a stress σ_f , applied to the specimen, can be computed and it is not equal to infinity when the crack length *a* approaches zero. It is true. However, these authors do not discuss a further consequences following from Eq.12. It is that the critical stress in front of the crack, σ_m, depends strongly on a crack length and for the crack length equal to zero, σ_f=σ_m=σ_u. Such a conclusion is not necessarily wrong but it needs experimental verification and it is against arguments of several authors, e.g. [1],[2],[21],[23]. They usually assume that σ_m is of the order (3÷5) σ_o. There is another observation during derivation of Eq.12 which should be pointed out. It concerns the formula (11) which was introduced into integrand (10). If instead of Eq.(11) the one, singular term for stresses in front of the crack is introduced into Eq.(10)

$$\sigma_{22}(r,\theta) = \frac{K_I}{\sqrt{2\pi r}} f_{ij}(\theta) + ...0(r^0) \tag{13}$$

the following relation is obtained:

$$\frac{\sigma_f}{\sigma_m} = \frac{1}{\sqrt{\frac{2a}{\Delta a}}} \tag{14}$$

In this case the critical external stress σ_f reaches the infinite value when a = 0. It is obvious that Eq.11 represents more than one term of the Williams series. Indeed, if the following relations x=a+r (Fig.6) and $K_I = \sigma\sqrt{\pi a}$ (Griffith crack) are introduced into Eq. 11 one obtains

$$\frac{x\sigma}{\sqrt{x^2 - a^2}} \Rightarrow \frac{K_I\left(1+\frac{r}{a}\right)}{\sqrt{\pi r\left(2+\frac{r}{a}\right)}} \tag{15}$$

Eq. 15 reduces to (13) for $r/a \ll 1$. An important conclusion follows from the above discussion for the Finite Fracture Mechanics. It is not enough to assume that the crack jump is finite. One should take into consideration more than one term in the Williams series. Thus, Eq. 7 is probably not precise, since to derive it one term was taken into account only. That such an assumption is very strong one has already been shown below Eq.8.

Let us reanalyze the Novizhilov formula (Eq.10) using more than one term from the Williams series. Actually we will use three terms, since the second one is equal to zero for the opening stress component, σ_{22}.

$$\sigma_{22} = \frac{K_I}{\sqrt{2\pi r}}\frac{1}{4}\left(3\cos\frac{\theta}{2} + \cos\frac{3\theta}{2}\right) + \frac{A_I\sqrt{r}}{\sqrt{2\pi}}\frac{3}{4}\left(5\cos\frac{\theta}{2} - \cos\frac{5\theta}{2}\right) \tag{16}$$

for θ=0

$$\sigma_{22} = \frac{K_I}{\sqrt{2\pi r}} + \frac{3A_I\sqrt{r}}{\sqrt{2\pi}} \tag{17}$$

When Eq.17 is introduced to (10) the following formula is obtained.

$$\sqrt{\frac{2}{\pi}}\sqrt{\Delta a}\left\{K_C\left[(a+\Delta a)^{1/2} - a^{1/2}\right] + A_C\left[(a+\Delta a)^{3/2} - a^{3/2}\right]\right\} = \sigma_m \Delta a \tag{18}$$

It follows from Eq.18 that the critical stress in front of the crack, σ_m is not a material constant and it changes since both K_C and A_C depend on the structural element geometry. Also, σ_m and Δa are not independent of each other. An example is shown in the Fig. 7. At the distance measured according to Eq.1 (which is $1.7\cdot10^{-2}$m) the influence of the in-plane constraint expressed by the A – term is essential. This influence is certainly even larger if the small scale yielding is accepted. For metals and alloys the purely linear elastic materials are not often met.

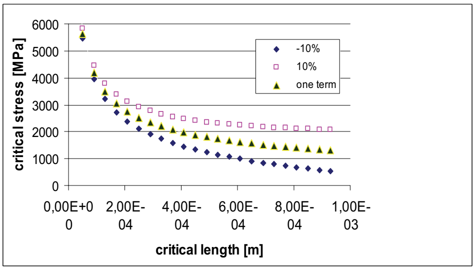

Fig. 7. Hypothetical average critical stress vs. crack jump. The K_C=100 MPa(m)$^{1/2}$, σ_o=600 MPa. The A term was assumed to provide the stress levels from 10% to -10% of the first term measured at the distance twice larger than the crack tip opening displacement.

If $K_I = \sigma\sqrt{\pi a}$ is introduced into Eq. 18 and the crack length a is assumed to be equal to zero the ultimate strength can be estimated as

$$\sigma_f = \sigma_m = \sigma_u = \sqrt{\frac{2}{\pi}} A_I \sqrt{\Delta a} \tag{19}$$

To assure the finite value of σ_f the value of A_I, which is function of external loading and geometrical dimensions of the specimen should depend on the crack length in the specific way, e.g. A_I~$(1+a/W)$.

It follows from Eq.18 and from the relation $K_I = \sigma\sqrt{\pi a}$ that

$$\frac{\sigma_f}{\sigma_m} = \frac{\sqrt{\pi/2}\cdot\eta - \frac{A_C}{\sigma_m}\sqrt{a}\left[(1+\eta)^{3/2} - 1\right]}{\sqrt{\pi}\left[(1+\eta)^{1/2} - 1\right]} \tag{20}$$

where $\eta=\Delta a/a$. If the right hand sides of Eqs 12 and 20 are compared one can receive the relationship between σ_m and A_C and Δa. Thus, the critical stress in front of the crack can be computed. Now, it is not a function of the critical stress intensity factor but it is function of the crack length, crack jump length and the external loading through the A_C term. This critical stress is not a material constant.

There is one important point in the theory of critical distances that should be stressed one more time. The very nice and simple relation (1) has been derived for the Griffith crack *only* and for very simple formulas (3), (5) and (11). For other geometries such a simple relation is not obtainable and a critical distance is different. When using Eq.4 and computing the strain energy for Griffith crack but using the Williams series instead of Eq. 3 the same result can be obtained for the one volume of integration only. For the crack length $2a$ one must integrate along the circle of the radius a, and the coordinate system located at the crack tip. Only one term for stress and strain must be used. Selection of such a domain of integration may even have a good justification but neglecting the second and higher terms does not. At the distance from the crack tip greater than $0.01a$ higher terms play an important role. The T – stress can not be neglected and it is widely known that this term plays an important role as the in-plane constraint measure.

Discussion

The theory of critical distances in fracture mechanics, where the critical distance is defined as Eq.1, provides interesting relations between various “critical stresses” in fracture and the crack lengths for short cracks in particular. In [3] author makes a short review of the experimental results for a wide class of fracture and fatigue problems. These problems are so different in a geometry of the test specimens, materials tested, shape and size of the defects used that looking for a unique theory is a very risky task. In fact the “critical distance” by definition differs by a factor 4 (Eq.1) from one case to another. Moreover, the critical stresses in (1) differ depending, among others, on the material by a factor up to three [24]. It means that the length must be corrected by a factor 9. In such a case it is not easy to accept a unique “critical length” parameter and a unique theory to explain all those experimental results summarized in [3]. It was shown, in the previous section, that two of four theories quoted and described in [3] are not convincing and they provide results which are not sufficiently exact. They are based on very strong assumptions. In fact, the fracture toughness entering Eq. 1 strongly depends on the in – plane constraint measure, the T parameter, for the short cracks. If it is so, how can this parameter be neglected in the derivation process of Eq.1? The third of four theories listed in [3], so called *point method PM* is not reasonable for metals and metallic alloys. The level of the stress components at the distance defined by Eq.1 is so low that the fracture is not likely to happen at this region. This distance has no reasonable physical meaning for the low in – plane constraint. The simple form of Eq. 1 is accidental and it is due to the simplicity of the Griffith crack geometry. For any other geometries the shape of Eq.1 must be different. Also the higher order terms in the Williams series should be used in derivation process and neglected when justified after derivation.

To finish this article with a positive accent it is *postulated* to redefine the *critical distance:*

$$\text{Critical distance} = \xi \cdot \zeta \frac{K_C^2}{E\sigma_o} \text{ or } = \xi \cdot \zeta \frac{J_C}{\sigma_o} \tag{22}$$

which is of the order of the crack tip opening displacement or the distance of the maximum opening stress location in front of the crack (elastic-plastic materials). The coefficient $\zeta = \frac{4}{\pi}$ for plane stress or $\zeta = \frac{4}{\pi}\frac{\left(1-\nu^2\right)}{\sqrt{3}}$ for plane strain. The coefficient ξ is to be determined but it should be of the order of 1 to 2. This coefficient may reflect the hypothesis that the opening stress in front of the crack should be greater than the critical one along the domain of certain length l_C [1], [2]. At this distance from the crack tip the higher terms in the Williams' series can be neglected without loosing the accuracy (see Fig. 7). Moreover, at this distance all processes of micro-crack or micro-voids nucleation and growth take place, at least in the steels. Also the experimental results as shown e.g. in [5] can still be well fitted to the theoretical curve. The Eq. (22) does not change the character of the Eq.(1). It is also proportional to K_I^2 or to J integral.

When (22) is introduced to (7) replacing the quantity $\Delta a/2$, G_C should be replaced by $J_C/(1+n)$, where n is Ramberg-Osgood power exponent and J_C is fracture toughness measured experimentally. In such a way one can receive the reasonable value of the critical external stress. The ratio $J_C/(1+n)$ follows from [13] as this part of energy dissipated which is spent on a new surface creation. In fact this ratio is proportional to the surface energy but the neglected coefficient is close to one.

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