Investigations on initiation and growth of fatigue cracks

Manuela Sander[a], Hans Albert Richard[b] and Janina Menke[c]

Universität Paderborn, Fachgruppe Angewandte Mechanik
Pohlweg 47-49, 33098 Paderborn, Germany

[a]sander@fam.upb.de, [b]richard@fam.upb.de, [c]menke@fam.upb.de

Keywords: Fatigue crack growth, crack initiation, structural durability, threshold stresses, short crack growth

Abstract. For a comprehensive description of the whole lifetime beginning with the crack initiation and ending with the failure it is required to merge the conventional concepts of structural durability and fracture mechanics and to extend them with the prediction of the crack initiation and the short crack growth. To check the reliability of the concepts experimental investigations with CTN specimen and notched round specimen have been performed. The comparison of the S-N-curves with simulation results show that the approaches can be conservative as well as non-conservative depending on the application. For the description of the crack initiation several methods and concepts exist, which can be divided into four groups: Threshold stress curve concepts, theory of critical distances, fatigue resistance curve approaches and the $\sqrt{area}$-concepts. The concepts are described and evaluated using numerical simulations.

Introduction

In order to ensure safety and structural mechanical reliability adequate concepts as well as methods for lifetime predictions of structures and components under cyclic loading should be used already in the phase of product development. Depending on their application the design criteria are very different. In aerospace, but also in other technical fields, two criteria are pursued: The Safe-life-criterion and the Damage-tolerant concept. In general these two types of design concepts lead in today´s practice often to lifetime predictions with a separated consideration of the crack initiation and the crack growth lifetime of machines or means of travel.

Conventional concepts of structural durability (stress- and strain-life approaches) allow statements about the lifetime up to an incipient crack. On the other hand with the conventional concepts of fracture mechanics and fatigue crack growth the final lifetime can be determined. An approach that accounts for both the crack initiation and propagation phase should result in a more efficient lifetime prediction. Thus, it is required to merge the concepts of structural durability and the concepts of fracture mechanics and to add the method for crack initiation and short crack growth.

Stress- and strain-based approaches

Using stress-based approaches the loading of a component or a structure is specified by nominal stresses. On the basis of the findings of *Palmgren* and *Miner* often the linear damage accumulation is applied, which add the damages of each cycle. The damage sum is defined as follows:

$$D = \sum_{i=1}^{k} \frac{n_{\mathrm{i}}}{N_{\mathrm{i}}}, \tag{1}$$

where n_i is the number of cycles on each level and N_i the number of failure cycles on the i-th level of a frequency distribution obtained from a S-N-curve. This implies that the damage of each cycle is independent of each other.

Many investigations [1-4] have shown that the so called *Miner´s* rule lead to both conservative and non-conservative results, thus many different modifications of this concept have been developed. These variants are based on modifications of the S-N-curve using an individual slope of the S-N-curve in the region of high-cycle-fatigue as well as below the nominal endurance limit.

The modified *Miner´s* rule proposed by *Haibach* uses an inclined line with the slope $2k$-1 beneath the endurance limit, where k is the slope of the S-N-curve in the finite life region.

The approach of *Liu* und *Zenner* [5] takes both fracture mechanical aspects and structural durability into account. With this concept the different behavior of short crack growth up to a initial crack length and of long crack growth is considered. Therefore *Liu* und *Zenner* define a reference S-N-curve, which slope is given by an average value of the slope k of the conventional S-N-curve and the slope m of the crack growth S-N-curve and the *Paris* exponent, respectively. For steels they propose a constant value of $m = 3.6$ for the slope of the crack growth S-N-curve. The S-N-curve in the finite life region is rotated around the maximum value of the frequency distribution and moreover a fictitious endurance limit $\sigma_D^* = 0{,}5\sigma_D$ is defined.

In Figure 1 a comparison of the experimentally and analytically determined S-N-curves for the standard load spectrum FELIX/28 is given. For the experimental investigations a round bar with a circumferential notch with a notch radius of 0,2 mm made of the steel 34CrNiMo6 is used. The analytical simulations have been performed with the program LMS FALANCS [6].

It becomes obvious that the approach of *Liu* und *Zenner* agree very well with experimental data. The modified *Miner´s* rule of *Haibach* rather lead to non-conservative results. This effect is also described for other loadings and specimen [3].

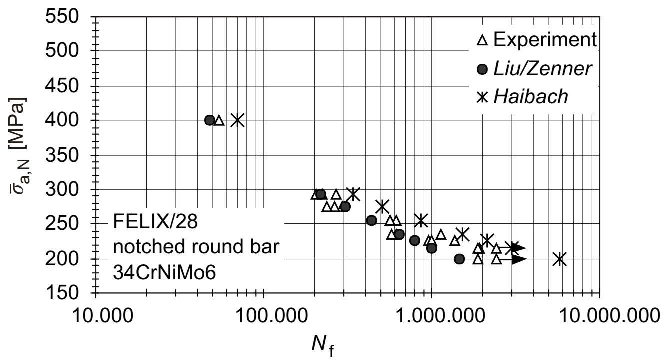

Figure 1: Comparison of the results of stress-based concepts with experimental data of notched round bars made of 34CrNiMo6 for the standard helicopter load spectrum FELIX/28 [3]

As mentioned *Liu* und *Zenner* assume that the slope of the crack growth S-N-curve matches with the *Paris* exponent m. For the investigation of this assumption crack growth S-N-curves for different materials depending on the R-ratio are determined using the crack growth simulation program NASGRO developed by the NASA. For these crack propagation S-N-curves the slopes are identified [3]. It becomes obvious that the slopes of theses curves agree very well with the *Paris* exponents of the determined crack growth curves (Fig. 2a). But, for the aluminum alloy, which has a double-S-curved characteristic of the crack growth curve, the *Paris* curve only approximates this region of the crack growth curve in average (Fig. 2b).

However, the proposal of a constant value for the slope of the crack growth S-N-curve of 3.6 is not valid. The *Paris* exponent depends on the material as well as on the *R*-ratio. For instance in Table 1 different determined slopes of crack propagation S-N-curves for C(T)-specimen are given.

Table 1: Slope of crack propagation S-N-curves using C(T)-specimen

R-ratio	34CrNiMo6	42CrMo4	EN AW-7075-T651
$R = 0{,}1$	2,7	3,1	4,1
$R = 0{,}5$	2,7	2,9	3,7

Against the backround of service loads with different mean stresses this is of great importance. The accuracy of the concept of *Liu* and *Zenner* is originated from the average of the slopes of crack propagation and conventional S-N-curve.

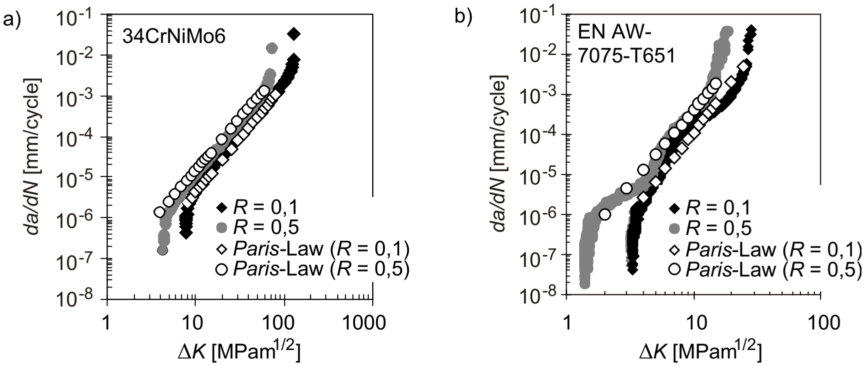

Figure 2: Comparison of the slope of the crack growth S-N-curve using the *Paris*-law with experimentally determined crack growth curves depending on the *R*-ratio for
a) 34CrNiMo6 and b) the aluminum alloy EN AW-7075-T651

Contrary to stress-based approaches strain-based concepts use the local elastic-plastic stresses and strains in the critical plane compared to the strain-life-curve. For the determination of the elastic-plastic stresses and strains e.g. *Neuber´s* rule or the ESED-concept of *Glinka* [7] are applied. Using these stresses and strains for each hysteresis the damage contribution by means of the strain-life-curve is calculated. The mean stress effect is considered by different damage parameters, like the damage parameter of *Smith*, *Watson* and *Topper*

$$P_{\mathrm{SWT}} = \sqrt{\sigma_{\max} \cdot \varepsilon_{\mathrm{a,t}} \cdot E} = \sqrt{(\sigma_{\mathrm{m}} + \sigma_{\mathrm{a}}) \cdot \varepsilon_{\mathrm{a,t}} \cdot E}\ . \tag{2}$$

A damage parameter on fracture mechanical basis is developed by *Vormwald* [8]. Using *Dowlings* approximated solution of the ΔJ-integral for a half-circular surface crack

$$\Delta J = \Delta J_{\mathrm{el}} + \Delta J_{\mathrm{pl}} = \left[1{,}24\frac{\Delta\sigma^2}{E} + \frac{1{,}02}{\sqrt{n'}} \cdot \Delta\sigma \cdot \Delta\varepsilon_{\mathrm{pl}}\right] \cdot a \tag{3}$$

the damage parameter P_{J} is defined as follows:

$$P_{\rm J} = \frac{\Delta J_{\rm eff}}{a} = 1{,}24\frac{\Delta\sigma_{\rm eff}^2}{E} + \frac{1{,}02}{\sqrt{n'}}\cdot\Delta\sigma_{\rm eff}\cdot\left[\Delta\varepsilon_{\rm eff} - \frac{\Delta\sigma_{\rm eff}}{E}\right], \tag{3}$$

where $\Delta\sigma_{\rm eff} = \sigma_{\max} - \sigma_{\rm cl}$ and $\Delta\varepsilon_{\rm eff} = \varepsilon_{\max} - \varepsilon_{\rm cl}$ regard the crack closure effect. *Vormwald* has shown that the crack is not closed at the same stress level where it opens, but at the same strains. Taking this effect into account *Vormwald* calculates the damage parameter by means of *Newman's* crack opening function.

For the investigations of the accuracy of the above mentioned approaches experiments have been performed with notched round bars as well as CTN-specimen. Fig. 3 shows e.g. a comparison of simulation results and experimental data of a notched round bar made from 34CrNiMo6 and loaded with the standard load spectrum FELIX/28. Within the scope of these investigations the damage parameter of *Smith*, *Watson* and *Topper* lead to both conservative and non-conservative lifetime predictions [3], because the slopes of the experimental S-N-curve and the simulated S-N-curve are very different. *Vormwald's* damage parameter lead in all investigated cases to more or less conservative results. Further details about this study are given in [3].

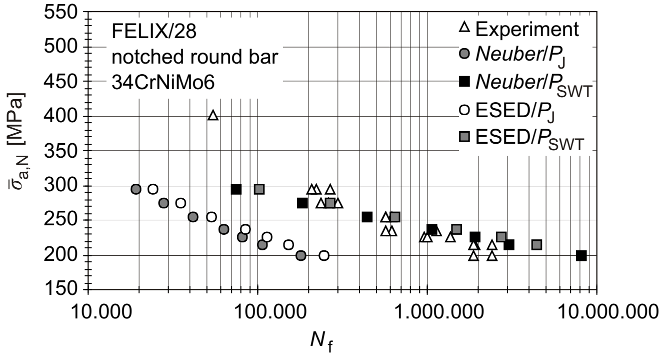

Figure 3: Comparison of the results of strain-based life concepts with experimental data of notched round bars made of 34CrNiMo6 for the standard helicopter load spectrum FELIX/28 [3]

For the calculation of the elastic-plastic stresses and strains *Neuber's* rule and the ESED-approach was applied. The differences in the lifetime prediction are very small. But the results of the ESED-approach tend to result in higher lifetime predictions independent of the used damage parameter.

Using the strain-based approaches lifetimes are predicted up to a initial crack, thus the S-N-curves have to be reduced by the crack propagation lifetime. Therefore the crack propagation lifetime of each corresponding stress level is subtracted from the failure lifetime with a failure probability of 50%.

In order to determine the propagation lifetime for the notched round bars some simulations have been performed with the program NASGRO. Due to the fact of the very sharp notch in the round bar, the propagation life prevails the initiation life. Hence the difference between a crack initiation S-N-curve and a failure S-N-curve can be neglected.

In the case of the used CTN-specimen the propagation life is determined with a C(T)-specimen, at which the initial crack length coincides with the notch depth of the CTN-specimen [9]. The difference between the failure lifetime with a failure probability of 50% and the propagation lifetime on each level defines the crack initiation S-N-curve. Further details about the investigations on CTN specimen e.g are given in [3, 9].

Crack initiation concepts

For the description of crack initiation several models and concepts exists, which can be divided into four groups:

- Threshold stress curve concepts,
- Theory of critical distances,
- Fatigue-resistance-curve concepts and
- $\sqrt{area}$-concepts.

The threshold stress curve concepts assume, that the stress level, at which a crack nucleates, depends on the crack length. For a crack length beneath a_0 the threshold stress converge asymptotical a constant value, which approximately equates the fatigue limit of an unnotched specimen. Above this limit crack length linear elastic fracture mechanics has to be apply. The most famous concept is developed by *Kitagawa* and *Takahashi* [10]. *El Haddad* et. al. [11] define the threshold stress limit curve as follows:

$$\Delta\sigma_{\rm th} = \frac{\Delta K_{\rm th}}{\sqrt{\pi \cdot (a + a_0)}} \qquad (4)$$

with

$$a_0 = \frac{1}{\pi}\left(\frac{\Delta K_{\rm th}}{\Delta\sigma_{\rm D}}\right)^2 . \qquad (5)$$

The underlying assumption of the theory of critical distances [12, 13] is that an average stress in front of a notch or a crack at a loading $\Delta K = \Delta K_{\rm th}$ equals the fatigue limit $\sigma_{\rm D}$. For simplicity *Taylor* reduces the calculation to a consideration of the stress at a single point or averaged over a given distance, area or volume. These approximations are referred to as point-, line-, area- and volume-methods [11].

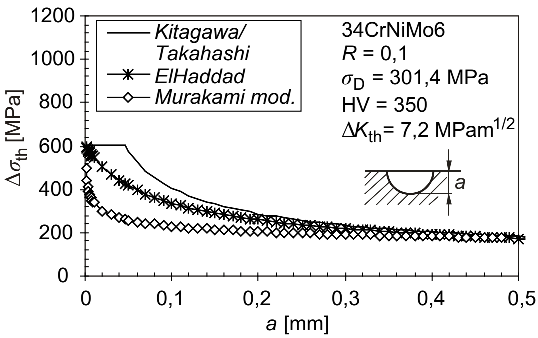

Figure 4: Crack initiation concepts for a half-circular surface-crack in the steel 34CrNiMo6 (R = 0.1)

Using the point-method the stresses at a distance $r = a_0/2$ are compared to the fatigue limit. By contrast the other methods use an average value evaluated by integration over the distance $r = 0 – 2a_0$ or over the area $r = a_0$.

The concepts of fatigue resistance curves [14, 15] propose that the fatigue crack growth threshold value in the regime of short cracks increases with increasing crack length until the constant value $\Delta K_{\rm th}$ for long crack is reached. This behavior is defined with so called fatigue resistance curves or R-curves.

Murakami has observed that for the consideration of small defects not the stress concentration factor is decisive, but the stress intensity factor of non-propagating cracks. Thus, for the evaluation of defects with different sizes and forms the parameter √*area* is used. The parameter *area* is the crack area projected on the plane perpendicular to the maximum principal stresses.

Figure 4 shows a comparison of the threshold stress curve approaches and the √*area*-concept applied to a half-circular surface crack in the steel 34CrNiMo6. Because the fatigue limit of the material is overestimated by the use of the original approach of the √*area*-concept taking the mean stress effect into account, the calculated threshold stresses in Fig. 4 are assumed as stress ranges in contrast to *Murakami´s* specifications. It becomes obvious that with this modification on the one hand the threshold stress for long cracks converge to the threshold value and on the other hand the fatigue limit is adequately reproduced. However, in the whole short crack regime the determined stresses are significantly smaller than those of the fatigue limit approach.

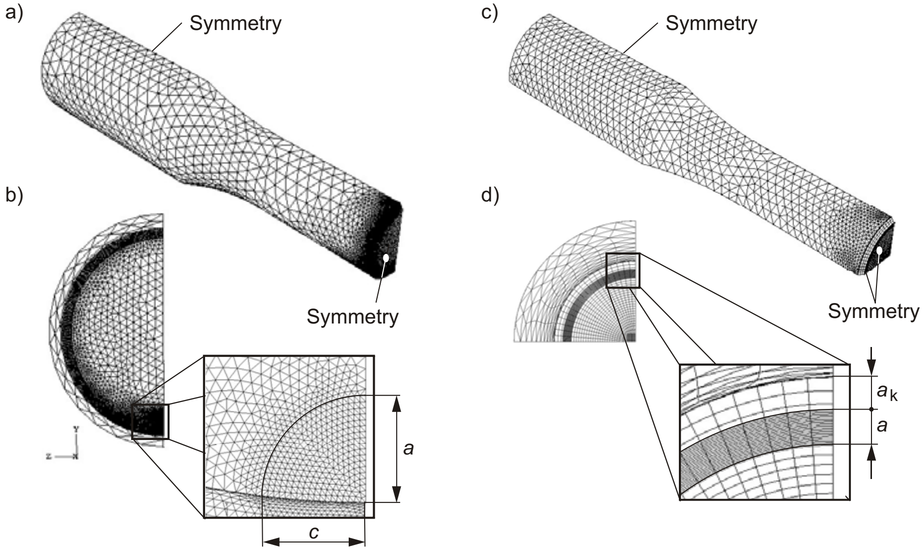

Figure 5: Three-dimensional FE-modeling of a notched round bar [3]
a) View of a quarter of the round bar with a circular surface crack
b) View in the *y*-*z*-plane in the notch root with a detail of the mesh refinement of the circular surface crack
c) View of a eighth of the round bar with a circumferential crack
d) View in the *y*-*z*-plane in the notch root with a detail of the mesh refinement of the circumferential crack

For the assessment of the theory of critical distances numerical simulations have been performed for a crack in a notched round bar (Fig. 5) [3]. The material parameters of the steel 34CrNiMo6 result in an *El Haddad* parameter of 0.03 mm. The results for a circumferential crack initiated in the notch root (Fig. 5c and d) show that not only between the limit stresses of the line- and area-method 10% differences are observable, but also between the threshold stresses of the point- and line-method. Within the scope of these investigations the threshold stresses of the line-method lead to 10% higher values than the limit stresses of the point-method. Because other investigations do not account for this difference, the choice of the correct method seems to be application dependent.

Instead of a circumferential crack also numerical simulations have been performed for a half-circular crack initiated in the notch root (Fig. 5a and b). Therefore different positions in front of the growing crack and the notch have been analyzed using the concept of critical distances (Fig. 6). The crack growth has been simulated in such a manner that the crack keeps a half-circular crack. Moreover the evaluation distance has been investigated. The results depending on the crack depth *a* are given in Fig. 6.

If a distance of $a_0/2$ in front of the deepest position of the crack front is used for the stress determination, the threshold stresses increase slightly after 0.03 mm crack growth, but then they stagnate. With increasing distance in front of the crack the threshold stresses rise as well and an increase over a distance of 0.2 mm is observable. The limit stresses near the surface of the notch are significantly smaller, which furthermore decrease with increasing crack length. As a reference line the analysis in a radial distance to the circumferential notch is given in Fig. 6.

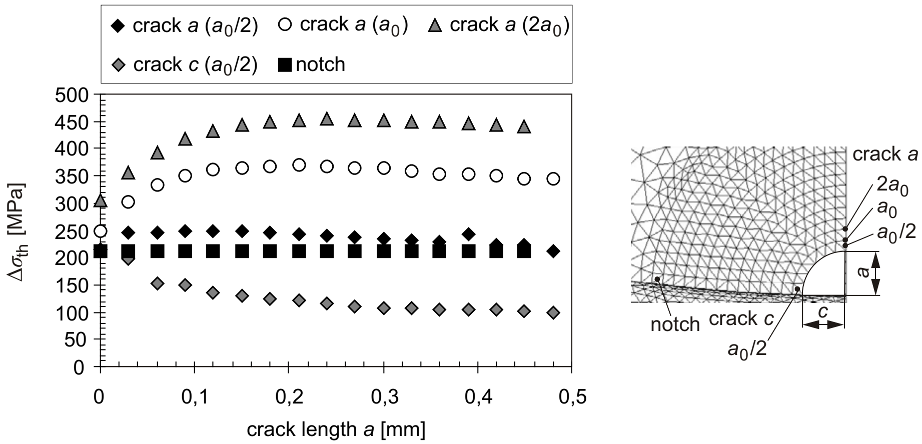

Figure 6: Threshold stresses determined with the theory of the critical distance for a circular surface crack depending on the evaluation distance and position [3]

From this stress distribution it can be concluded that the half-circular crack growth, which has been assumed in the numerical simulation, is not continued in reality. According to the stress evaluation the crack will grow firstly in circumferential direction, because the limit stresses are at a lowest level. These observations coincides with conventional fracture mechanical analysis and fractographical investigation .

Conclusion

Within the scope of this paper investigations of many concepts and their results have been described. The accuracy of the results depends on numerous factors, like the material or the geometry. Moreover it has been demonstrated that no method or concept exists, which can thoroughly be applied by all means.

References

[1] Schijve, J.: Fatigue of Structures and Materials. Kluwer Academic Publisher, Dordrecht, 2001.

[2] Schütz, W.; Zenner, H.: Schadensakkumulationshypothesen zur Lebensdauervorhersage bei schwingender Beanspruchung – Ein kritischer Überblick. Teil 1: In: Zeitschrift für Werkstofftechnik, 1973, Nr. 1, S. 25-33. Teil 2: In: Zeitschrift für Werkstofftechnik, 1973, Nr. 2, S. 97-102.

[3] Sander, M.: Sicherheit und Betriebsfestigkeit von Maschinen und Anlagen. Springer-Verlag, Berlin, 2008.

[4] Berger, C.; Eulitz, K.-G.; Heuler, P., Kotte, K.-L.; Naundorf, H.; Schütz, W., Sonsino, C. M.; Wimmer, A.; Zenner, H.: Betriebsfestigkeit in Germany – an overview. In: Intern. J. Fatigue, Vol. 24, 2002, pp. 603-625.

[5] Zenner, H.; Liu, J.: Vorschlag zur Verbesserung der Lebensdauerabschätzung nach dem Nennspannungskonzept. In: Konstruktion, 44, 1992, S. 9-17.

[6] LMS FALANCS, Version 2.11, LMS International, Leuven, Belgien, 2006.

[7] Glinka, G.: Calculation of inelastic notch-tip strain-stress histories under cyclic loading. In: Eng. Frac. Mech., Vol. 22, 1985, pp. 839-854.

[8] Vormwald, M.: Lebensdauervorhersagen verbessern. In: Materialprüfung, 45, 2003, S. 92-99.

[9] Sander, M.; Menke, J.; Richard, H. A.: Experimental investigations on crack initiation and threshold values. CD-ROM Proc. of the Intern. Conference on Crack Paths, CP 2006, Parma.

[10] El Haddad, M.H.; Topper, T.H.; Smith, K.N.: Prediction of non propagating cracks. In: Eng. Frac. Mech. Vol. 6, 1970, pp. 379-392.

[11] Kitagawa, H.; Takahashi, S.: Applicability of fracture mechanics to very small cracks or the cracks in the early stage. In: Prod. of the 2nd Intern. Conf. on Mechanical Behavior of Materials, Boston, 1976, pp. 627-631.

[12] Taylor, D.: Geometrical effects in fatigue: a unifying theoretical model. In: Intern. J. Fatigue, Vol. 21, pp. 387-394.

[13] Fujimoto, Y.; Hamada, K.; Shintaku, E.; Pirker, G.: Inherent damage zone model for strength evaluation o f small fatigue cracks. In. Eng. Frac. Mech., Vol. 68, 2001, pp. 455-473.

[14] Pippan, R.; Tabernig, B.; Gach, E.; Riemelmoser, F.: Non-propagation conditions for fatigue cracks and fatigue in the very high-cycle regime. In: FFEMS, Vol. 25, 2002, pp. 805-811.

[15] Tanaka, K.; Akiniwa, Y.: Resistance-curve method for predicting propagation thresholds of short fatigue cracks at notches. In: Eng. Frac. Mech., Vol. 30, 1988, pp. 863-876.

[16] Murakami, Y.: Metal Fatigue: Effects of small defects and non-metallic inclusions. Elsevier, London, 2002.