Fatigue Life Analysis of Damaged Structural Component Using Strain Energy Density Method

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Abstract. In this paper, a fatigue life prediction model using the strain energy density concept has been proposed. The computation crack growth model is based on the strain energy density method in which crack growth rate is expressed in terms of low cycle parameters. Initial damages are asummed in critical parts of structural components such as stress concentration areas. Special attenion is focused to study wing skin as damaged aircraft structural component. The stress intensity factor (SIF) was determined using analytical/numerical approaches. Finite element method (FEM) is used to derive analytic expressions for stress intensity factors that are necessary for crack growth analyses. The predictions crack growth results are compared with experimental data. A good agreement was obtained between the predictions and the corresponding experimental data.

Introduction

In the analysis related to fatigue behavior of real structural components under service loading, one of the fundamental issues is the evaluation and formulation of adequate relations which could describe fatigue life. Fatigue as a very complex process can be considered and analyzed as: 1) crack initiation phase and 2) crack propagation phase. In this paper, the computational model for calculation of the crack growth behavior of damaged structural components was developed.

Over the last few decades, researchers have proposed many physical models for prediction of fatigue crack growth behavior. It is well known that some of these models are considered as conventional methods [1], while other as energy based methods [2-4]. Special attention should focus on energy based methods as in these concepts are considered both elastic and elastic-plastic behavior of material near the crack tip. One of such energy-based models is method based on strain density. This method has been formulated for the first time by Rice (known as the HRR fields) [4], and later further examined and updated. The method is based on the fact that energy density depends on the stress and plastic strain near the crack tip. Additionally, with energy based models the trend is to associate the fatigue damage with some fundamental cyclic properties, e.g. to those obtained in low cyclic fatigue [5-9]. As a result of including the low cyclic fatigue properties [10] for prediction of the fatigue crack growth behavior, energy based models are becoming more important since the low cyclic fatigue properties are available in literature for different types of materials.

In this paper authors, have formulated and considered the energy based model using strain energy density concept for the fatigue crack growth prediction under axial loading. Additionally, by using the wing skin (as an example) the authors presented procedure for evaluation of equation for corrective function (needed for crack growth estimation) in the case when the stress intensity factors were obtained by using Finite element method

Formulation of Strain Energy Density Method

The study of fatigue life prediction is an important research aspect of fatigue failure and safe design criteria for structural components so in this section, the energy based method for fatigue life estimation is formulized.

As energy based model and strain energy density method are considered, it is possible to start from the following fact: Energy absorbed per unit growth of crack is equal to the plastic energy dissipated within the process zone per cycle. This concept is expressed by:

$$W_c\,\delta a = \omega_p, \tag{1}$$

where W_c is energy absorbed till failure, ω_p- the plastic energy and a- the crack length. Relation for energy absorbed till failure could be determined if the relation for uni-axial elastic-plastic behavior of materials under cyclic loading (stress-strain curve) is known, like Ramberg-Osgood equation:

$$\Delta\varepsilon = \frac{\Delta\sigma}{E} + 2\left(\frac{\Delta\sigma}{2k^{/}}\right)^{\frac{1}{n^{/}}}, \tag{2}$$

where $k^{/}$ is the cyclic strength coefficient, $n^{/}$- the cyclic strain hardening exponent, E - the modulus of elasticity, $\Delta\varepsilon/2$ is strain amplitude and $\Delta\sigma/2$ is stress amplitude.

Then the energy absorbed till failure W_c is the area below the cyclic stress-strain curve and is expressed by:

$$W_c = \frac{4}{1+n^{/}}\sigma_f^{/}\varepsilon_f^{/}, \tag{3}$$

where $\sigma_f^{/}$ is cyclic yield strength and $\varepsilon_f^{/}$ is fatigue ductility coefficient.

The plastic energy dissipated in the process zone ω_p could be determined by integration of equation for the cyclic plastic strain energy density in the units of Joule per cycle per unit volume [3] from zero to the length of the process zone ahead of crack tip d^* and as result the plastic energy dissipated in the process zone becomes:

$$\omega_p = \left(\frac{1-n^{/}}{1+n^{/}}\right)\frac{\Delta K_I^2\,\psi}{E\,I_{n^{/}}}, \tag{4}$$

where ΔK_I is the stress intensity factor, ψ - constant depending on the strain hardening exponent $n^{/}$, $I_{n}^{/}$ - the non-dimensional parameter depending on $n^{/}$.

The stress intensity factor is the factor which includes the geometry of structural component and load in calculation of service life for crack propagation phase. The stress intensity factor can be determined by using analytical and/or numerical approaches. Usually, for simple cases of geometry analytical approach could be used, while for more complicated geometry and loading cases it is necessary to use numerical methods. In this work the finite element method in the framework of the program package MCS/NASTRAN has been used for calculation discrete values of stress intensity factors (SIF) as well as analytical approach. Discrete values of SIF are used to derive analytic expressions of SIF that are necessary in crack growth analyses.

By substituting Eq. (3) and Eq. (4) in Eq. (1) it gives:

$$\frac{da}{dN} = \frac{(1-n^{/})\psi}{4E\,I_{n^{/}}\sigma_f^{/}\,\varepsilon_f^{/}}(\Delta K_I - \Delta K_{th})^2, \tag{5}$$

where ΔK_{th} is the range of threshold stress intensity factor and is function of stress ratio i.e.

$$\Delta K_{th} = \Delta K_{th0}(1-R)^{\gamma}, \tag{6}$$

ΔK_{th0} is the range of threshold stress intensity factor for the stress ratio R = 0 and γ is coefficient (usually, $\gamma = 0.71$).
Eq. (5) can be used to predict fatigue crack growth rate and with integration of Eq. (5) it is possible to obtain the number of cycles to failure:

$$\int_0^N dN = \frac{4 E I_{n'} \sigma_f' \varepsilon_f'}{(1-n')\psi} \int_{a_0}^{a_c} \frac{da}{(\Delta K_I - \Delta K_{th})^2}. \tag{7}$$

Eq. (7) indicates that the required number of cycles which causes a crack propagation from the initial length a_0 to the critical crack length a_c can be explicitly determined, if σ_f', ε_f' and n' are known. σ_f', ε_f', and n' are actually material parameters. These are the low cyclic fatigue properties and they can be obtained experimentally.

Evaluation of corrective function using finite elements

Corrective function is the function of geometry and it appears in the equation of stress intensity factor:

$$K_I = Y S \sqrt{\pi a}, \tag{8}$$

where Y is corrective function, S – nominal stress and a – the length of crack.
Corrective function can be given in analytical form (equation obtained from graphics/tables) but also as a relation that was obtained by numerical simulation of stress intensity factor using Finite Element method. Numerical simulation is used for cases with complex structural components when there are no available analytical equations (or graphics/tables) in handbooks.
Since in the present work wing skin with crack and two holes was considered as the damage structural component, in this section, for the same geometry, we will perform evaluation of equation for corrective function.
Prior to evaluation of equation for corrective function it necessary to determine stress intensity factor using numerical simulation. For the numerical simulation of stress intensity factor the FEM-program package MSC/NASTRAN has been used. One of the constructed finite element models is shown in Fig.1 and other obtained values of stress intensity factors are listed in Table 1 for given geometry and material characteristics (see example 2).

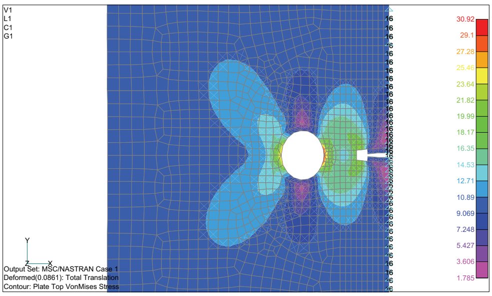


Fig. 1 Finite element model of wing skin with crack between two holes (a = 2.5 [mm], S_{max} = 30.92 [daN/mm^2])

Table 1 Calculation of stress intensity factors using FEM

	a [mm]	K_I^{FEM} [daN/mm$^{3/2}$]
1	2.0	14.39
2	2.2	15.14
3	2.4	15.84
4	2.6	16.52
5	2.8	17.21
6	3.0	17.93
7	3.2	18.69
8	3.4	19.50
9	3.6	20.36
10	3.8	21.26

Since we determined stress intensity factors for discrete values of crack length it is possible for each one of them to calculate adequate values of corrective function. All those obtained values are used for evaluation of polynomial. Polynomial expression of fourth and fifth degree obtained by this method:

$$Y_4 = 0.152 + 1.2883\,a - 0.68483\,a^2 + 0.15667\,a^3 - 0.01267\,a^4 \qquad (9)$$

$$Y_5 = 0.90288 + 0.09774\,a - 0.0058\,a^2 - 0.0053\,a^3 - 0.0007\,a^4 + 0.00047\,a^5 \qquad (10)$$

In the following section, equations (9) and (10) will be used as corrective functions, i.e. in the equation for stress intensity factor (8), in order to determine crack growth life of structural component.

Numerical examples

The validity of presented strain energy density method can only be assessed through a comparison with experimental data, which is the focus of this section. In addition, stress intensity factors were calculated by using analytical and numerical approaches. In the case where numerical simulation was used for the stress intensity factor for crack growth life estimation, the evaluated polynomial expressions were used for corrective function, which include geometry of structural element.

Example 1. Estimation of fatigue crack growth life to failure for CC specimen

In this example fatigue crack growth life prediction was considered. Structural element is a central cracked plate made of 2219 T851 Al Alloy. External loading is axial with constant amplitude. Material characteristics Al Alloy under cyclic loading are: $\sigma_f^{/} = 613$ MPa; $\varepsilon_f^{/} = 0.35$; $n^{/} = 0.121$; $k^{/} = 710$ MPa; $S_y = 334$ MPa; E = 7.1 10^4 MPa; K_{IC}=120 MPa m$^{1/2}$; $I_n^{/} = 3.067$; $\psi = 0.95152$ and <u>ΔK_{th0} = 8</u> MPa m$^{1/2}$. Geometry characteristics for a central cracked plate are: w = 152.4 mm; a_0 =7.6 mm. Needed analytical relation for the stress intensity factor of the central cracked plate is:

$$K_I = \left(1 + 0.256\left(\frac{a}{w}\right) - 1.152\left(\frac{a}{w}\right)^2 + 12.2\left(\frac{a}{w}\right)^5\right) S\,\sqrt{\pi\,a}, \qquad (11)$$

where S is the nominal stress; w – width of structural element; a - the length of crack and polynomial expression in the bracket is a corrective function for the central cracked plate;
With calculation, first was determined stress intensity factor K_I, and after that number of cycles to failure, N, using Strain energy density method.

Number of cycles predicted and presented in Fig.2.a and Fig.2.b are obtained by integration of equation (5) and by using equation (6) for ΔK_{th}. In the case of relation for K_I equation (11) was used, since in this example central cracked plate specimen was considered.

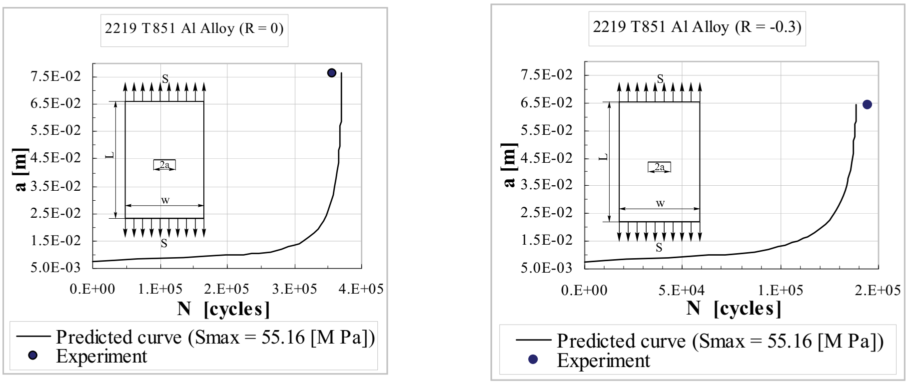
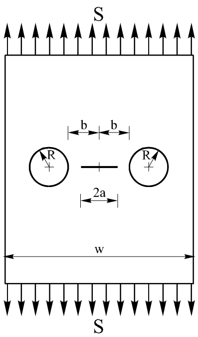


Fig. 2 Crack length versus number of cycles to failure N

Crack propagation experiments were carried out for stress ratios of R = 0 and R = -0.3. Central cracked plate with different stress ratios were subjected to fatigue up to failure. The tests were conducted in air at room temperature on MTS machine.
As shown in Fig. 2.a and Fig. 2.b, predictions are compared with experimental data and a good agreement was obtained.

Example 2. Crack growth prediction of damaged wing skin
In this example, fatigue life estimation of wing skin with crack between two holes was carried out. For stress analysis finite element method was used. The structural element was subjected to constant amplitude axially loading. Material used in this example is the same as previous (2219 T851 Al alloy). Geometry characteristics for a plate with crack between two holes are: R = 2 mm; b = 6 mm; w = 26 mm; a_0 =2 mm.
Since, in example 1, formulated procedure for estimation of fatigue crack growth life was verified, now we can start with determination of a number of cycles for crack propagation phase. The stress intensity factor was obtained by using analytical and numerical approaches.



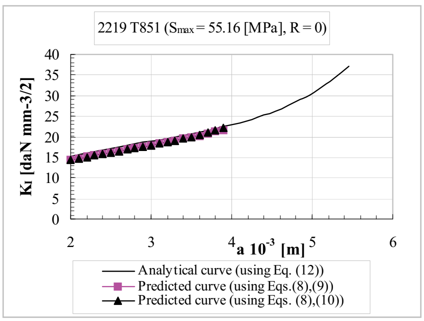


Fig. 3 Geometry of wing skin with crack and two holes **Fig. 4 The stress intensity factor versus crack length**

Analytical relation for stress intensity factor used in this example is:

$$K_I = \left(1.08899 + 0.04369\left(\frac{a}{b}\right) - 1.77302\left(\frac{a}{b}\right)^2 + 9.21212\left(\frac{a}{b}\right)^5 - 15.8683\left(\frac{a}{b}\right)^4 + 9.48718\left(\frac{a}{b}\right)^5\right) S\sqrt{\pi a} \quad (12)$$

In addition to equation (12) evaluated corrective functions (polynomial expressions) (9) and (10) were used which can be substituted in equation (8) for the stress intensity factor. This way, two new relations for the stress intensity factor were obtained.

Obtained values for stress intensity factors using equation (12) or one of polynomial expressions (9), (10) in equation for stress intensity factor (8) are presented in Fig. 4.

By using equation (7) with relation (12) or one of polynomial expressions (9) or (10) as corrective function in equation (8) it is possible to find number of cycles for fatigue crack propagation. Obtained results are presented in Fig. 5 for nominal stress S_{max}= 55.16 MPa.

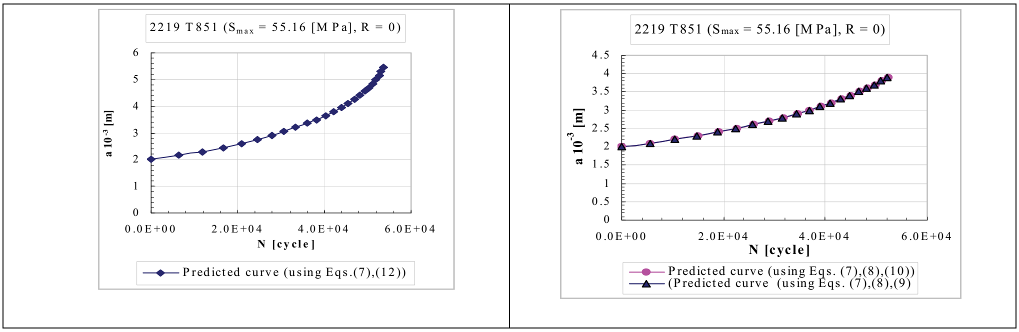


Fig. 5 Crack length versus number of cycles to failure N

Predictions of number of cycles to failure obtained by using analytical (Eq. (12)) are in a good agreement with predictions determined using equations (9) or (10) as corrective functions for stress intensity factor (8) and implies as conclusion that this approach could be used. This is an important fact in engineering applications as there are complex structural components without available analytical relations for corrective functions, i.e., stress intensity factor and the numerical simulation (FEM) must be used for calculation of stress intensity factors and after that with obtained values for stress intensity factors it is possible to evaluate an adequate equation for corrective function.

Example 3. Fatigue crack growth analysis of damaged wing skin under biaxial loading

In example 2, good correlation was obtained for crack growth behavior ad fatigue life predictions by using both analytical relation and numerical simulation (FEM) for the stress intensity factor and it showed that both approaches could be successfully used. Additionally, in this example the number of cycles to failure will be determined but only by using FEM for calculation of stress intensity factor. The reason for selecting FEM for calculation of stress intensity factor is due to the fact that the external loading is biaxial. The same material and geometry were used, as in example 2 (R = 2 mm; b = 6 mm; w = 26 mm; a_0 =2 mm).

In order to determine the number of cycles to failure it is necessary to formulate an adequate polynomial expression for corrective function. In this example the new polynomial expression obtained after calculation of the stress intensity factor for discrete values of crack length is evaluated. The evaluated new polynomial expression i.e. corrective function for geometry of structural component, shown in Fig. 6, subjected to biaxial loading is:

$$Y_4 = 2.50202 - 5.95245\left(\frac{a}{b}\right) + 10.29208\left(\frac{a}{b}\right)^2 - 9.38549\left(\frac{a}{b}\right)^3 + 4.10202\left(\frac{a}{b}\right)^4. \tag{13}$$

After the corrective function is formulated, it is possible to determine the number of cycles to failure but certainly by using equations (7) and (8). Obtained predictions for the case of biaxial loading as well as for uniaxial (see example 2) are presented in Fig 7. Presentation of both predicted curves in the same figure was done in order to perform comparison of number of cycles to failure.

The analysis of these predicted curves for biaxial and axial loading (see Fig.7) showed that biaxial load significantly reduced fatigue life of structural component.

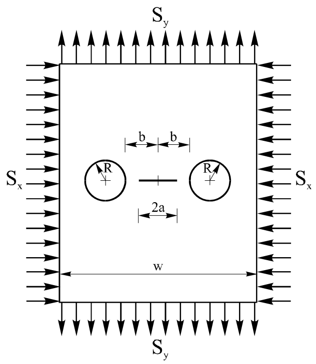


Fig. 6 Geometry of wing skin with crack between two holes subjected to a biaxial load

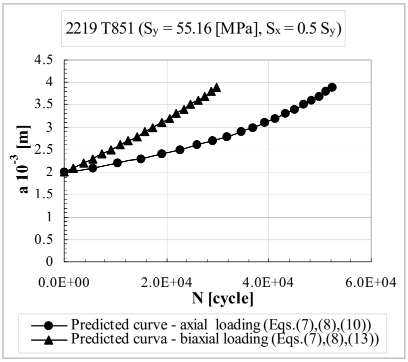


Fig. 7 Crack length versus number of cycles: Comparisons uniaxial with biaxial loads

Practically, here is illustrated the complete methodology for derivation of a new expresses of stress intensity factors using FEM. These expresses are used in connection with strain energy density method to estimate fatigue life of cracked structural components.

Conclusion

Based on the results from this study the following conclusions can be drawn.

1. The use of the Strain energy density method is efficient for fatigue crack growth prediction under cyclic loading in damaged structural components.
2. Strain energy density method is easy for engineering applications since it does not require any additional determination of fatigue parameters (those would need to be separately determined for fatigue crack propagation phase), and low cyclic fatigue parameters are used instead.
3. In the case of complex structural components, when it is necessary to use the numerical simulation (Finite Element Method), an approach for evaluation of corrective function was recommended. Needed corrective function for fatigue crack life estimation gives predictions for number of cycles to failure close to predictions obtained when analytical relation for corrective function was used.
4. All obtained results for number of cycles to failure show that the proposed approach can predict fatigue life of structural components.
5. Comparison between predicted curves for biaxial and axial loading implies as conclusion that biaxial loading could significantly reduce fatigue life to failure.

6. Practically, here is presented the general computation procedure for fatigue life analysis of cracked structural components based on combining FEM to derive analytic expressions for SIF together with strain energy density method for crack growth analysis.

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