Investigation of Elastic and Strength Properties of Nanoobjects

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Keywords: Atomic force microscopy, elastic and strength properties, nanoobjects.

Abstract. Using this mechanical model, analytical approach and numerical methods, we obtained the dependence elastic properties and dimension of nanoobject from the cantilever oscillation amplitude.

Introduction.

Elastic characteristics of nanoobjects being studied using analytical approach and numerical methods. The dependence of elastic stiffness and strength properties is determined from the cantilever oscillation amplitude in the atomic force microscopy.

Atomic force microscopy is a system contains a cantilever-tip and a sample (nanoobject) (Fig. 1).

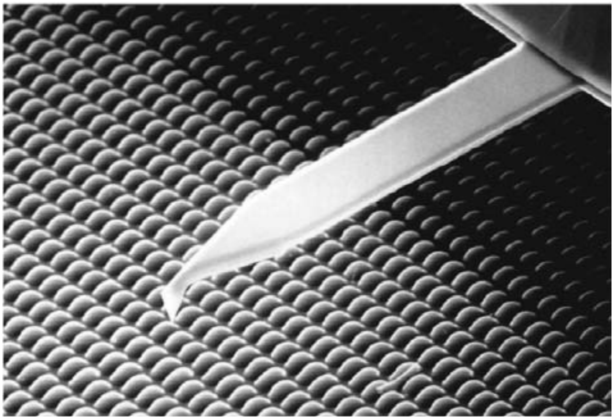


Fig. 1. Atomic force microscopy [1].

AM-AFM is a dynamic force microscopy mode where the cantilever-tip is excited at a fixed frequency, usually near or at the free resonance frequency. The oscillation amplitude is used as a feedback parameter to image the sample topography. The scan technique with using AM is: the cantilever-tip is on the object surface, the cantilever oscillates comparatively object. When cantilever-tip is contact the object surface, the oscillation amplitude changed. This changing depended from elastic and strength object characteristics. Consequently, when we process data about cantilever-tip location, we can obtain data about surface relief, elasticity, strength, viscosity.

Using analytical approach and numerical methods, in this paper we obtain elastic characteristics of nanoobject on basis of cantilever oscillation amplitude. The driving force's frequencies agree with resonance frequencies of cantilever free oscillations (Fig. 2): $\omega_0^2 = \frac{c}{m}$

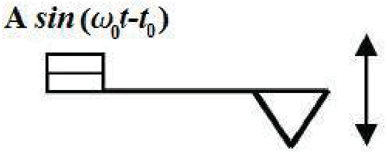


Fig. 2. The driving force action.

In this work we consider cantilever-tip like mass point on the spring with stiffness C. The object's model is a without mass point on the spring with c_1 (Fig 3).

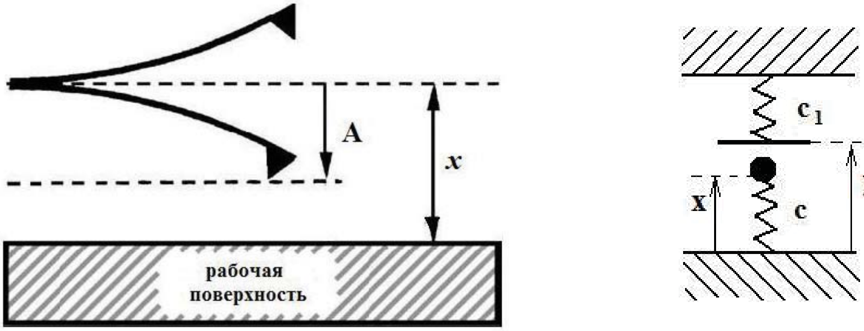


Fig 3. Mechanical model.

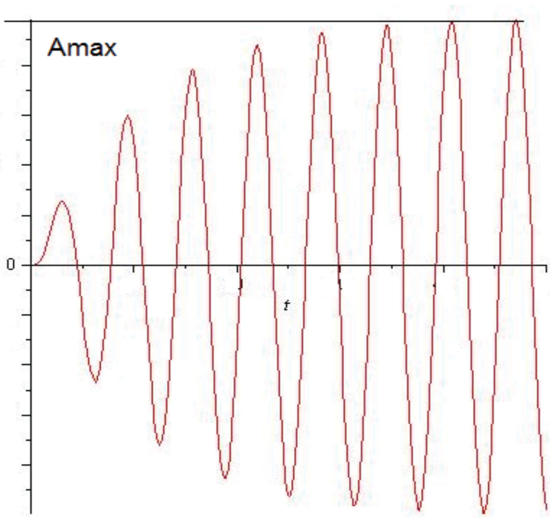


Fig 4. The oscillations of the cantilever-tip and the driving force.

The oscillations of the cantilever-tip and the driving force are shown in Fig. 4. The oscillations reach the stationary regime after several cycles of vibration.
The system's motion can be composed of two phase:

1) The oscillations of the cantilever-tip with object.
2) The oscillations of the cantilever-tip without object.

The oscillations of the cantilever-tip and the driving force are described by nonlinear second-order differential equation (1):

$$m\ddot{x} + b\dot{x} + cx = A\sin(\omega_0 t - t_0), \tag{1}$$

where A and ω_0 – amplitude and frequencies of the driving force, m – cantilever's mass, c and b– system stiffness and viscosity. Under contact the cantilever-tip with the object stiffness is added to the system (eq. 2):

$$m\ddot{x} + b\dot{x} + cx = A\sin(\omega_0 t - t_0) + c_1(y - x) \tag{2}$$

Numerical approach

The numerical solution of equations (1, 2) is shown on Fig. 5. The ho is object height.

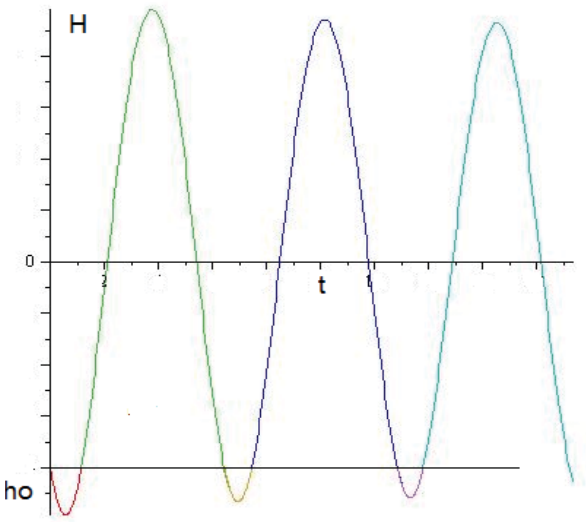


Fig. 5. The oscillations of the cantilever-tip with object.

The dependence nanoobject height (with different stiffnesses) from the cantilever oscillation amplitude is shown on Fig.6. As we can see, the cantilever oscillation amplitude is decrease with nanoobject height is raise. The Amax is maximal cantilever oscillation amplitude in the stationary regime.

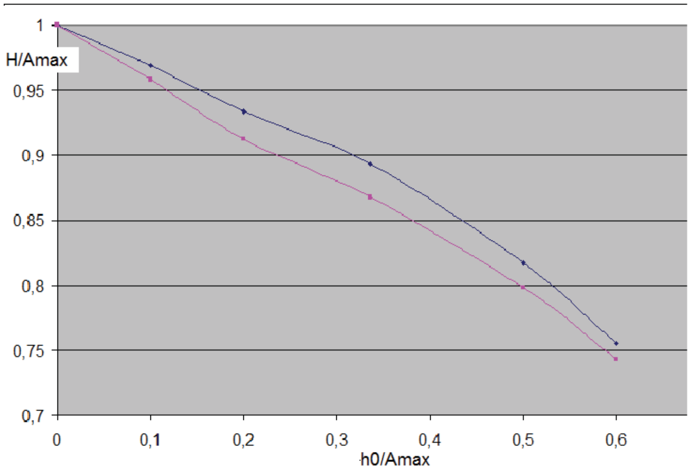


Fig.6. Dependence nanoobject height from the cantilever oscillation amplitude

Analytical approach

Let introduce nondimensional time: $\tau = \omega_0 t$. It easy to show the equation (1, 2) can be written as equation (3, 4)

$$\xi'' + 2\nu\xi' + \xi = \sin(\tau - \tau_0) \tag{3}$$

$$\xi'' + 2\nu\xi' + \chi^2\xi = \sin(\tau - \tau_0) + (\chi^2 - 1)\eta \tag{4}$$

We used the following notations:

$$\xi = \frac{m}{A}\omega_0^2 x, \quad \eta = \frac{m}{A}\omega_0^2 y, \quad \nu = \frac{b}{2\omega_0 m}, \quad \chi^2 = \frac{c + c_1}{c} \approx 1 + \nu,$$

ν – small parameter.

The functions ξ_1 and ξ_2, are the solutions of the equations (3,4):

$$\begin{cases} \xi_1 = \zeta_1 + C_{11} f_{11}(\tau) + C_{12} f_{12}(\tau) \\ \xi_2 = \zeta_2 + C_{21} f_{21}(\tau) + C_{22} f_{22}(\tau) \end{cases} \tag{5}$$

where C_{11}, C_{12}, C_{21}, C_{22} – integration constants. The oscillations are determined by functions ξ1 and ξ2:

$$\zeta_1 = \frac{1}{2\nu}\sin(\tau - \tau_0 - \frac{\pi}{2}), \; f_{11} = e^{-\nu\tau}\sin(\sqrt{1-\nu^2}\tau), \; f_{12} = e^{-\nu\tau}\cos(\sqrt{1-\nu^2}\tau) \tag{6}$$

$$\zeta_2 = \frac{1}{2\nu\chi}\sin(\tau-\tau_0-\frac{\pi}{2})+\frac{\chi^2-1}{\chi^2}\eta,\ f_{21}=e^{-\nu\tau}\sin(\sqrt{\chi^2-\nu^2}\tau),\ f_{22}=e^{-\nu\tau}\cos(\sqrt{\chi^2-\nu^2}\tau) \quad (7)$$

The type of the solution is shown on Fig.7. In the moment $\tau=\tau 1$ a transition from first phase to second take place. We try stationary periodical solution.

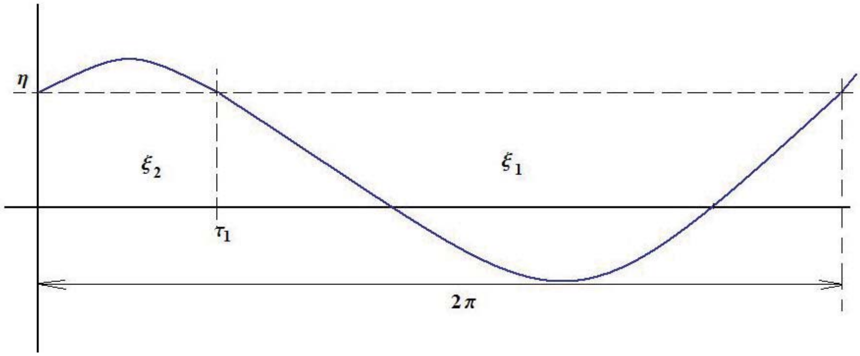


Fig. 7. Type of the solution.

In the moment $\tau=\tau_1$ the first phase of motion finish.

We have 6 unknown parameters (integration constants C_{11}, C_{12}, C_{21}, C_{22}, τ_1 and τ_0) and we have boundary conditions:

The condition of continuity of the solution:

$$\xi_2(0)=\xi_1(2\pi)=\eta$$

$$\xi_2'(0)=\xi_1'(2\pi)$$

The condition of periodicity of the solution:

$$\xi_2(\tau_1)=\xi_1(\tau_1)=\eta$$

$$\xi_2'(\tau_1)=\xi_1'(\tau_1)$$

Считая ν малым параметром, из этих уравнений (без условия равенства η в начальный, конечный и переходный моменты времени) получаем

$$C_{12}(1-2\pi\nu)=C_{22}+\frac{\nu}{1+\nu}\eta,$$

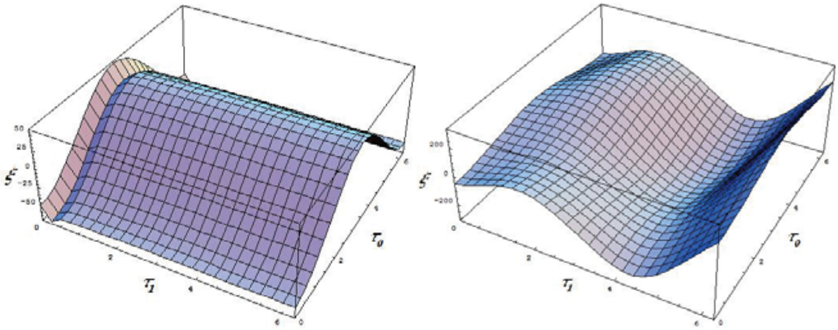
$$C_{12}(1-2\pi\tau_1)\cos[\tau_1]+C_{12}(1-2\pi\tau_1)\sin[\tau_1]=$$

$$=\frac{\nu}{1+\nu}\eta+C_{22}(1-2\pi\tau_1)\cos\left[\tau_1\left(1+\frac{\nu}{2}\right)\right]+C_{21}(1-2\pi\tau_1)\sin\left[\tau_1\left(1+\frac{\nu}{2}\right)\right],$$

$$C_{11}(1-2\pi\nu)-C_{12}\nu=C_{21}\left(1+\frac{\nu}{2}\right)-C_{22}\nu,$$

$$\cos[\tau_1]\{C_{11}-C_{12}\nu\}-\sin[\tau_1]\{C_{12}+C_{11}\nu\}=$$
$$=\cos\left[\tau_1\left(1+\frac{\nu}{2}\right)\right]\left\{C_{21}\left(1+\frac{\nu}{2}\right)-C_{22}\nu\right\}-\sin\left[\tau_1\left(1+\frac{\nu}{2}\right)\right]\left\{C_{22}\left(1+\frac{\nu}{2}\right)+C_{21}\nu\right\}. \qquad (8)$$

For determination τ1 and τ0 we plot surfaces $\xi_2(0)=\eta$ $\xi_2(\tau_1)=\eta$ (Fig. 8). On Fig. 9 we can see a crossing surfaces $\xi_2(0)=\eta$ $\xi_2(\tau_1)=\eta$ with plane ξ=η.



$\xi_2(0)$ $\xi_2(\tau_1)$

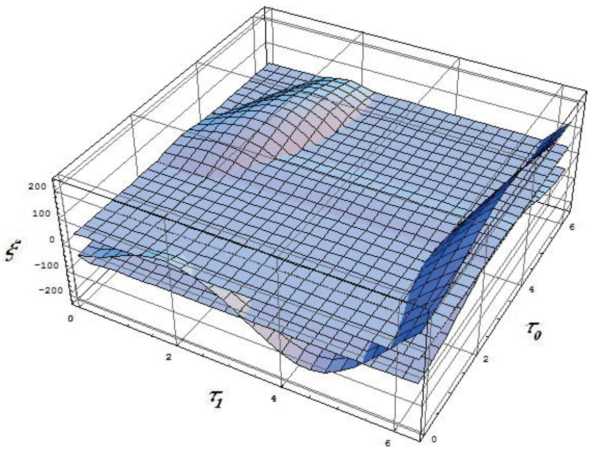
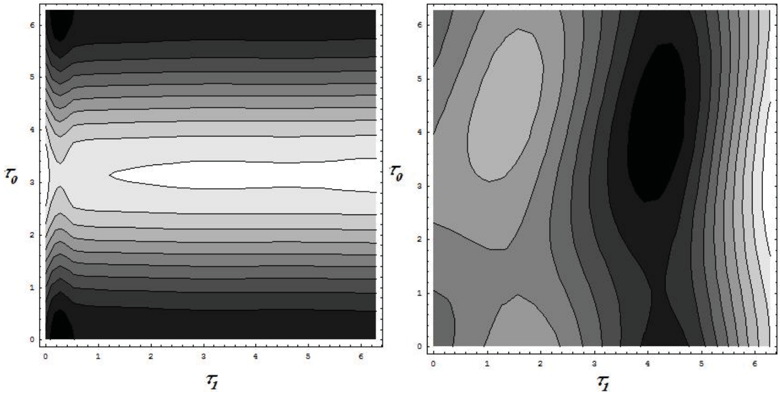


Fig. 8. Surfaces $\xi_2(0)=\eta$, $\xi_2(\tau_1)=\eta$.

In this case the value η=0,7A.



$\xi_2(0)$ $\xi_2(\tau_1)$

Fig. 9. Level line (surfaces $\xi_2(0) = \eta$, $\xi_2(\tau_1) = \eta$)

The coordinates of crossing all surfaces (Fig. 8) let us obtain values of parameters τ_1 and τ_0. We can change the parameter η (nanoobject height) and obtain graphic series, which determine solution.

Summary

Using this mechanical model, analytical approach and numerical methods, we obtained the dependence elastic properties and dimension of nanoobject from the cantilever oscillation amplitude.

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