Instability Analysis for a C(T) Specimen Undergoing Stable Crack Extension and the Crack Growth Law Concept

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Abstract. The fracture toughness of a ductile material may be measured as a point value, or as a complete fracture toughness resistance curve, as per ASTM Standard Method E 1820. In the latter option, a *J*-based resistance curve — the *J-R* curve — is obtained from a single specimen fracture test, in which the actual crack length may be measured from elastic compliance changes.
Donoso, Zahr and Landes proposed an alternative method of construction of the *J-R* curve that makes use of a postulated crack growth law relating stable crack extension, Δa, to plastic displacement, v_{pl}. In this method, an analytical treatment of both the J-Δa curve and the force-displacement, *P-v*, curve, is possible even when the only inputs are the *P-v* curve and the initial and final crack length values; that is, when no crack extension measurements concurrent with the fracture test are available.

Based upon the crack growth law concept, and on the Common and the Concise Format Equations developed by Donoso and Landes, two alternative ways of generating values of crack size along the *P-v* curve are presented. These methods, designated as the "*intercept*" method, and the "*corrected force*" method, are examined by using known crack extension data obtained by the unloading compliance technique on a C(T) fracture specimen. The role of the maximum load on the crack growth law is emphasized, in the light of the relations obtained from an instability analysis of the *P-v* curve for a ductile material that undergoes stable crack extension. *P-v* curves obtained with these methods of crack extension evaluation are compared with those experimentally obtained from a C(T) fracture test specimen. The results are encouraging, and suggest that the method could be applied to cases in which there are only measured initial and measured final crack sizes available.

Introduction

The construction of a *J-R* curve for a ductile material requires consideration of the stable crack extension process, and the appropriate correction for the incremental crack growth that ensues. According to Standard ASTM E1820 [1], the *J-R* curve is obtained from a single specimen test, in which the actual crack length is measured concurrently by the unloading compliance method, or other similar techniques, including the normalization method of Appendix A15 of Standard E1820.

Donoso, Zahr and Landes (DZL) proposed an alternative method of construction of the *J-R* curve by postulating a crack growth law that relates the measured stable crack extension, Δa, to the plastic component of the displacement, v_{pl} [2]. The original method allowed for an analytical treatment of both the J-Δa and the force-displacement (*P-v*) curves, when the sole inputs were the experimental *P-v* curve and the *measured* initial and *measured* final crack sizes, a_o and a_f. Thus, the DZL method made it possible to generate crack size values as a function of force and displacement, even when no crack extension measurements made simultaneously with the fracture test were available.

In this work, the instability analysis that constitutes the foundation of the method outlined above will be reviewed. Of paramount significance is the existence of a maximum force on the force-displacement plot for a specimen that displays stable crack extension. In fact, the amount of stable crack growth is of relatively little importance before the force attains its maximum value; in other words, most of the crack extension that develops during the test up to Δa_{max} – a quantity defined in Standard E1820 [1] – occurs for displacements larger than that at which maximum force is attained.

Two methods for evaluating the crack size at any point along the *P-v* curve beyond maximum force will be presented. The first is based upon the intercept of the actual *P-v* curve with curves with constant crack size constructed with the Common and Concise (*C&C*) Format Equations. The second is a *C&C* improved version of the normalization technique included in Standard E1820. The results obtained with these methods are compared to crack extension values measured by unloading compliance on a 1T-C(T) specimen, and related to those obtained from the maximum force solution based upon the instability condition of a fracture specimen that undergoes stable crack extension .

The P-v curve and the instability criterion

A fracture toughness test of a ductile material, in which there is stable crack growth, is carried normally to a displacement beyond that at which maximum force is attained. In Figure 1, three schematic *P-v* curves have been drawn for a C(T) specimen. The first curve, labeled as“1” or “*a variable*”, represents the behavior of a specimen that has initial crack size a_o, shows stable crack extension, and for which maximum force P_{max} is attained at the total displacement value v_c. The test is finished at a point on the curve labeled as “v_f, P_f, a_f”, meaning that displacement and force at the test final point are (v_f, P_f), and the final crack size is a_f. The total stable crack extension at the end of the fracture test is $\Delta a = a_f - a_o$.

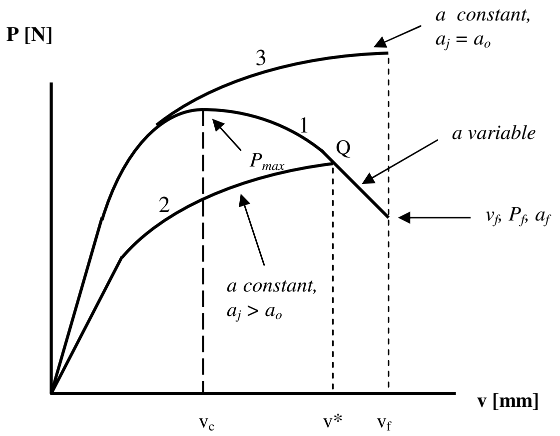

Figure 1.- Schematic force-displacement curves for variable and constant crack size.

A second curve, designated as “2” or “*a constant*, $a_j > a_o$”, intersects the “*a variable*” curve at point *Q*, which has a displacement v^*. This curve shows the behavior of a blunt-notch specimen with an initial crack size a_j, intermediate between initial and final crack sizes (i.e., $a_f > a_j > a_o$), and for which there is no crack growth. In this case, the force always increases, and the displacements may be calculated from the Common Format Equation [3] for the plastic component, and from the Concise Format [4] for the elastic part. The first method for the evaluation of crack size to be shown presently, the so-called “*intercept*” method, is based on constructing curves of the “*a constant*” type, like curve “2” of Figure 1, and has been introduced earlier as a powerful tool in the construction of *J-R* curves for C(T) specimens in which the variation of crack size during the test is not known [5].

The third curve in Figure 1 stems from the "*a variable*" curve, and is labeled "3" or "*a constant,* $a_j = a_o$". It represents the behavior the original specimen with initial crack size a_o would have if the crack did not grow during the test; that is, much like curve "2", curve "3" represents the behavior of a blunt-notch specimen with initial crack size a_o. The second method for the calculation of crack sizes, designated as the "*corrected force*" method, is based on the construction of this single curve of the "*a constant*" type, with crack size $a_j = a_o$.

Both the "*intercept*" as well as the "*corrected force*" methods rely heavily upon the Common Format Equation, CFE, proposed by Donoso and Landes [3] to describe the force-plastic displacement relationship for a blunt-notch fracture specimen. The CFE relates the applied force P with two variables representing the non-linear deformation of a fracture specimen that has a stationary crack: v_{pl}/W, the normalized plastic component of the force-line displacement, and b/W, the normalized ligament size (ligament size b in lieu of the crack size a). The CFE also includes a term that takes into account the out-of-plane constraint, Ω^*, and is written as:

$$P = \Omega^* \cdot B \cdot C \cdot W \cdot (b/W)^m \cdot \sigma^* \cdot (v_{pl}/W)^{1/n} . \tag{1}$$

where B is the specimen thickness; C and m are the geometry function parameters, and σ^* and n are material properties, obtained from normalized force-normalized displacement data [5, 6].

For a blunt-notch specimen, the crack size is constant, and v and P become the variables of the calibration function, at constant crack size. When the specimen has a sharp crack, however, and the deformation is accompanied by stable crack extension, the crack size a also becomes a variable, so that a separate relation between a and v_{pl} is needed. For such purpose, Donoso, Zahr and Landes proposed a "*crack growth law*" [2] to account for the relation between stable crack extension Δa, and plastic displacement v_{pl}. The assumed crack growth law is a two-parameter power law equation relating the change in normalized crack size, $\Delta a/W$, with normalized plastic displacement, v_{pl}/W:

$$\frac{\Delta a}{W} = l_0 \left(\frac{v_{pl}}{W} \right)^{l_1} . \tag{2}$$

In Eq. (2), l_o is a coefficient to be determined by calibration with the final point data (v_f, P_f, a_f) and l_1 an exponent, which for C(T) specimens is of the order of 2.0 [2].

The crack extension may also be written in terms of the change in ligament size, i.e., $\Delta a = b_o - b$, where b_o is the initial ligament size (b_o is equal to W minus the initial crack size, a_o). Thus, Eq. (2) gives the following expression for the current ligament size, b:

$$\frac{b}{W} = \frac{b_o}{W} - l_o \left(\frac{v_{pl}}{W} \right)^{l_1} . \tag{3}$$

Substitution of Eqs. (2) and (3) into Eq (1) yield the following expression for the CFE in terms of only the plastic displacement:

$$P = DBCW \left(\frac{b_o}{W} - l_o \left(\frac{v_{pl}}{W} \right)^{l_1} \right)^m \left(\frac{v_{pl}}{W} \right)^{1/n} . \tag{4}$$

Equation (4) is the relation for the force *vs.* plastic displacement curve of the DZL model when there is stable crack growth, in which D is the product of the parameters σ^* and Ω^*. The format of

Eq. (4) clearly suggests the existence of a maximum value for P in terms of plastic displacement. Following the guidelines of ASTM E1820 [1], the fracture toughness test data will often be given as load *vs.* total displacement. Also, as a complement to the P-v data, the stable crack extension values are presented as a function of total displacement. The alternative look at the J-R curve construction proposed by DZL includes the crack growth law, Eq. (2), and the relation between force and plastic displacement for a crack growth situation, Eq. (4). Equations (2) and (4) relate crack extension Δa, and force P, respectively, to the plastic component of the displacement; the elastic displacement, on the other hand, may be obtained with the use of the Concise Format [4].

The value of the maximum force, that is, the condition for instability when there is stable crack extension, is obtained by differentiating Eq. (4) with respect to plastic displacement, then solving for $dP/dv_{pl} = 0$. With Eqs. (2) and (3), the solution for the normalized critical crack size, a_c/W, is

$$\frac{a_c}{W} = \frac{1 + mnl_1\left(a_o / W \right)}{1 + mnl_1} \ . \tag{5}$$

Alternatively, the solution for the critical ligament size, b_c/W, is

$$\frac{b_c}{W} = \frac{b_o}{W}\left[\frac{mnl_1}{1 + mnl_1} \right] . \tag{6}$$

Equations (5) and (6) suggest a relevant truth: when in a fracture test of a ductile material the specimen undergoes stable crack extension, at the point the force reaches its maximum value, the crack size has increased from the initial value a_o to a_c, given by Eq. (5). Conversely, the ratio of the ligaments, b_c / b_o (*i.e.*, critical to initial ligament size), is given by Eq. (6); that is, by the expression $mnl_1/(1+mnl_1)$. For a 1-T C(T) specimen of a material that has ideally $n = 6.0$, and $l_1 = 2.0$, the ratio $b_c/b_o = 0.964$, which means that, when $P = P_{max}$, the current ligament size is 3.6% smaller than the initial ligament size. If $a_o/W = 0.6$, and $W = 50$ mm, then the value of the stable crack extension at maximum force, $\Delta a_c = b_o - b_c$, would be of the order of 0.72 mm.

The assessment of instability, on the other hand, requires evaluating both the maximum force and the critical plastic displacement. Substitution of Eq. (6) into Eq. (3) gives, for the critical plastic displacement:

$$\left(\frac{v_{pl}}{W} \right)_c = \left(\frac{b_o / W}{l_o (1 + mnl_1)} \right)^{1/l_1} . \tag{7}$$

The value of the maximum force, P_{max}, is then obtained by substitution of Eq. (7) into Eq. (4):

$$P_{\max} = DBCW \left(\frac{b_o}{W} \right)^{(m+1/nl_1)} \left[\frac{mnl_1}{1+mnl_1} \right]^m \left[\frac{1}{l_o} \left[\frac{1}{1+mnl_1} \right] \right]^{1/nl_1} . \tag{8}$$

Equation (8), a solution for the maximum force obtained from instability considerations when there is stable crack growth, depends on the material properties (D, n); on the specimen geometry (C, m); on the constraint (Ω^*, included in the term D), on specimen size (B, W), on the initial ligament size (b_o), and on the crack growth law parameters (l_o, l_1). Eq. (8) may either serve to predict the value of P_{max}, or better yet, to obtain the value of the parameter l_o, with l_1 known, as explained in earlier works [2]. This latter option will be presented here.

The intercept method

In order to present both crack size evaluation methods, the force-displacement data from a ASTM A 533B 1-T C(T) specimen tested at room temperature (CT-10) will be used [7], which has been characterized by D = 335 MPa and n = 5.38 [6]. Fig. 2(a) (left side) shows the experimental P-v curve for the C(T) specimen for which there is stable crack extension. Three "*a constant*" curves are shown: the uppermost curve corresponds to $a = a_o$ (30.22 mm); the lower one, to $a = a_f$ (34.17 mm), and the intermediate curve ($a = a_1$) corresponds to the crack size at one of the experimental points, at a total displacement v of 1.9 mm.

In Fig. 2(b), the experimental, the $C\&C(a_o)$, and the $C\&C(a_f)$ curves have been drawn, together with six "*a constant*" curves. These six "*a constant*" type curves are characterized by the fact that their crack sizes are equally spaced between a_o and a_f. Thus, for example, the curve denoted $C\&C(a_3)$, has a crack size a_3 = 32.2 mm, and crosses the experimental curve at $v \approx 2.4$ mm. The intercepts of these six curves with the experimental curve have been evaluated, and will be shown presently in a plot of crack size *vs.* total displacement.

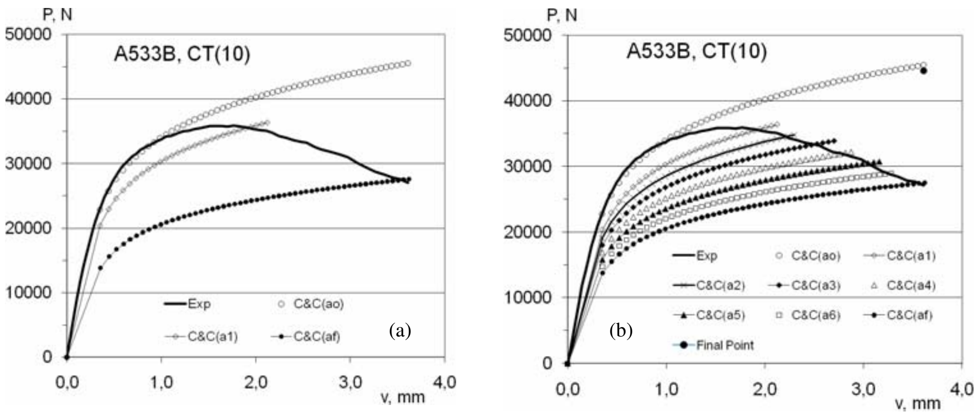

Figure 2.- P-v plots of the A 533B 1-T C(T) specimen with $a_o/W = 0.6$. On the left, the experimental curve with three *a constant* curves; on the right, six intercept *a constant* curves.

The corrected force method

Figure 3 shows normalized P-v plots for the A 533B specimen. The abscissa in Figs. 3(a)/(b) shows the normalized displacement, v/W, whereas the ordinate shows the normalized force, P_n, obtained by dividing P by the geometry function of the CFE, $G(b)$. The geometry function G corresponds to that part of Eq. (1) in between the constraint term Ω^* and the deformation function, and is defined as:

$$G = B \cdot C \cdot W \cdot (b/W)^m . \tag{9}$$

The experimental P-v data, for which the force has been normalized by the function designated as "$G(b_o)$", with $G(b_o) = B \cdot C \cdot W \cdot (b_o/W)^m$, displays the corresponding maximum in normalized force, shown by the solid arrow in Fig. 3(a). In this case, b_o is the initial ligament size. On the other hand, if the force is normalized by the geometry function "$G(b_j)$", where b_j is the actual ligament size that changes from b_o to $b_f = W - a_f$, the resulting curve continuously rises as shown by the solid line.

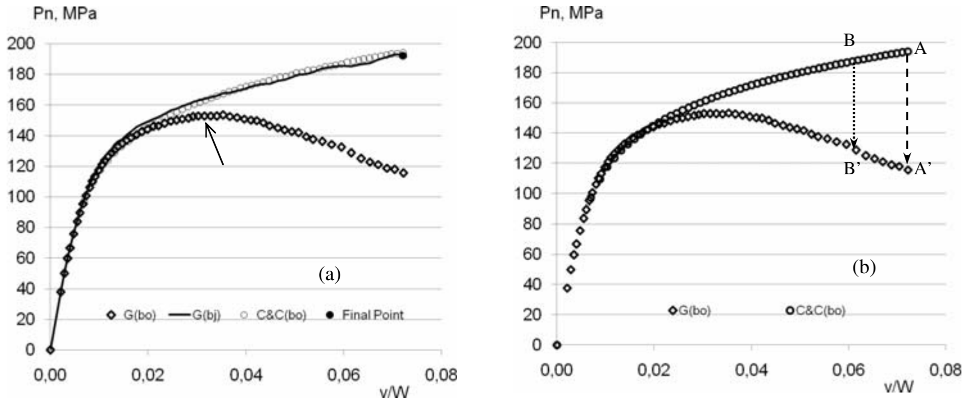

Figure 3.- Normalized *P-v* data to show the corrected force method.

The third curve shown on Fig. 3(a), labelled "*C&C*(b_o)", is the same *C&C*(a_o) curve of Fig. 2, constructed with the *C&C* Formats for the initial ligament size, and normalized with the function $G(b_o)$. The graphic evidence shows that the curve obtained by normalizing the experimental force data by $G(b_j)$, is absolutely coincident with the *C&C*(b_o) curve, normalized with $G(b_o)$. Fig. 3(a) also includes the "final point", which is coincident with the last point of the *C&C*(b_o) curve. This point is the same point with coordinates (v_f, P_f) and final crack size a_f in Fig. 1, but it has been corrected by the *force ratio* of the $G(b_o)$ and the $G(b_j)$ curves, at the same displacement v_f, as will be shown.

This fact is used in Fig. 3(b), with only the normalized experimental curve, $G(b_o)$, and the normalized *C&C* curve for the initial ligament size, *C&C*(b_o), instead of the $G(b_j)$ curve. This relation is shown in Fig. 3(b) by the segmented arrow pointing from the *C&C*(b_o) curve towards the experimental one (A-A'), for the final point. For any point B with displacement $v_c < v_j < v_f$ (see Fig. 1), and ligament $b_o > b_j > b_f$, a counterpart B' exists such that the experimental value of *P*, P_{xp}, normalized by $G(b_j)$, should be equal to the CFE value of the force, $P_{C\&C}$, normalized by $G(b_o)$, *i.e.*:

$$P_{C\&C}/(b_o/W)^m = P_{xp}/(b_j/W)^m. \tag{10}$$

The solution for the unknown value of the ligament, b_j, at point B' is, thus:

$$b_j = b_o(P_{xp}/P_{C\&C})^{1/m}. \tag{11}$$

The overall results of calculating the "unknown" crack sizes with the *intercept* and the *corrected force* methods are shown in Fig. 4(a), in the format of crack size *vs*. total displacement, and in Fig. 4(b), with the variables of the crack growth law only, Eq. (2), i.e., $\Delta a/W$ and v_{pl}/W. The *a-v* plot of Fig. 4(a) shows the original experimental points (continuous line), the results obtained with the intercept method (o), shown on Fig. 2, and those obtained by the corrected force method (▲), for random displacements coincident with those of the experimental data.

The arrow on Fig. 4(a) marks the experimental point at which $P = P_{max}$. The results obtained with both methods, being aware of the fact that the crack sizes are known point-by-point along the experimental *P-v* curve, are certainly good and encouraging. Earlier results obtained for this specimen used a *calibration method* with the final point, and $l_1 = 2.0$, giving for the coefficient of the crack growth law $l_o = 22.36$ [2]. Table 1 shows a comparison of the parameters of the crack growth law for our four modes of calculation, including those obtained from maximum force.

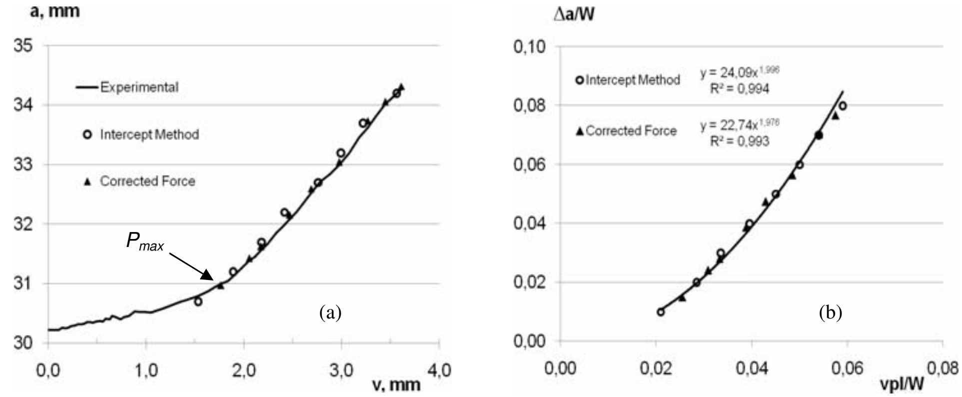

Figure 4.- Results of the intercept and corrected force methods, shown against experimental data (left). On the right, the crack growth law plot for the intercept and corrected force methods.

Method	l_o	l_1
Intercept	24.09	1.996
Corrected force	22.74	1.976
Maximum force, Eq. (8)	23.41	2.0
Calibration (final point)	22.36	2.0
Mean values	*23.15*	*1.993*

Table 1.- Crack growth law parameters for various methods.

In Table 1, the third method refers to solving for l_o from Eq. (8), which shows P_{max} as a function of several parameters. As explained earlier, Equation (8) is the solution for maximum force obtained from instability considerations when there is stable crack growth. As such, it depends on material properties and constraint, specimen geometry, specimen size, initial ligament size, and on the crack growth law parameters l_o, l_1. Solving for l_o requires knowledge of all these parameters [2, 5], and this includes l_1. Thus, the maximum force solution shown in Table 1 implies using $l_1 = 2.0$.

The last line of Table 1 has been included in order to show the similarity between the results for the crack growth law parameters obtained by the various methods. Without resorting to a statistical analysis, one might venture that the "best" solution is given by the maximum force relation, Eq. (8). This would be quite convenient were it not for the fact that l_1 is not always close to 2.0 [5]. In fact, for some C(T) specimens, the exponent l_1 has been found to be of the order of 1.5, and in cases even closer to unity [7], which may rule out this method as a "universal" method.

Figure 5 shows a comparison between the corrected force, the intercept, and the maximum force methods. Based upon the crack growth law of Eq. (2), and the CFE, Eq. (4), the *P*-*v* data have been re-calculated and plotted against the experimental points. The crack growth law parameters of Table 1 have been used to calculate the current value of the crack size along the *P*-*v* curve, as explained elsewhere [2, 5]. It seems apparent that the three methods yield most encouraging results, taking into account the fact that the model produces a smooth curve, as compared to the experimental set of data. If $l_1 \neq 2.0$, the intercept method should be a first choice to evaluate crack extension in a test for which it is difficult or impossible to have concurrent measurements of crack size available.

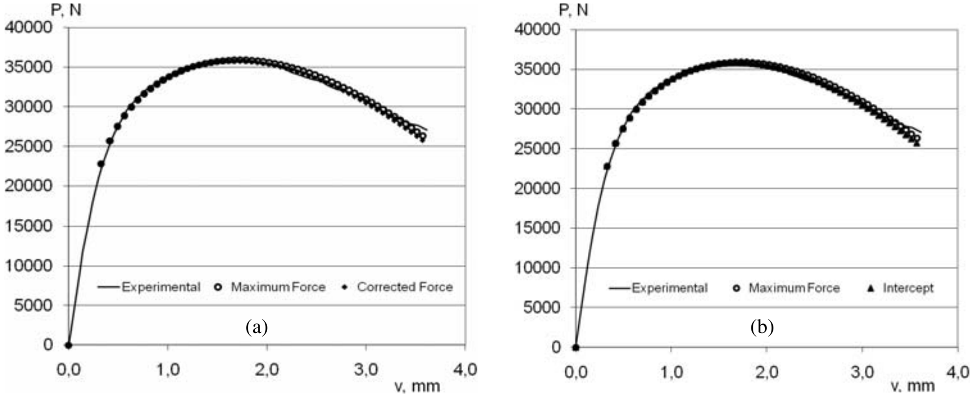

Figure 5.- DZL model curves constructed with the crack growth law parameters obtained from *maximum force* and *corrected force* (left) and *maximum force* and *intercept* methods (right).

Concluding Remarks

Two ways of evaluating the crack size in a test for which there are no measurements available, have been introduced: the *intercept* and the *corrected force* methods. The crack growth law parameters of a C(T) specimen of known crack extension values were evaluated with these methods, comparing well with those given by *calibration* with the final experimental point, and with the solution given by the *maximum force*, obtained from an instability analysis. The *intercept* method appears to be the least dependent on the progress of crack extension, as opposed to the *maximum force* solution, which requires $l_1 = 2.0$, a value which might well be a material-dependent variable.

Acknowledgments

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