Fatigue Life Assessment of Structural Component with a Non-uniform Stress Distribution Based on a Probabilistic Approach

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Abstract. A probabilistic approach is developed to take into account the stress gradient effect on fatigue life for structural metallic component. This approach is based on the weakest link concept that determines, for a given lifetime, the probability distribution of the fatigue strength. The new concept determines the probability that the fatigue life N will be smaller than a specific fatigue life N_i. This new concept allows calculating fatigue life and its scatter for any arbitrary loading level. A Weibull type probability distribution is used in the proposal, its parameters become functions of local equivalent stresses whatever the stress level. The proposed approach was applied to calculate the number of cycles to crack initiation for a failure probability of 5%, 63% and 95%. The calculated lifetimes were compared with lifetimes obtained from experiments carried out on notched cruciform and round specimens, made of two different steels, under constant amplitude loading.

Introduction

Effect of alternating stresses in structural elements is commonly combined with effect of heterogeneous stress distribution. These two effects lead to complex mechanisms of fatigue failure. Two of them should be clearly distinguished. The first mechanism concerns the case when a particular fatigue crack, responsible for failure, reaches its critical length under the influence of heterogeneous stress field. The methods that take into account this mechanism are based on averaged stresses over a material domain. The considered domain could be volume [1], plane [2, 3] or their simplification such as line domain [4] or point domain [5]. The second mechanism concerns the case when the fatigue failure could start at any elementary domain in element but the probability of such event depends on the local stress history. Thus, the fatigue failure of the whole element is a function of cyclic volumetric stress distribution and dimensions of structural element. This mechanism is usually taken into account by probabilistic methods based on the weakest link concept.

This paper concerns the second mechanism of fatigue failure, for which the authors presents a probabilistic method of fatigue life assessment related to structural elements based on the weakest link concept [6, 7]. Contrarily to the conventional determination of the fatigue strength probability distribution for a given fatigue life (a number of cycles to failure), the proposed method hereafter emphasizes in the determination of the probability that the fatigue life N is less than a specific life N_i. Such approach allows determining fatigue life for any probability level. The calculations are based on the Weibull probability distribution which parameters become functions of local equivalent stresses.

A Weibull based mathematical model

This paragraph presents the weakest link concept and its application for formulating a two-dimensional distribution of fatigue failure probability allowing calculates the fatigue life for any arbitrary probabilities. Foundations of the weakest link concept, being the base of the Weibull theory, were formulated in the twenties of the 20th century. Principal assumptions of the weakest link concept are as follows: (i) the element includes statistically distributed defects; (ii) failure is going to occur in a certain elementary area (link) of the element that contains the „most harmful defect" (in fatigue, and according to the authors failure is seen as crack initiation); (iii) the probabilities of failure in each link are independent.

From experiments it appears that for identical elements (at the macroscopic scale) loaded by time dependent forces $F(t)$, the logarithm of the number of cycles N to crack initiation is a random variable with a given failure probability density distribution p_f. In the successive elements (specimens) "the most harmful defect" exhibits different features, and thus the crack initiation occurs under different number of cycles N_i. In the case of heterogeneous stress field, the given element is divided into sub domains. The probability that a crack will not occurs in the interval [0, N_i] means that crack initiation will not occur in any elementary sub domain (weakest link concept). Indicating that $P_s^{(i)}$ is the survival probability means that the sub domain (i) will not initiate a crack in the number of cycles interval [0, N_i] then the survival probability P_s for the whole element is the product of all the individual probabilities $P_s^{(i)}$

$$P_s = \prod_{i=1}^{i=k} P_s^{(i)} \,, \tag{1}$$

where k is the number of sub domains (links). Assuming exponential form of the survival probability $P_s^{(i)} = e^{-f(\sigma)}$ leads to the substitution of product Π in Eq. (1) by continuous summation (integration) $P_s = P_s^{(i)} \cdot P_s^{(i+1)} ... = e^{-f(\sigma^{(i)})} \cdot e^{-f(\sigma^{(i+1)})} ... = e^{-f(\sigma^{(i)}) - f(\sigma^{(i+1)}) ...}$. Weibull [6] proposed such a form of survival probability distribution in 1939. The usual (Weibull) form of failure probability P_f=1-P_s is as follows

$$P_f = 1 - e^{-\frac{1}{\Omega_0}\int_\omega g(\sigma) d\omega}, \text{ where } g(\sigma) = \left(\frac{\sigma}{\sigma_u}\right)^m \text{ or } g(\sigma) = \left(\frac{\sigma - \sigma_0}{\sigma_u}\right)^m, \tag{2a, 2b, 2c}$$

where Ω_0 is the volume or surface reference domain; $g(\sigma)$ is a function called by Weibull 'risk of rupture' which form depends on material properties. Weibull proposed function $g(\sigma)$ with two (Eq. 2b) or three (Eq. 2c) parameters, where σ_0, σ_u, m are parameters of stress shift, stress scale and shape, respectively. Owing to different material properties on surface and in volume due to, e.g. manufacturing process, Weibull considered individual failure probability for volume V ($\omega = V$ in Eq. 2a) and surface A ($\omega = A$ in Eq. 2a). In case of fatigue processes in uniaxial loaded element with homogeneous stress distribution σ_a the failure probability is two dimensional function of stress σ_a and fatigue life N, P_f=$F(\sigma_a, N)$. In other words, fatigue life scatter obtained under the same stress amplitude σ_a or the same fatigue life may be achieved under different stresses σ_a. Such two dimensional function was considered by Weibull [7], however the mathematical expression was not proposed. When the failure of the element is assigned to a specific fatigue life N_i then the failure probability P_f is reduced to stress function only. This concept is very popular to determine the fatigue limit [8-12] of element under heterogeneous stress distribution. In such case, the specific fatigue life N_i is assigned to lifetime (usually around 10^6-$5 \cdot 10^6$ cycles) which defines the beginning of "infinite" fatigue regime. Thus, the failure probability is defined as $P_f(N<N_i)$, where N_i is the fatigue life of the considered specimen.

The present paper is focused on a probability model able to asses the fatigue life (to crack initiation) of structural element under heterogeneous stress distribution for a requested probability level. The general form of two-dimensional probability distribution is analogous to the Weibull expression (Eq. 2a). However, the former stress function of 'risk of rupture' $g(\sigma)$ becomes also the function of lifetime N and the failure probability takes the general form as follows

$$P_f = 1 - e^{-\frac{1}{\Omega_0}\int_\omega h(N,\sigma)d\omega}. \tag{3}$$

The failure probability P_f (before a given lifetime N_i) increases with increasing the stress level σ but P_f (for a given σ) also increases with the number of cycles N. The longer structural element is in service the failure probability is higher.

Some researchers [13] lean towards a view that the Weibull distribution describes the scatter of fatigue life (under a given stress amplitude σ_a) in logarithmic scale fairly well. It could be expresses by the following equation

$$P_f = 1 - e^{-\left(\frac{\log(N)}{\mu}\right)^m}, \tag{4}$$

where μ is the lifetime scale parameter and m is the shape parameter. Because the magnitude of fatigue life scatter depends on the stress level, σ_a then the distribution parameters μ and m should be a function of stress amplitude. The correct conditioned (on stress level) scale parameter μ must enable to perform comparison of fatigue life scatters obtained under different stress amplitudes σ_a. In this connection, the scale parameter μ takes the form $\mu = \log(N_f)$, where N_f is the characteristic fatigue life (reference) for a given stress σ_a. The value of N_f is determined from the reference fatigue curve, e.g. σ_a-N_f. For a constant scale parameter μ=log(N_f) the parameter m responds for the shape of the distribution that is for fatigue life scatter. Therefore, the shape parameter m is a factor reflecting first, manufacture quality of the element. Secondly, the shape parameter must reflect the relation between the scatter band of fatigue lives and stress levels. Under loading equal to the ultimate tensile strength, the fatigue life does not exhibit any scatter compared with the fatigue scatter (in fatigue sense, $N_f \rightarrow 1$ of loading cycle). On the other hand, under loading in the fatigue limit regime some specimen fails, and some others have unlimited fatigue life. It means that scatter of fatigue lives depends on stress amplitude which could be related to log(N_f). This relation is modeled by a proposed simple function (Eq. 5) –where p is a constant parameter. The mathematical relationship between σ_a and N ($\sigma_a = \sigma_a(N)$) is known by the empirical reference SN curve (Basquin equation for instance)

$$m(\sigma_a) = m(N_f) = \frac{p}{\log(N_f)}. \tag{5}$$

In other words, the coefficient p can be seen as a quality factor of both element manufacturing process and material (internal defect for instance). Finally, the failure probability distribution of the element, before N under the stress amplitude σ_a, takes the form:

$$P_f = 1 - e^{-\frac{1}{\Omega_0}\int_\omega \left(\frac{\log(N)}{\log(N_f)}\right)^{\frac{p}{\log(N_f)}} d\omega}, \text{ where } \omega = V \text{ or } \omega = A. \tag{6}$$

In the case of uniform stress distribution, Eq. (6) reduces to Eq. (7)

$$P_f = 1 - e^{-\left(\frac{\log(N)}{\log(N_f)}\right)^{\frac{p}{\log(N_f)}}} . \quad (7)$$

For instance, Fig. 1a shows a two-dimensional distribution of failure probability obtained from Eq. (7), using the fatigue reference Wöhler curve (σ_a-N_f) of 18G2A steel (Tab. 1) with p = 580.

Fig. 1b illustrates experimental fatigue tests data used to identify the reference curve σ_a-N_f along with scatter band obtained for P_f=0.05 and P_f=0.95. Experimental tests were stopped when the number of cycles reached $5\cdot10^6$ with specimens without any fatigue crack, which corresponds to $\sigma_a=\sigma_a^*$=175.4 MPa. The same assumption was done in the determination of the probability distribution P_f, thus $P_f(\sigma_a<\sigma_a^*) = 0$ (Fig. 1a).

(a) (b)

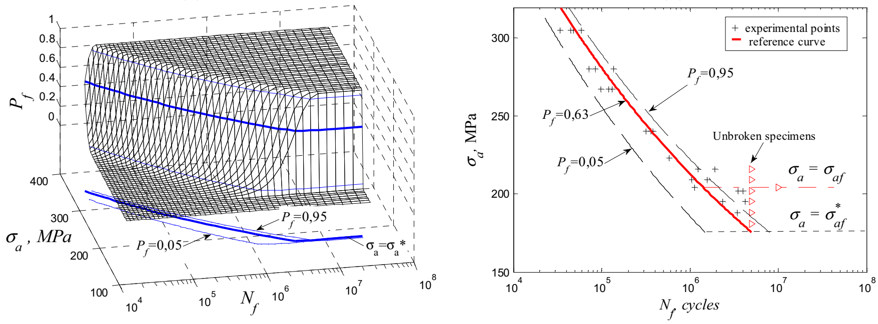

Fig. 1. (a) Simulated two-dimensional distribution of failure probability P_f for the element made of 18G2A steel (p=580); (b) Fatigue reference curve σ_a-N_f with experimental points and fatigue scatter bands for P_f=0.05 and P_f=0.95 (p=580)

Crossing the two-dimensional distribution $P_f(\sigma_a, N_f)$ by a horizontal plane P_f=const., the fatigue S-N curve σ_a-N_f for P_f=const. probability is obtained. An important point is to realize that the conventional reference curve σ_a=N_f obtained by fitting experimental data with the least square technique tests corresponds to $P_f \approx 0.63$ if the probability distribution expressed by Eq. (7) is assumed.

Implementation of the two-dimensional probability distribution for fatigue life calculations

Let us assume that the cracks occurring on the free surface of the element are responsible for failure ($\omega = A$ and $\Omega_0 = A_0$ in Eq. 6). If the parameters of two-dimensional probability distribution (Eq. 6) are known, the procedure of the fatigue life assessment for a structural element with heterogeneous stress distribution is as follows:

- The free surface of the considered element is divided into sub domains of sizes $A^{(i)}$ which allows appropriate integration process (Fig. 2a).
- In each sub domain $A^{(i)}$ multiaxial stress state $\sigma_{kl}^{(i)}(t)$ (where k, l are tensorial indexes) is reduced to an equivalent stress amplitude $\sigma_{eqa}^{(i)}$ (where eqa means the equivalent amplitude) state by using a multiaxial fatigue crack initiation criterion.
- The equivalent stress amplitude $\sigma_{eqa}^{(i)}$ and the fatigue reference S-N curve σ_a-N_f are used for calculating the number of cycles to failure $N_f^{(i)}$ for each sub domain $A^{(i)}$ (Fig. 2b). Then, the survival probability distribution $P_s^{(i)}$ is determined (Fig. 2b) as follows $P_s^{(i)}(N) = e^{-\frac{1}{A_0}\left(\frac{\log(N)}{\log(N_f^{(i)})}\right)^{\frac{p}{\log(N_f^{(i)})}} A^{(i)}}$.

- For each fatigue life N, exponents of the natural logarithm are summed over all the sub domains $A^{(i)}$ and the survival probability distribution $P_s(N)$ for the whole structural element is obtained.
- The fatigue life calculation N_{cal} is performed for $P_f(N_{cal})$=0.63. Fatigue life for any other probability level, i.e. the scatter of results, can be calculated in a similar way.

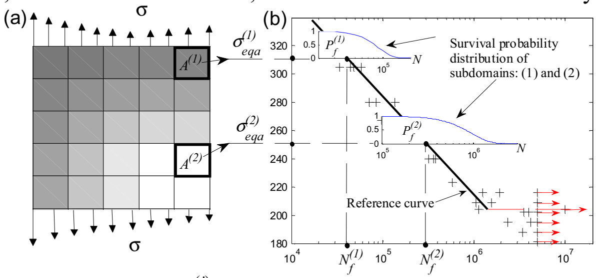

Fig. 2. (a) The separated sub domains $A^{(i)}$ of the element, (b) distributions of survival probability $P_s^{(i)}$ of particular sub domains against the fatigue reference curve

Identification of the parameters of two-dimensional failure probability distribution

Taking advantage of empirical analytic equation of the reference curve σ_a-N_f, the two-dimensional distribution (Eq. 6) has only two parameters to be identified, i.e. A_0 and p. The reference surface area A_0 is the free surface area of the specimen applied for determination of the reference curve σ_a-N_f.

The parameter p responsible for the distribution of fatigue life scatters can be determined from the tests of specimens having the same distribution of defects (kind and morphology) as the considered element. However, manufacturing qualities of elements and specimens are usually different. In such a case, distribution parameters should be fitted based on one series of tests of a real element subjected to simple fatigue loadings. Such procedure was applied, for example, by Delahay and Palin-Luc [11] for identifying the parameters of the fatigue limit probability distribution. In the present paper, the authors applied different values of the parameter p to find the best correlation between experimental fatigue lives N_{exp} and the calculated fatigue lives N_{cal}.

The experimental tests and results

Experimental results obtained from testing two steels and different specimen geometries were used for analyzing and verifying the proposed probabilistic method.

In the first experiments, cruciform specimens made of 18G2A steel (Fig. 3 and Table 1) with a central hole as stress concentrator were subjected to biaxial fatigue loading. The experiments were performed with the cooperation of a few researchers from Opole University of Technology; the original results are published in [14].

Cyclic properties of the tested steel, i.e. relation between the number of cycles to failure N_f and the stress amplitude σ_a as well as parameters of the cyclic hardening curve are given in Table 1 from tests on smooth specimens. The fatigue tests on notched specimens (Fig. 3) were carried out under force control with: $F_x(t)=F_{xa}\sin(2\pi ft)$, $F_y(t)=F_{ya}\sin(2\pi ft-\delta)$ with the same frequency on each direction (f = 13 Hz) and similar amplitudes F_{xa} and F_{ya} with a phase shift δ=180° (Tab. 2). Table 2 contains also the numbers of cycles to crack initiation N_i corresponding to the crack length a_i. The crack length a_i is the length of the first registered crack. The crack lengths were identified from pictures made with an optical microscope (magnification 7x) and a digital camera (0.0085 mm/pixel). The pictures of specimen surface near the hole were periodically taken in order to detect number of cycles N_i when crack initiation occurs and to analyse the crack growth rate.

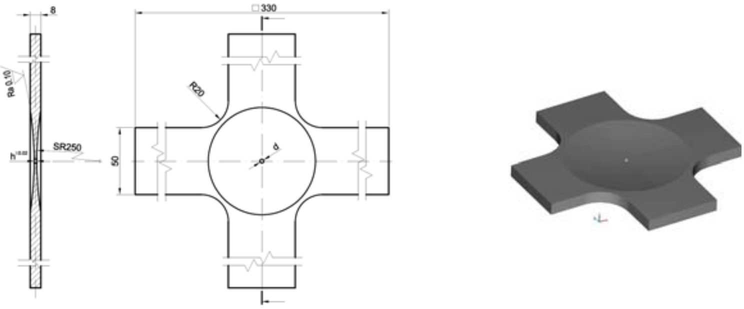

Fig. 3. Geometry of a cruciform specimen in 18G2A steel

Table 1. Cyclic properties of 18G2A steel under fully reversed tension on smooth specimens

$\sigma_a = \sigma_{af}\left(N_\sigma / N_f\right)^{1/m_\sigma}$			$\varepsilon_a^p = \left(\sigma_a / K'\right)^{1/n'}$		A_0
σ_{af}, [GPa]	m_σ	N_σ, [cycles]	K', [MPa]	n'	[mm^2]
204	8.32	$1.426 \cdot 10^6$	1323	0.207	1256

Indices: *af* – fatigue limit, *a* – amplitude, *p* – plastic part

Table 2. Test conditions and results

Specimen	d, [mm]	h, [mm]	F_{xa}, [kN]	F_{ya}, [kN]	N_i, [cycles]	a_i, [mm]
P02	3.0	1.40	13.30	13.10	39700	0.22
P03	3.0	1.54	13.50	13.30	31100	0.37
P04	3.0	1.86	13.55	13.30	60048	0.07
P05	2.5	1.50	10.21	9.90	246695	0.25
P07	3.0	1.75	11.20	10.80	140700	0.20
P08	2.4	1.20	9.30	9.10	167050	0.10

The second set of experimental results were taken from the work of Fatemi et al. [15]. The circumferentially notched round bar (Fig. 4) made of vanadium-based micro alloyed forged steel, in both the as-forged (AF) and quenched and tempered (QT) conditions were subjected to tension-compression loading. In the AF condition two notch radius were tested R=0.529 mm or R=1.588 mm which generate the following stress concentration factors in tension $K_t = 2.8$ and $K_t = 1.8$, respectively. Under the QT condition only one specimen geometry with notch radius R=1.588 mm ($K_t = 1.8$) was tested. The properties of the reference curve are presented in Table. 3.

The fatigue life of the notched and smooth (reference) specimens were defined as the number of cycles endured until the specimen failure in two parts. However, observations carried out with a traveling microscope (magnification 30x) shows that macro-crack growth life was not a significant part of the total life. Therefore, the number of cycles up to crack initiation could be assumed equal to the total fatigue life.

Fig. 4. Geometry of a round notched specimen, where R=0.529 mm (K_t=2.8) or R=1.588 mm (K_t = 1.8) from [15].

Table 3. Cyclic properties of AISI 1141 steel in two conditions: (AF) and (QT)

	$\varepsilon_a = \sigma_f'/E(2N_f)^b + \varepsilon_f'(2N_f)^c$					$\varepsilon_a^p = (\sigma_a/K')^{1/n'}$		A_0
State	E, [GPa]	σ_f', [MPa]	ε_f'	b	c	K', [MPa]	n'	[mm^2]
AF	200	1296	1,026	-0,088	-0,686	1205	0,122	162
QT	212	765	1,664	-0,041	-0,704	1133	0,134	162

Numerical simulations and results

The strain and stress distribution in the specimens were calculated using the 3D finite element analysis COMSOL software with an elastic-plastic constitutive model [16]. In all the computations, a cyclic constitutive model with linear kinematic hardening was applied. The material hardening was identified from the cyclic hardening curve (from half-life hysteresis loops) expressed by the Ramberg-Osgood $\varepsilon_a^p = (\sigma_a/K')^{1/n'}$ relationship. The plasticity condition was defined by the conventional Huber-Mises-Hencky hypothesis. The Lagrange elements (tetrahedrons) of order 2 with a high mesh density in the vicinity of the notch were used in the computations.

The criterion of maximum normal stresses (cruciform specimens) or strains (round specimens) on the critical plane was assumed as the criteria of multiaxial fatigue failure. The equivalent stresses or strains are calculated according to the following equations

$$\sigma_{eq}(t) = \sigma_n(t) = \sigma_{ij}(t)n_i n_j, \quad \varepsilon_{eq}(t) = \varepsilon_n(t) = \varepsilon_{ij}(t)n_i n_j, \tag{8}$$

where n_i is the unit normal vector to the plane experiencing the maximum normal stress $\max_{t,\mathbf{n}} \sigma_n(t)$ or strain $\max_{t,\mathbf{n}} \varepsilon_n(t)$. Surfaces of the finite elements were understood as sub domains $A^{(i)}$ described in paragraph 3. Fatigue lives to crack initiation N_{cal} were calculated for three probability levels: P_f={0.05; 0.63; 0.95} and for different values of the parameter p. The upper P_f=0.95 and lower P_f=0.05 bands of failure probability are marked by plus (+) in Fig. 5. The fatigue life for the failure probability P_f=0.63 is indicated by filled marker (• or ♦). Additional scatter band (×2.0) around the solid line N_{cal}=N_{exp} of perfect results consistency is also shown in Fig. 5. For 18G2A steel the best fatigue lives correlation is attained for p = 580 (Fig. 5a). For AISI 1141 steel in both condition (AF) and (QT) the best agreement between the calculated and experimental fatigue lives is obtained for p = 340 (Fig. 5b). It must be noted that independently on the notch radius R (Fig. 4) and specimen state (AF) or (QT) the best agreement between N_{exp} and N_{cal} is obtained for a single value of p=340. Since the surface quality of all specimens is similar in these cases, the obtained results are in agreement with assumption that p coefficient is a manufacturing quality factor.

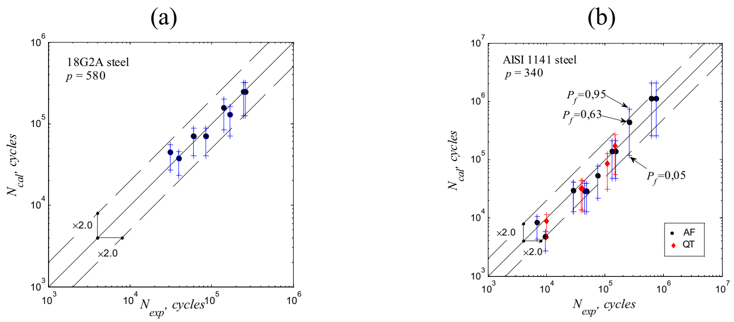

Fig. 5. Comparison of the experimental fatigue lives N_{exp} with the calculated lives N_{cal} for (a) cruciform specimens in 18G2A steel with p = 580 and (b) round specimens in AISI 1141 steel with p = 340

Conclusions

The authors proposed a procedure for determining the two-dimensional failure probability distribution P_f-N-σ of structural metallic elements and its application for the calculations of the fatigue life of such elements. The presented approach allows calculating fatigue life at any requested probability levels. The presented probability distribution according to Eq. (6) has a general form; it could be used for different fatigue damage parameters (stress, strain or energy). Generally, the paper presents the two-dimensional distribution P_f – fatigue life – fatigue damage parameter.

The calculated fatigue life N_{cal} is in good agreement with experimental fatigue life for notched round specimens made of AISI 1141 steel (with p=340) and for the notched cruciform specimen made of 18G2A steel (with p=580).

The proposed probability distribution function (Eq. 6) to fatigue crack initiation needs to determine only one additional parameter p. Such a simple form is suitable for the considered two types of notched specimens. However, it should be expected that other elements made of the same steels but with different quality (manufacturing process) would reveal other values of parameter p.

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