Finite Element RCC-MR creep analysis of a circumferentially cracked cylinder under combined residual stress and mechanical load

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Abstract. In this paper, finite element (FE) creep analyses for a circumferential crack in a 316H/316H girth weld of a cylinder at a temperature 550°C are addressed. In the analyses, *J(t)* and *C(t)* are evaluated for both the welding residual stress and combined residual stress and mechanical load. The problems have then been reanalysed using the newly extended *J(t)* and *C(t)* estimates of Ainsworth & Dean (A & D) and compared with the FE results. Good agreement has been found provided that the correct value of elastic follow-up factor is used.

Introduction

Creep crack growth has to be considered in a residual life assessment using R5 [1] for defective components operating at elevated temperature. The rate of creep crack growth, $\dot{a}$, is correlated with an appropriate parameter, either *C(t)*, a function of time, *t*, or C^*, see e.g. [1, 2], as

$$\begin{aligned} \dot{a} &= D_0 C(t)^q && \text{for transient creep} \\ \dot{a} &= D_0 C^{*q} && \text{for steady - state creep} \end{aligned} \tag{1}$$

where D_0 and q are material constants. Both *C(t)* and C^* are creep crack tip parameters measuring the amplitude of the singularity of the creep crack tip fields. The former is appropriate for describing transient creep whereas the latter is applied to steady-state creep. For primary and combined primary and secondary loadings, *C(t)* approaches C^* as *t* increases. At the beginning of creep, the crack tip fields are actually controlled by *J*, an elastic-plastic fracture mechanics parameter. In R5 Volume 4/5 [1], advice on the evaluation of *C(t)* and C^*, based on the reference stress method, is given in Appendix A2 for primary load under elastic-plastic creep conditions and in Appendix A3 for combined loading and elastic-creep behaviour. Recent work [3] by Ainsworth & Dean (will be referred as "A & D" in the rest of this paper) has extended the Appendix A3 method for combined loading to elastic-plastic-creep behaviour. The A & D method [3] has been validated for power-law creep deformation [4] using the elastic-plastic creep finite element (FE) cases analysed by Lei [5] for circumferentially cracked cylinders under combined residual stress and mechanical load and the elastic-creep case analysed by Kim [6] for a centre-cracked plate under displacement controlled loading. In this work, the A & D method is further validated for the RCC-MR creep deformation law [7] using elastic-plastic creep FE analyses.

Background

Evaluation of J and C(t)

The methods for *J* and *C*(*t*) prediction developed by A & D [3] were based on the Ramberg-Osgood stress-strain relationship and power-law creep deformation with a stress exponent *n*. However, previous work shows that these methods may be used for other strain hardening materials and creep laws.

For a general case of combined primary and secondary loading, J can then be expressed as a function of time, t, by [3]

$$J/J_0 = \left[1 + \varepsilon_c\left(\sigma_{ref}^p\right)/\varepsilon_{ref}^0 - (Z-1)\left(\sigma_{ref} - \sigma_{ref}^0\right)/\left(E\varepsilon_{ref}^0\right)\right]\sigma_{ref}/\sigma_{ref}^0 \tag{2}$$

where σ_{ref} and σ_{ref}^p are the total reference stress and the reference stress for primary load only, respectively, σ_{ref}^0 and ε_{ref}^0 are the initial reference stress and strain at $t = 0$, $J = J_0$ at $t = 0$, $Z \geq 1$ is the elastic follow-up factor, $\varepsilon_c\left(\sigma_{ref}^p\right)$ is the accumulated creep strain due to the primary reference stress at time t and E is the Young's modulus. Note that the current reference stress, σ_{ref}, is generally a function of time.

For combined primary and secondary loading and the RCC-MR creep deformation law [7], the following expression for $C(t)$ can be obtained from [3]

$$\frac{C(t)}{\left(C_0^*\right)_m} = \frac{\sigma_{ref}\dot{\varepsilon}_c\left(\sigma_{ref}\right)}{\sigma_{ref}^0\dot{\varepsilon}_{c2}\left(\sigma_{ref}^0\right)}\frac{\left(\varepsilon_{ref}/\varepsilon_{ref}^0\right)^{n+1}}{\left(\varepsilon_{ref}/\varepsilon_{ref}^0\right)^{n+1} - \sigma_{ref}^0/\left(E\varepsilon_{ref}^0\right)} \tag{3}$$

where ε_{ref} is the total (elastic + plastic + creep) reference strain at time t and reference stress $\sigma_{ref}(t)$, $\dot{\varepsilon}_c\left(\sigma_{ref}\right)$ is the creep strain rate corresponding to σ_{ref}, $\dot{\varepsilon}_{c2}\left(\sigma_{ref}^0\right)$ is the secondary creep rate calculated based on the initial reference stress, σ_{ref}^0, and $\left(C_0^*\right)_m$ is a normalisation for $C(t)$, which can be expressed as

$$\left(C_0^*\right)_m = \sigma_{ref}^0\dot{\varepsilon}_{c2}\left(\sigma_{ref}^0\right)R', \; R' = \left(K^p/\sigma_{ref}^p\right)^2 \tag{4}$$

where K^p is the elastic stress intensity factor (SIF) for primary load. Eq. (4) is independent of time and can be used for both combined loading and residual stress only. For RCC-MR creep law, it is proposed in [3] to use the stress exponent of the primary creep law (cn_1) in place of n in Eq. (3) or to estimate n using

$$n = q/(1-q) \tag{5}$$

where q is the exponent in the creep crack propagation law (Eq. (1)).

Relaxation of the reference stress for combined loading

When crack growth is not significant, the basic reference stress rate expression in Section A3.4.3 of R5 Volume 4/5 [1] still applies, that is

$$\dot{\sigma}_{ref} = -\left(\dot{\varepsilon}_c\left(\sigma_{ref}, \varepsilon_c\right) - \dot{\varepsilon}_c\left(\sigma_{ref}^p, \varepsilon_c\right)\right)E/Z \tag{6}$$

where $\dot{\sigma}_{ref}$ is the reference stress rate and $\dot{\varepsilon}_c\left(\sigma_{ref}, \varepsilon_c\right)$ and $\dot{\varepsilon}_c\left(\sigma_{ref}^p, \varepsilon_c\right)$ are the creep strain rates for total and primary reference stresses, respectively, evaluated at the current creep strain, ε_c. The reference stress may then be obtained by solving Eq. (6) for given initial conditions, $\sigma_{ref} = \sigma_{ref}^0$ at $t = 0$.

Determination of the initial reference stress and strain

In this paper, the initial reference stress is calculated directly using the FE method as follows. The reference stress for combined loading may be determined by performing cracked body elastic-plastic FE analysis when the J value for the combined loading, J_{com}, is available. A cracked body FE analysis under pure mechanical load may be performed and J evaluated. Adjusting the applied mechanical load, P, until $J = J_{com}$, the reference stress for combined loading is then obtained from

$$\sigma^0_{ref} = \left(P\left(J_{com}\right)/P_L\right)\sigma_y \tag{7}$$

where $P(J_{com})$ is the load corresponding to $J = J_{com}$ and P_L is the limit load of the cracked body. The initial reference strain, ε^0_{ref}, can then be obtained from the stress strain relationship of the material.

FE analyses

Finite element creep analyses are performed for an external full-circumferential crack in the girth weld of a cylinder under welding residual stress and combined residual stress and mechanical load. The commercial finite element package ABAQUS [8] is used in the analysis.

Geometry and FE models

The weld considered is a girth weld of 316 weld material connecting two iso-thickness cylinders made of 316H. The dimensions of the cylinders are given in Fig. 1, where R_i and R_o are the inner and outer radii, w is the wall thickness and a is the crack length measured from the outside surface of the cylinder. The weld joint can be idealised as an axisymmetric mechanical model. 8-noded, axisymmetric elements (ABAQUS [8] element type CAX8R) are used to model the joint. The FE mesh used in the analyses is shown in Fig. 2. Only half of the joint is modelled because of the symmetry about the plane through the weld centre. A focused mesh arrangement is employed, with 30 element rings around the crack tip (Fig. 2), with the size of the first element ahead the crack tip being equal to 5×10^{-3}mm, in order to capture the singular stress and strain fields ahead of the crack tip. Various crack depths (a/w = 0.1~0.4) have been analysed. However, only the results for a/w = 0.2 are presented in this paper for brevity.

Material properties

To simplify the problem, the whole joint is considered to be a homogenous structure made of 316H parent material operating at a temperature of 550°C. At 550°C, the Young's modulus, E, of the material is 160 GPa and Poisson's ratio, ν, is 0.298 as given in R66 [9]. The true stress-true strain relationship used in the FE analyses is shown in Fig. 3. The 0.2% proof stress, σ_y, determined from the stress-plastic strain curve (Fig. 3) for the given temperature, is 162 MPa. The RCC-MR creep law [7] for 316H steel is considered in the FE analyses and can be expressed as follows for the temperature region of $480^\circ C \le \theta \le 700^\circ C$:

$$\dot{\varepsilon}_c = \begin{cases} 0.01 C_1 C_2 t^{C_2 - 1} \sigma^{cn_1} & \text{for } t \le t_{fp} \\ C\sigma^{cn} & \text{for } t > t_{fp} \end{cases} \tag{8}$$

where $\dot{\varepsilon}_c$ is the creep strain rate in 1/hour, t is the time in hours, σ is the uni-axial stress in MPa and C, C_1, C_2, cn and cn_1 are material constants, which are a function of temperature, θ, and t_{fp} is the time at the end of primary creep, in hours. Under conditions of constant stress, it is determined as follows for the temperature region of $480^\circ C \le \theta \le 700^\circ C$:

$$t_{fp} = \left(100C/\left(C_1 C_2\right)\right)^{1/(C_2-1)} \sigma^{(cn-cn_1)/(C_2-1)} \quad (9)$$

The RCC-MR creep law material constants for 316H stainless steel at 550°C used in the analyses are from R66 [9] and are shown in Table 1. The RCC-MR creep law with the constants given in Table 1 has been implemented into an ABAQUS v6.3 user subroutine, CREEP, and this has been used in the FE analyses.

Loads

The residual stresses considered in this work are the as-welded stresses in a girth weld in the cylinder. The residual stresses are taken from a FE welding simulation [10]; these stresses are mapped onto the uncracked body FE model used in this work, together with the equivalent plastic strains, as initial conditions. The normalised axial and hoop residual stress distributions, σ_{yy}/σ_y and $\sigma_{\theta\theta}/\sigma_y$, respectively, along the crack plane for the uncracked body are plotted in Fig. 4, against the normalised distance from the inside surface of the cylinder.

The mechanical load is applied in the form of an axial membrane stress, σ^a, at the ends of the cylinder. In this work, a constant axial stress, σ^a =28.5 MPa, is applied. The mechanical load level is measured by L_r, the ratio of applied axial membrane stress, σ^a, to limit axial membrane stress, σ_L^a, as follows

$$L_r = \sigma^a / \sigma_L^a \quad (10)$$

where the limit membrane axial stress σ_L^a is estimated according to reference [11].

Calculation of J and C(t)

$C(t)$ is calculated using the in-built contour integration facility in ABAQUS on 30 contours around the crack tip. The $C(t)$ values presented in this paper are those obtained on the 3rd contour. Since $C(t)$ is path-dependent in the transient creep period, the correct value should be calculated at the crack tip. However, the $C(t)$ value obtained on the 3rd contour near the crack tip is a good representation of the crack tip value. When steady-state conditions are achieved, $C(t)$ should become path-independent and equal to C^* for primary load. The ABAQUS in-built J computation does not evaluate J correctly when residual stresses exist [12]. Accordingly, the J-integral is calculated by an alternative ABAQUS post-processing program, developed by Lei [12], on 30 contours around the crack tip. J values presented in this report are the average of all values obtained on the 3rd to 30th contours.

Analyses and results

Initial reference stress estimation

The normalised applied mechanical load, L_r, J-integral for the secondary stress from elastic-plastic FE analyses, J^s, and the J-integral for combined loading from the elastic-plastic FE analyses, J_{com}, are shown in Table 2. The initial reference stress is estimated using the method described above and the results are summarised in Table 2.

Determining the elastic follow-up factor Z

The value of the elastic follow-up factor, Z, is required in solving the relaxation equation, Eq. (6), to obtain the current reference stress, $\sigma_{ref}(t)$. The values of the Z factor used in the calculations of this paper are selected such that the FE J results for long-term creep are well predicted by Eq. (2). The Z values obtained for the cases considered in this paper are also given in Table 2.

Prediction of J(t) and C(t)

$J(t)$ is predicted using Eqs. (2) and (6) and the corresponding Z values listed in Table 2. The predicted $J(t)$ values for the two cases in Table 2 are normalised by J_0 and plotted in Figs. 5(a) for residual stress only and 6(a) for combined residual stress and tension, together with the FE results (labelled "FE"), against the normalised time, $t/(t_{red})_m$, where $(t_{red})_m$ is the redistribution time for an equivalent mechanical load corresponding to $\sigma_{ref}^p = \sigma_{ref}^0$ and is defined, for the RCC-MR creep law (Eq. (8)), following the principle given in [2], as

$$(t_{red})_m = \begin{cases} \left(100/\left(C_1 E\left(\sigma_{ref}^0\right)^{cn_1-1}\right)\right)^{1/C_2} & \text{for } t_{red} \le t_{fp} \\ t_{fp} + \left(\sigma_{ref}^0/E - 0.01 C_1 t_{fp}^{C_2}\left(\sigma_{ref}^0\right)^{cn_1}\right)/\left(C\left(\sigma_{ref}^0\right)^{cn}\right) & \text{for } t_{red} > t_{fp} \end{cases} \tag{11}$$

C(t) is predicted using Eqs. (3) and (6) with n estimated using Eq. (5) with q = 0.891, which is taken from reference [13] for the upper bound creep crack growth rate law of 316H parent, leading to n = 8.17. The normalised $C(t)$ (normalised by $\left(C_0^*\right)_m$, defined by Eq. (4)) values predicted using Eqs. (3) and (6) are plotted in Fig. 5(b) for the case of pure residual stress and in Fig. 6(b) for the case for combined loading in log-log coordinates, against the normalised time. Corresponding FE results ($C(t)$), are also normalised and plotted in Figs. 5(b) and 6(b) (labelled "FE"), for comparison.

Discussion

J prediction

For the case of pure residual stress, see Fig. 5(a), the long-term FE results are accurately predicted by Eq. (2) when the Z value shown in Table 2 is used. This is expected because the Z factor was calibrated using the FE J for long-term creep. However, for short-term creep, the FE results are underestimated. This is probably due to a very strong elastic follow-up at the beginning of creep, corresponding to a high Z value. In the prediction, a Z value for long-term creep was used and, therefore, the short term J was significantly underestimated.

For the case of combined loading, good agreement between predicted and FE J values can be seen in Fig. 6(a), where the Z value shown in Table 2 was used.

The above discussion shows that the Z factor may be a function of time. The Z value at the beginning of creep may be much higher than that for the long-term creep. Currently, the Z factor is assumed to be a constant in the relaxation equation (Eq. (6)). It is possible that a variable Z factor may result in more accurate predictions of J in the early stages of creep.

C(t) prediction

For the case of pure residual stress, Fig. 5(b), the predicted *C(t)* values using Eq. (3) are accurate for long-term creep compared with the FE results. However, for short-term creep ($\left(t/t_{red}\right)_m < 1$), the FE results are underestimated. For combined loading, Fig. 6(b), the predictions for long-term creep are very close to the FE results, but slightly non-conservative.

When using Eq. (3) for a non-power-law creep material, the exponent, n, is unavailable. It is proposed in [3] that the exponent can be estimated using the exponent of the creep crack growth law of the material via Eq. (5) or directly replaced by the stress exponent of the primary creep law. In this paper, three values of exponent, n, are used to predict *C(t)*, one uses Eq. (5), another uses the primary creep law exponent, cn_1, and the third uses the secondary creep law exponent, cn. Figures 7(a) and (b) compare the results for residual stress only and combined loading, respectively. From the figures, a smaller value of n gives higher *C(t)*. However, slight difference can be seen between the results for n = 4.18 and 8.2. Obviously, $C(t)$ predicted using Eq. (3) is insensitive to n for the RCC-MR creep law.

Evaluation of Z factor

The elastic follow-up factor, Z, strongly effects the J prediction using Eq. (2) and $C(t)$ prediction using Eq. (3) due to its influence on the relaxation of the reference stress. Recently, a FE method for determining the Z factor for displacement controlled loading has been proposed [14]. However, there is currently no independent method available for the evaluation of Z factor for cracks in residual stress fields. In this work, Z values are obtained using the J prediction equation (Eq. (2)) via the best match between the FE results for long-term creep and predicted values. However, in engineering practice, J is generally unavailable. Future work should consider developing independent methods for evaluating the Z factor for residual stress problems and combined loading.

Summary

In this paper, FE creep analyses have been performed for a circumferential crack in a 316H/316H girth weld of a cylinder at a temperature 550°C. Both the welding residual stress and mechanical load have been considered. A real material stress-strain curve and the RCC-MR creep law for a temperature of 550°C have been used in the analyses. The problems have then been reanalysed using the newly extended $J(t)$ and $C(t)$ estimates of Ainsworth & Dean [3] and compared with the FE results. Conclusions drawn from this investigation are as follows.

1. The $J(t)$ estimate proposed in [3] is accurate for long-term creep for both pure residual stress and combined residual stress and mechanical load when the value of the elastic follow up factor, Z, for long-term creep is used. However, this approach under-estimates $J(t)$ for short-term creep. This may be due to variations in the elastic follow-up factor, Z, during creep.
2. The $C(t)$ estimate proposed in [3] is accurate for long-term creep for both pure residual stress and combined loading when the long-term creep Z value is used. However, the $C(t)$ predictions become non-conservative for short-term creep ($(t/t_{red})_m < 1$).

Acknowledgment

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Table 1 RCC-MR creep law material constants of 316H at 550°C [9]

C	C_1	C_2	cn	cn_1	Note
5.29×10^{-26}	2.9618×10^{-12}	0.42131	8.2	4.18	

Table 2 The initial reference stresses and the Z values

Case*	L_r	J^s (N/mm)	J_{com} (N/mm)	σ^0_{ref}/σ_y	Z
a/w = 0.2, RS only	0	9.70	9.70	0.953	1.7
a/w = 0.2, RS + AT	0.2	9.70	12.54	0.972	2.3

* RS – Residual stress; AT – Axial tension.

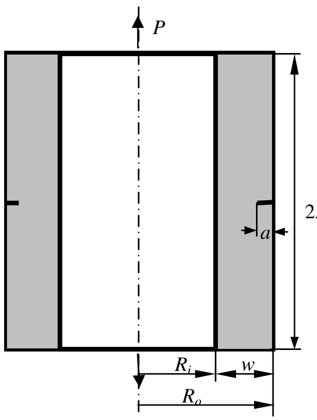


Fig. 1 Geometry and dimensions of the externally circumferentially cracked cylinder (R_i = 235.75 mm, w= 34.9 mm, $L \approx$ 250 mm, a/w = 0.2)

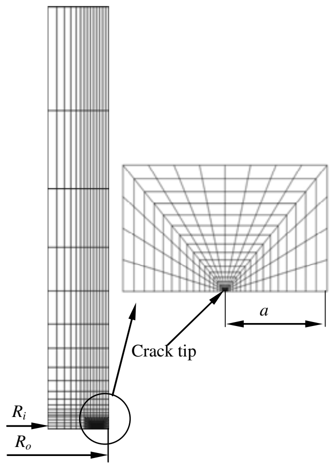
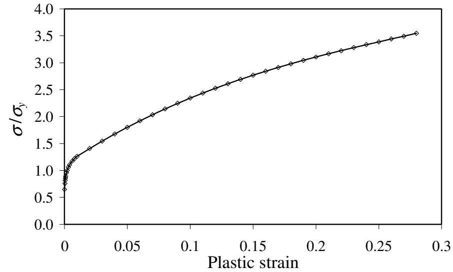


Fig. 2 Typical FE mesh used in the majority of analyses (a/w = 0.2)



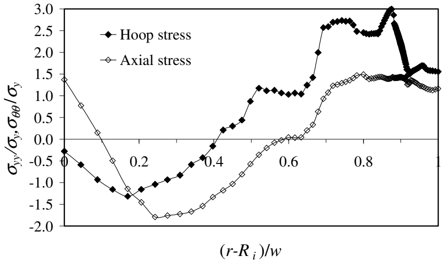


Fig. 3 Material true stress-true plastic strain curve

Fig. 4 Uncracked body residual stress distributions through the thickness of the cylinders on the crack plane at $t = 0$ (r is the radius variable)

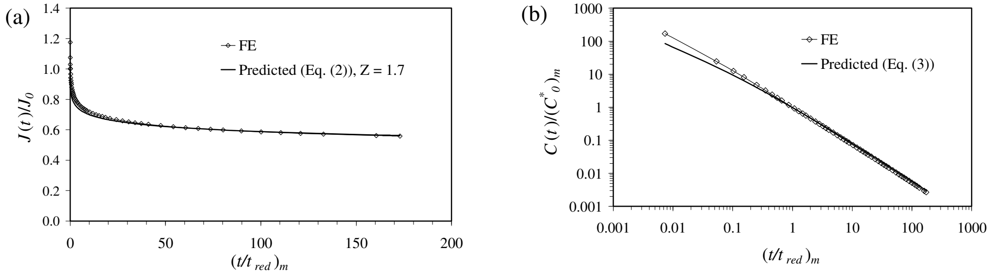


Fig. 5 Comparison of J and $C(t)$ between predicted values and the FE results ($a/w = 0.2$, residual stress only) (a) $J(t)$ and (b) $C(t)$

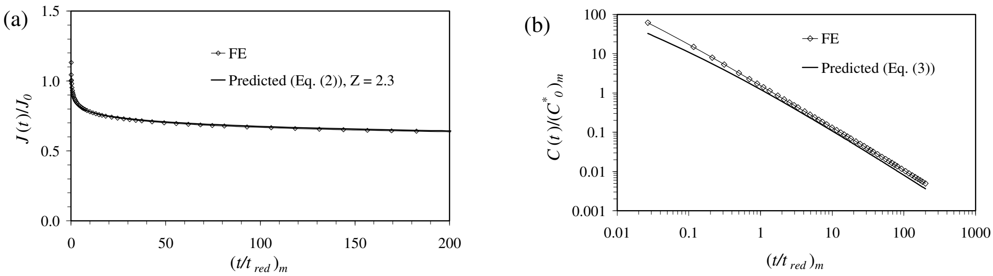


Fig. 6 Comparison of J and $C(t)$ between predicted values and the FE results ($a/w = 0.2$, combined residual stress and axial tension) (a) $J(t)$ and (b) $C(t)$

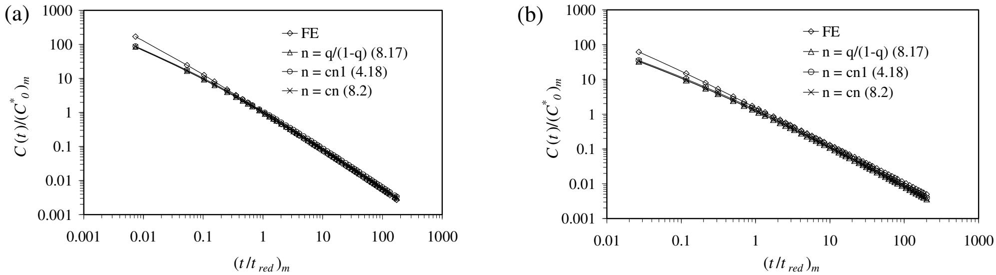


Fig 7 Comparison of $C(t)$ between the FE results and values predicted using various n values for $a/w = 0.2$, (a) residual stress only and (b) combined residual stress and tension