Identifying Cracks in Gears Using Wavelet Analysis

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Abstract. A crack in the tooth root, which is the least desirable damage caused to gear units, often leads to failure of gear unit operation. Gear units with real damages or faults, which had been formed with the aid of numerical simulations of real operating conditions, were used for fault analyses presented in this article. A laboratory test plant was used. It is possible to identify damages by monitoring vibration. The influences that a crack in a single-stage gear unit has on produced vibrations are presented. A fatigue crack in the tooth root causes important changes in tooth stiffness whereas, in relation to other faults, changes of other dynamic parameters are more expressed. Time signals were obtained by experiments and different analysis methods were applied. Amplitudes of time signal vibration are, by frequency analysis, presented above all as a function of frequencies in a spectrum using hybrid procedure to determine the level of non-stationarity of operating conditions primarily of a rotational frequency. Also in relation to a non-stationary signal, signal analysis was performed, using the family of Time Frequency Analysis tools, including Wavelets and Joint Time Frequency Analyses. Typical spectrogram and scalogram patterns, which result from reactions to faults or damages, indicate the presence of faults or damages in a very reliable way.

Introduction

The aim of maintenance is to keep a technical system (gear-unit) in the most suitable working condition, whereas its purpose is to discover, to diagnose, to foresee, to prevent and to eliminate damages. When speaking of modern maintenance, however, its purpose is not only to eliminate failures but also to identify the stage before a sudden failure of system operation. The objective of diagnostics is to determine the current condition of the system and, in addition to that, the location, shape and reason of damage formation. To define incorrect operation, possible damages, their location and to eliminate damages, the following diagnostic values are used: different signals, condition parameters and other indirect signs. It is possible to identify the form of damage on the basis of deviations from the undamaged gear system.

Time-Frequency Analysis of Vibrations

A gear unit is a complex dynamic model, comprising elements that enable transmission of rotating movement. Its movement, however, is usually periodical, whereby faults and damages represent a disturbing quantity or impulse. Local and time changes in vibration signals indicate disturbances. As a result it is possible to expect time-frequency changes [2]. This idea is based on kinematics and operating characteristics.

In signals, individual frequency components often appear only occasionally. Classical frequency analysis of such signals does not suffice to establish the time when certain frequencies appear in the spectrum. Time-frequency analysis, on the other hand, makes it possible to describe in what way frequency components of non-stationary signals change with time; in addition to that, it makes it possible to define their intensity levels.

Fourier, adaptive and wavelet transforms and Gabor expansion represent various time-frequency algorithms [1]. The basic idea of all linear transforms is to carry out comparison with elementary function determined in advance [4]. On the basis of various elementary functions it is possible to acquire different signal presentations.

Based on frequency modulation, it is possible to obtain different resolutions of frequency axis in concern to windowed Fourier transform. Based on time scaling, it is possible to acquire different resolutions of frequency axis in regard to wavelet transform. Changes in width and height of the wavelet function directly influence time-frequency distribution of the wavelet function. Different resolutions of frequency axis can be acquired by changing the time scale.

Adaptive Method and Vibration Analysis

Many authors had been developing algorithms without interference parts that reduce usability of individual transforms as opposed to Cohen's class, on the other hand, Qian [4] enhanced the adaptive transform of a signal to a large extent.

Adaptive transform of a signal x(t) is expressed as follows:

$$x(t)=\sum_{p} B_p \cdot h_p(t) \tag{1}$$

where analysis coefficients are defined on the basis of the following equations

$$B_p = \langle x, h_p \rangle \tag{2}$$

whereby similarity between the measured signal x(t) and elementary functions hp(t) of transform is expressed.

A time-dependent adaptive spectrum can be defined in the following way

$$P_{ADT}(t,\omega)=\sum_{p} \left|B_p\right|^2 \cdot P_{WV} h_p(t,\omega) \tag{3}$$

The basic issue of linear presentations is to select elementary functions. Relating to a Gabor expansion, a set of elementary functions comprises time-shifted and frequency modulated prototype window function w(t). When speaking of wavelets, scaling and shifting of a mother wavelet ψ(t) enable the acquisition of elementary functions. In these two examples, structures of elementary functions are defined in advance. Concerning adaptive representation, elementary functions are rather demanding.

Adaptive transform is, generally, independent from the choice of elementary functions because it allows arbitrary elementary functions. In principle, elementary functions, used for adaptive representation of a signal with equation (1), are very general. Yet, in practice that is not always the case. In order to emphasize time dependence of a signal, it is desirable that elementary functions are localised in concern to time and frequency. Additionally, it is desirable that they are able to use the presented algorithm in a relatively simple manner. Very favourable features are associated with a Gauss type signal, which is, in relation to adaptive representation, considered a basic choice.

Wavelet Analysis

The continuous wavelet transform of function $x(t) \in L^2(\Re)$ at the time and scale is expressed as follows [4]:

$$W\,x(u,s) = \langle x, \psi_{u,s} \rangle = \int_{-\infty}^{+\infty} x(t) \cdot \frac{1}{\sqrt{s}} \cdot \psi^* \left(\frac{t-u}{s} \right) \cdot dt = x(t) \otimes \overline{\psi}_s(t) \quad (4)$$

In relation to the continuous wavelet transform, the observed function x(t) is multiplied by a group of shifted and scaled wavelet functions. Thus, their time and frequency dissemination changes. Time and frequency dissemination of the continuous wavelet transform changes simultaneously. The continuous wavelet transform is very sensitive to local non-stationarities since locally limited functions (wavelets) are applied in order to analyse the observed function x(t).

Gabor wavelet function represents an approximately analytical wavelet function, obtained on the basis of a frequency modulation of the Gauss window function [4]:

$$\psi_{Gabor}(t,\sigma,\eta) = \frac{1}{\sqrt[4]{\sigma^2 \cdot \pi}} \cdot e^{-\frac{t^2}{2\cdot\sigma^2}} \cdot e^{i\cdot\eta\cdot t} \quad (5)$$

Practical Example

The test plant of the Computer Aided Design Laboratory of the Faculty of Mechanical Engineering, University of Maribor, was used to perform the measurements. The test plant is presented in Fig. 1; a one-stage helical gear unit can be observed at the spot where vibrations were measured.

A single stage gear unit was used. A helical gear unit with straight teeth was integrated into the gear unit. Under constant loads, tests were carried out and vibrations were measured directly with accelometers, fixed on the housings. A carburised spur gear pair of module 4 mm was in each gear unit, the pinion had 19 and the wheel 34 teeth. The presented results are related to a nominal pinion torque of 20 Nm and nominal pinion speed of 1200 rpm (20 Hz), which is a very typical load condition for this type of gear units in industrial applications.

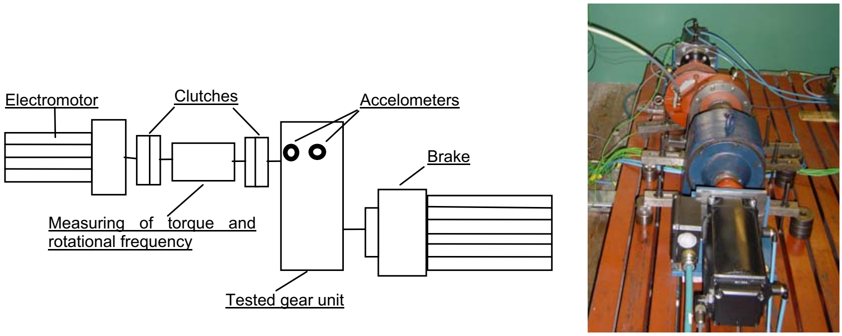

Figure 1: Test plant and a part of measuring equipment

Measurements of a gear unit with a fatigue crack in the tooth root of a pinion were carried out; the operating conditions were such as they are normally associated with this type of a gear unit. A standard gear pair (presented in Fig. 2), with the teeth quality 6, was used but with a crack in the tooth root of a pinion.

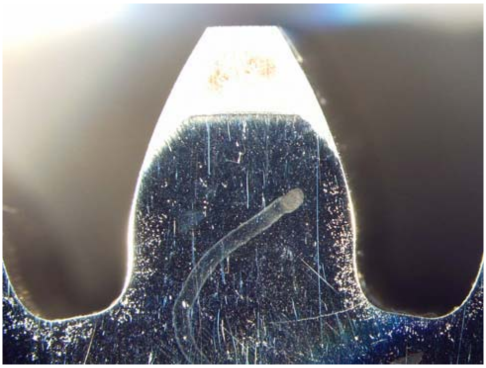

Figure 2: Pinion with a crack in the tooth root

The crack in Fig. 2 on one of the teeth is 4.8 mm long. The whole measurement process and analysis preparations are presented in [5].

The features of elementary functions are limited. Consequently, adaptive spectrogram has a fine adaptive time-frequency resolution. Time-frequency resolution of the transform is adapted to signal characteristics. As an elementary function, Gauss function (impulse) and linear chirp with Gauss window can be used. If a linear change in the rotational frequency of a gear unit results in linear chirps composing a signal, an adaptive spectrogram can be used to find out in what ways a possible frequency modulation is reflected in the time-frequency domain. If non-linear frequency modulation is present, this may be a problem as a spectrogram may include a certain level of distortion; adaptive representation is namely approaching non-linear modulation in the form of a linear combination of chirps with linear frequency modulation. This makes the time needed for transform calculation longer, together with the increased data quantity and the number of cycles necessary to search for an adequate elementary function.

The signal of measured values was 1 s long and composed of, on an average, 12500 measuring points. Spectrograms regarding Gabor and window Fourier transforms are given for comparison, with the length of the window being 700 points, which is 10% more that the length of the period of one rotation of a gear pair. The time required for adaptive spectrogram calculation is at least 15 times longer than for the Fourier transform. But the resolution of the adaptive transform, is, on an average, 1.7 times better. The spectrogram evaluation can be based on an average spectrogram representing an amplitude spectrum of a Fourier transform of a measured signal and on the observation of pulsating frequencies of individual frequency components.

Gabor spectrogram is presented in Fig. 3; no rhythmic pulsation of harmonics is evident. Typical frequencies are the exception, determined on the basis of power spectrum. Some pulsation sources can be observed although they are not very expressed in relation to adaptive spectrogram (Fig. 4), which features a higher level of energy accumulation in the origins. It is of particular interest to monitor the increase or decrease (complete disappearance) in appropriate frequency components with rotational frequency of 20 Hz. This is typical of the 3rd harmonic of mesh frequency. 1530 Hz is expressed only in relation to the presence of a crack. This phenomenon is much more expressed in the adaptive spectrogram (Fig. 6) than in the Gabor spectrogram (Fig 5). In Fig. 6, pulsation (the area, marked with a continuous line) is expressed, reflecting a single engagement of a gear pair with a crack in one rotation of a shaft. Similarly, sources that denote pulsating portions of individual components with the frequency of 20 Hz are indicated between the 6th and the 9th harmonics (area marked with a dashed line).

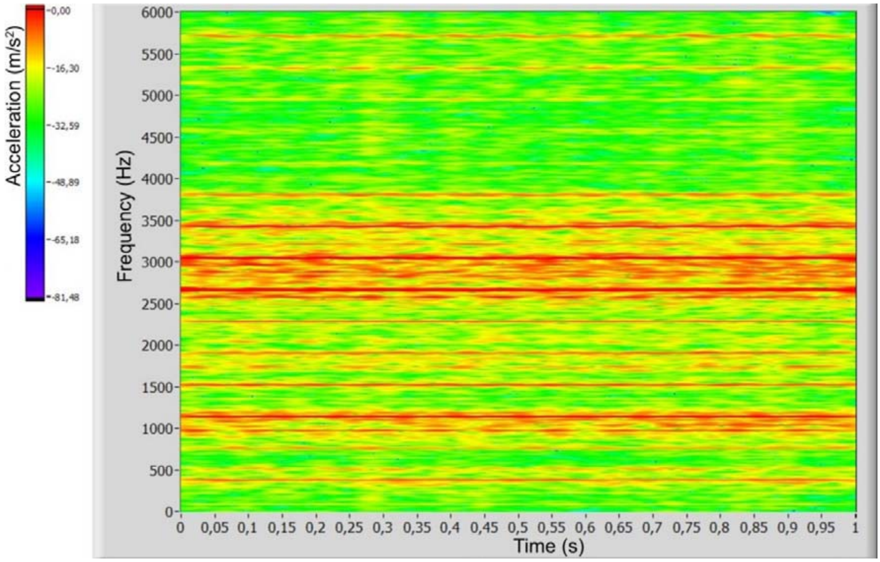

Figure 3: Gabor's spectrogram of a faultless gear unit

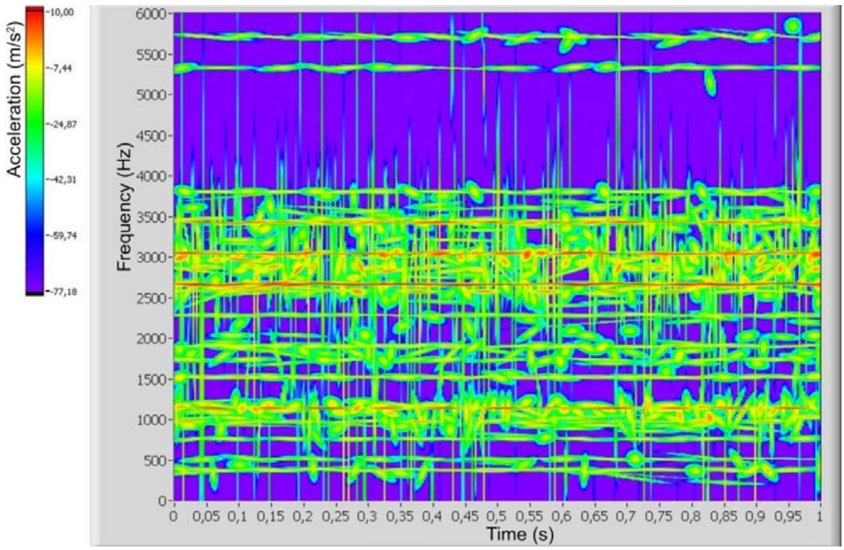

Figure 4: Adaptive spectrogram of a faultless gear unit

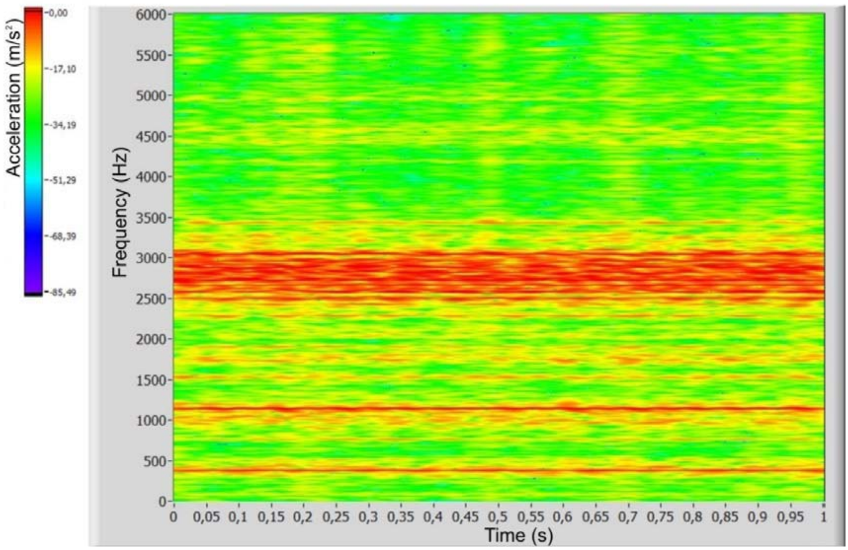

Figure 5: Gabor's spectrogram of a gear unit with a pinion with a crack

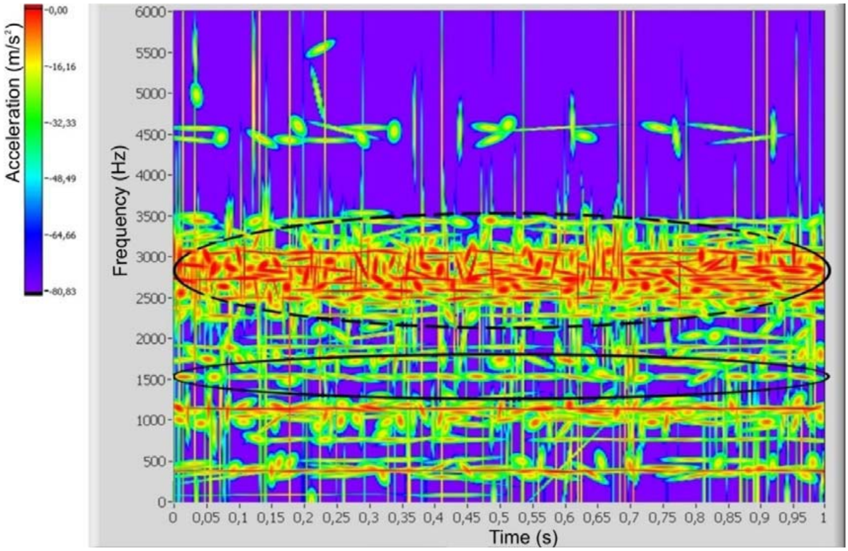

Figure 6: Adaptive spectrogram of a gear unit with a pinion with a crack

A scalogram of analytical wavelet transform with Gabor wavelet function represents normalised and square values of amplitudes of wavelet coefficients. The representation is carried out in a time-frequency domain because the connection between the scale and frequency is established. For technical diagnostics this is favourable because it is much simpler to find adequate characteristics in time-frequency domain (frequency scalogram) than in time-scale domain (scalogram). The transform matches the Parseval characterstic of energy preservation on the basis of normalization. This means that the energy of wavelet transform equals the energy of the original signal in time domain.

Wavelet analysis is suitable primarily for non-stationary phenomena with local changes. Therefore, the analysis was performed for an example of defining the condition relating to the presence of a crack in a tooth root. The purpose was to identify the location of the crack.

For analysis, the analytical continuous wavelet transform, with parameters η = 6 and σ = 1, was used. The highest frequency in the signal (6250 Hz) was acquired using Nyquist frequency and the frequency of sampling the measured time signal. Afterwards the scale type for constant frequency distribution was determined based on the known connection between scale and frequency. The representation of the frequency scalogram is given in the form of wavelet coefficients or their square values. Owing to their representation method, they are more suitable for observation. For analysis, only a part of the signal, representing one whole rotation of the tooth, i.e. of a pinion with a crack, of 50 ms, was applied.

It is clear from the figures in the frequency scalogram that, in relation to the faultless gear, there are no particularities in expressed components that would indicate local changes. This applies for a square representation (Fig. 7) of wavelet coefficients. The matter is quite different in relation to the analysis of the signal produced by a gear with a crack: a local change in wavelet coefficients, in time at the value of 11 ms, is clearly evident in frequency scalograms (Fig. 8). It is possible to observe a local change, i.e. the presence of transients, in relation to the tooth with the crack in its root. If the wavelet length is 50 ms, representing one rotation of the pinion, and if there are 19 teeth along the circumference, the increased amplitude is located at 11 ms. It belongs to the fourth tooth in the direction of rotation from the reference positional point of the gear unit.

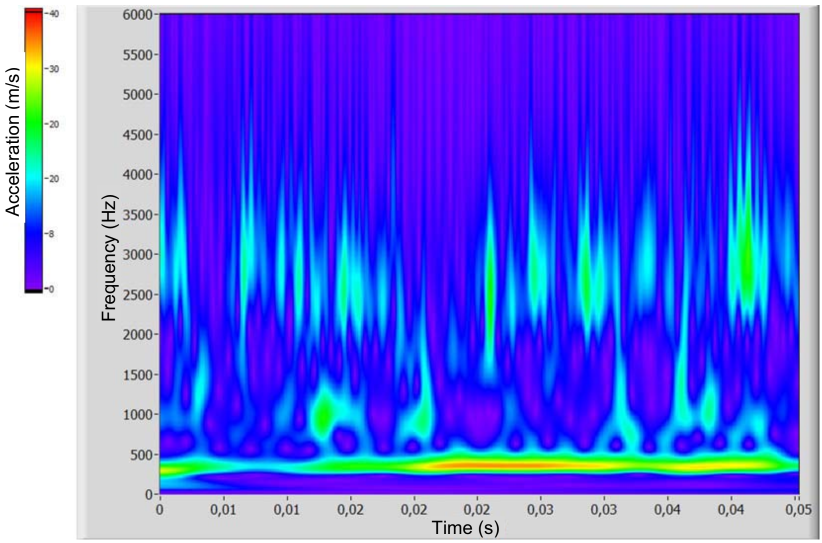

Figure 7: Average twodimensional Gabor frequency scalogram of square wavelet coefficient of the reference gear unit

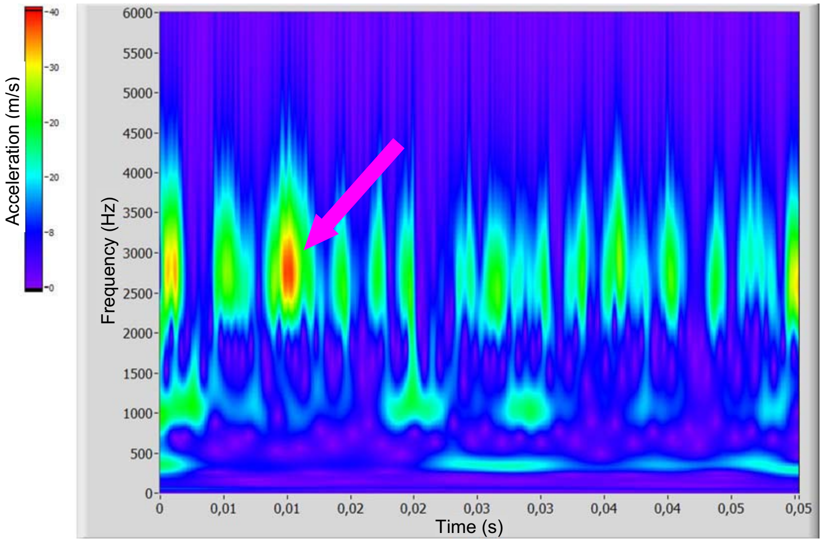

Figure 8: Average twodimensional Gabor frequency scalogram of square wavelet coefficient of the gear unit with a gear with a crack in a tooth root

Conclusions

Vibration analysis, the purpose of which is to detect a fault in industrial gear units, is presented; the described methods can increase the safety of operation and, as a result, the reliability of monitoring operational capabilities can be improved. Life cycle of a gear unit can be monitored more reliably by using appropriate spectrogram samples and a clear presentation of the pulsation of individual frequency components. Along with the average spectrum, they represent a criterion for evaluating the condition of a gear unit. A reliable prediction is possible on the basis of adaptive time-frequency representation. The representation is clearer and without increased dissemination of signal energy into the surroundings. Wavelet transform makes it possible to identify changes in a very short time and to determine the presence of a damage at the level of an individual tooth.

It is possible to monitor the actual condition of a device and its vital component parts in relation to life cycle design. This can, by means of an adequate method or criterion, considerably influence the operational capability. Naturally, the reliability of operation is controlled to a great extent if faults and damages are detected in time. Very reliable fault detection undoubtedly makes a positive contribution to the prediction of the remaining life cycle of a gear unit.

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