Fatigue in PMMA: the effect of notches and pores predicted using the TCD

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Abstract. We studied notch fatigue behaviour in a polymer (PMMA) which contained a significant amount of porosity. An unusual feature of the experimental data was that certain types of notches caused no reduction in the fatigue limit compared to that measured in plain specimens. By conducting a critical distance analysis we were able to explain this behaviour and also to derive the fatigue strength of pore-free material and to predict the effect of a wide range of notches on the high-cycle fatigue strength. This work sheds light on the fatigue behaviour of porous materials and also has important practical applications relating to the material's use in medical devices.

Introduction

A well known approach to the prediction of high-cycle fatigue in notched components was pioneered by Smith and Miller some 30 years ago [1]. The principle is illustrated in fig.1a, which shows schematically the effect of the notch stress concentration factor K_t on the nominal fatigue limit $\Delta\sigma_{on}$ for a series of notches in which K_t is varied by changing the notch root radius, ρ, whilst keeping the notch depth, D, constant. Two regimes of behaviour are apparent. If $K_t<K_t^*$ then the fatigue limit (or fatigue strength in the high-cycle regime) can be adequately described by:

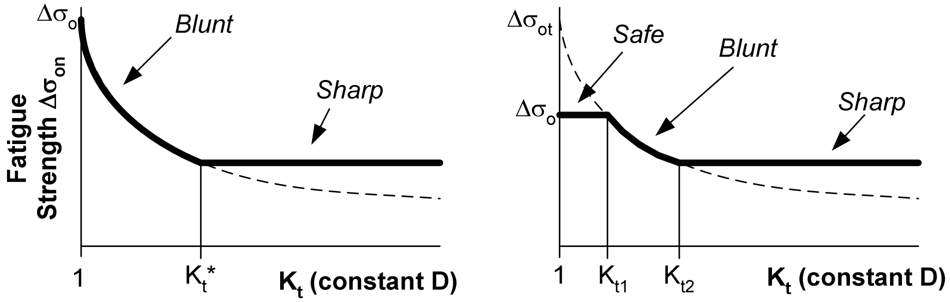
$$\Delta\sigma_{on} = \frac{\Delta\sigma_o}{K_t} \tag{1}$$

Here the effect of the notch is simply to reduce the fatigue limit of the material $\Delta\sigma_o$ (measured using plain specimens) by the factor K_t. However, if $K_t>K_t^*$ the fatigue limit becomes constant, equal to that of a sharp crack of the same length, therefore the standard fracture mechanics equation applies:

$$\Delta\sigma_{on} = \frac{\Delta K_{th}}{F\sqrt{\pi D}} \tag{2}$$

Here F is the crack geometry factor. We can conveniently refer to these two regimes as "Blunt Notch" and "Sharp Notch" respectively. The Smith & Miller approach is not exact – it tends to underestimate the fatigue limit in some cases, especially close to K_t^*, but it is accurate enough for many practical applications and provides a useful insight into the general effect of a notch.

In the present investigation we examined a brittle polymer, PMMA (polymethylmethacrylate). In its normal, solid form (when it is known as Perspex) it shows typical notch-fatigue behaviour which can be described by the Smith & Miller approach, but we found different behaviour when we tested a porous form of PMMA. In this form the material typically contains about 10% porosity, consisting of spherical pores with diameters up to about 1mm. Fig.2 shows the results of a porosity analysis which we conducted by examining pores visible on fracture surfaces. In this condition the material has important medical applications which will be described below in the Discussion. When we tested this material in specimens containing various different notches we found behaviour which is illustrated schematically in fig.1b. Instead of two regions, there were three. For very low values of K_t, the measured fatigue strength (at 10^5 cycles to failure) remained constant, equal to $\Delta\sigma_o$ (when any reduction in cross-section due to the notch was taken into account), i.e. the notch itself had no effect on fatigue behaviour. We call this regime "Safe Notch": in addition the data showed evidence of the Blunt and Sharp regions, as normal. We refer to the two transition values between these regions as K_{t1} and K_{t2} as shown on figure 1b.



(a) (b)

Figure 1: Schematics showing (a) typical notch fatigue behaviour as described by Smith & Miller; (b) behaviour demonstrated by our porous PMMA material.

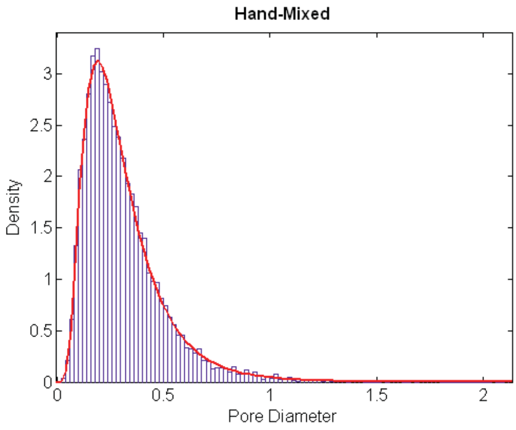


Figure 2: Pore size distribution results (pore diameter in mm).

We hypothesized that this unusual behaviour was due to the porosity, specifically that pores were acting as crack initiation sites, causing a reduction in the plain fatigue limit. But, if so, why was the fatigue limit not affected by the presence of a notch with $K_t<K_{t1}$? The aims of the present work were to investigate this problem and to develop a general model to predict the role of porosity at all K_t values.

The Theory of Critical Distances

The Theory of Critical Distances (TCD) is an approach for predicting fatigue and fracture in the presence of stress concentration features of all kinds. It has been applied to a wide variety of materials (for a full discussion see a recently published book on the subject [2]), including extensive use for predicting fatigue limits in metals (e.g.[3]), but, until recent work by the present authors [4], it has not been applied to study fatigue in polymers. The TCD makes use of a material property with units of length, the so-called critical distance, L. In the case of high-cycle fatigue it can be shown that L is related to $\Delta\sigma_o$ and ΔK_{th} through the following relationship:

$$L = \frac{1}{\pi}\left(\frac{\Delta K_{th}}{\Delta\sigma_o}\right)^2 \qquad (3)$$

This critical distance can be used in various ways to predict material behaviour. The simplest approach (and one which we have shown is very accurate for predicting fatigue in metals [3]) involves finding the elastic stress at a distance L/2 from the notch root. Fatigue failure is predicted to occur if the stress range at this point is equal to the plain specimen value $\Delta\sigma_o$. We applied this approach to experimental data from the present material. Fig.3 shows plots of elastic stress range as a function of distance for two different notches: a sharp notch (D=2mm, ρ=0.1mm, K_t=11) and a circular hole (Radius = D = 2.2mm, K_t=3.1). Both are loaded at their respective fatigue strengths. If the TCD is applicable, then the point of intersection of these two lines will occur at a distance of L/2 and a stress of $\Delta\sigma_o$. In this case the intersection occurred at (25MPa, 0.1mm). This stress, however, is much larger than the fatigue strength which we measured from plain specimens, which was 14.4MPa.

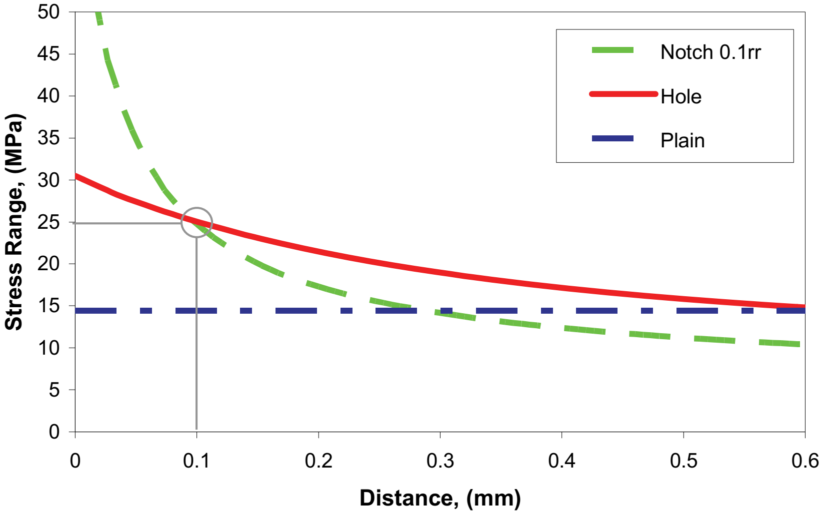


Figure 3: Stress/distance plots for a sharp notch and a circular hole, both loaded at their fatigue strengths (at 10^5cycles). The intersection of these two lines defines the critical distance and critical stress. Also shown is the fatigue strength of plain specimens.

The True Fatigue Strength

To explain this anomaly, we hypothesised that the stress range of 25MPa which we calculated from the TCD was actually the true fatigue limit of the material, i.e. the fatigue limit which this material would have if it did not contain any porosity. To test this hypothesis we manufactured specimens containing no pores. This is difficult to do in this material because pores arise during curing as a result of trapped air and evaporating monomer. Using commercial Perspex was not an option because our material differs in other ways - it has a lower molecular weight and a microstructure containing unreacted beads of PMMA – both of which affect its fatigue behaviour. We made pore-free specimens by a two-stage process: first we applied a vacuum during curing to suck out any pores; this reduced the overall porosity to around 2% but left a small number of quite large pores. Then we machined specimens from selected areas of material in which there were no pores present. The fatigue strength of this material was found to be 22MPa, which is quite close to our predicted value of the true fatigue strength, $\Delta\sigma_{ot}$ = 25MPa.

Application to Notches

Fig.4 shows the results of an extensive test programme in which we measured the fatigue strength at 10^5 cycles for specimens containing a wide range of different notches, including edge notches with K_t values from 1.4 to 11, circular holes, and small hemispherical depressions. In each case we predicted the fatigue strength using the TCD, obtaining elastic stress analyses from finite element models (ANSYS). We used the value of L/2 measured above and the true fatigue strength $\Delta\sigma_{ot}$. We also made a second prediction in which we simply assumed that failure would occur when the nominal stress range (on the remaining cross section) was equal to the fatigue strength of plain specimens, $\Delta\sigma_o$. It is clear from the results of this analysis that it was possible to predict the fatigue strength with good accuracy in all cases, using either the TCD or the simple nominal-stress approach. The latter approach gave good predictions for the four bluntest notches, which had K_t factors of from 1.4–2.4. For K_t=2.4 (4mm radius notch) the two predictions were almost identical.

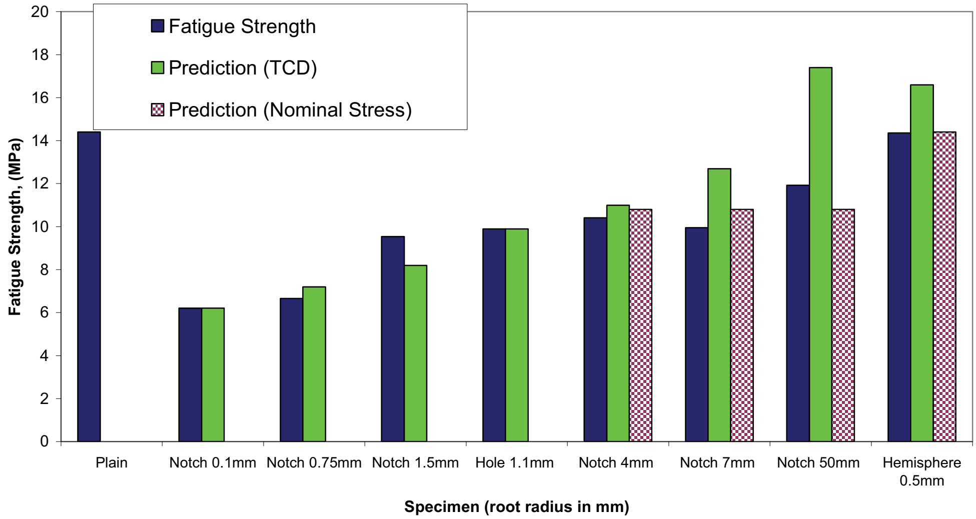


Figure 4: Measured fatigue strength for 9 different specimen types, with predictions using the TCD and also using the nominal stress on the reduced cross section. This shows that the four notches on the right are “Safe Notches”, causing no reduction in fatigue strength.

We can conclude that all notches which had a nominal fatigue limit of $\Delta\sigma_o$ must lie below K_{t1} in fig.1b, i.e. these are all Safe Notches by our terminology. In these cases the notch plays no role in the fatigue failure. This was obvious for the small hemispheres: in their case the fatigue failures initiated elsewhere in the specimens. For the other notches failure inevitably occurred in the vicinity of the notch because it had a significant effect in reducing the local cross section of the specimen, but our analysis shows that the stress-concentration of the notch itself had no effect.

Discussion

This material is used in a wide range of medical applications, particularly in orthopaedic surgery, where it is known as bone cement. In artificial joints such as the hip and knee it is used as a kind of grouting cement, to fill the irregular gaps between bone and metallic implants such as the femoral stem (in the hip joint) or tibial tray (in the knee joint). It is also used to replace lost or poor-quality bone, for example in the vertebrae of people suffering from osteoporosis. Fatigue cracking in this material is a major cause of long-term failure in these situations, so an understanding of the fatigue behaviour of this material is crucial. For many years it has been believed that porosity reduces the material's fatigue strength, and this has been shown in data from test specimens [5,6], so a great deal of effort has been put into reducing porosity, through a variety of mixing techniques. However, follow-up studies on patients receiving artificial joints have found no correlation between porosity and failure rates [7], suggesting that porosity is not really a problem in practice.

Our analysis helps to shed light on this issue and to resolve this anomaly. We have shown that porosity only has an effect if the notch K_t factor is less than a certain value, K_{t1}. For notches with higher K_t factors the material behaviour could be predicted using the true fatigue strength, i.e. by assuming that there was no porosity present. When used in surgical applications, this material experiences relatively high stress concentrations and stress gradients; for example, cracking in the bone cement surrounding a hip implant usually initiates at the interface between the cement and metal stem, whose sharp corners cause local stress concentrations. This explains why the negative effects of porosity, which are so obvious in plain test specimens, are not experienced in service.

In general we might expect complex interactions between notches and porosity, but we have shown, at least for the present material, that this is not the case; in fact, the behaviour in all cases can be described by one of two failure mechanisms, as follows:

1) If the notch is sufficiently sharp, then porosity will have no effect, because failure will be dictated by cracking in a small region close to the notch. Normally this region will not contain any pores, but even if it does the only effect of a pore will be to slightly change the local geometry of the notch (its root radius or length). This will have no effect if $K_t > K_{t2}$, and even if K_t lies between K_{t2} and K_{t1} its effect will be small unless the notch itself is very small.
2) If the notch is sufficiently blunt ($K_t < K_{t1}$) it will have no effect, because failure will occur from pores located elsewhere in the specimen.

In order to discover the exact effect of any given notch or stress concentration feature, it is necessary to carry out an elastic stress analysis using finite element modelling. However, we can derive some simple, approximate rules, as follows. We assume that Smith/Miller behaviour applies for all notches greater than K_{t1}, thus we can use equations 1-3 above, but taking the fatigue strength to be $\Delta\sigma_{ot}$ rather than $\Delta\sigma_o$ in equations 1 and 3. Values for the two critical K_t values can then be simply derived, as follows:

$$K_{t1} = \frac{\Delta\sigma_{ot}}{\Delta\sigma_o} \tag{4}$$

$$K_{t2} = F\sqrt{\frac{D}{L}} \tag{5}$$

The value of K_{t1} is useful because it defines notches which have no effect on fatigue behaviour and therefore can safely be left in the structure. Using equation 4 we would predict a value of 25/14.4 = 1.74. Experimentally we found Safe Notch behaviour in all notches with $K_t<1.74$ and also for some other notches with K_t up to 2.4. This suggests that equation 4 gives a conservative estimate of the limits of this region. The value of K_{t2} is also useful because it defines the worst possible notch. So if the structure already contains such a notch, there is nothing to be lost by increasing its K_t factor even more, because this will not reduce the fatigue strength.

K_{t2} is dependant on three factors, D, ρ and F; however it is worth noting that the K_t factors of many notches can be approximated by the well-known formula derived by Neuber, which is exact for elliptical notches:

$$K_t = 1 + 2\sqrt{\frac{D}{\rho}} \tag{6}$$

Comparing equations 5 and 6 we can see that K_{t2} is characterised by a notch root radius which is of the same order of magnitude as the critical distance, L, so this will be a useful approximate guide for the transition between the Blunt Notch and Sharp Notch regimes.

Conclusions

1) Porous PMMA (bone cement) displays a regime of “safe notches” in addition to the usual two regimes “blunt notches” and “sharp notches”. The limits of this new regime can be predicted accurately using the theory of critical distances (TCD); an approximate formula can also be derived.
2) The TCD can also predict the fatigue strength of pore-free PMMA, using only data from tests conducted on porous PMMA.
3) This analysis sheds light on the fatigue behaviour of this material and of porous materials in general; in particular it explains why porosity is not a problem for bone cement in service, even though it reduces the fatigue strength of test specimens.

References

[1] R.A.Smith and K.J.Miller: Int.J.Mech.Sci. 20 (1978) 201-206.

[2] D.Taylor: *The Theory of Critical Distances* (Elsevier, UK, 2007).

[3] L.Susmel and D.Taylor: Int.J.Strain Anal. 38 (2003) 443-452.

[4] D.Hoey and D.Taylor: Int.J.Fatigue 30 (2008) 989-995.

[5] S.Ishihara, A.J.McEvily, T.Goshima, K.Kanekasus, T.Nara: J.Mat.Sci., Materials in Medicine 11 (2000) 661-666.

[6] L.Cristofolini, C.Minari and M.Viceconti: Fat.Fract.Eng Mater.Struct. 23 (2000) 953-957

[7] H.Malchau, P.Herberts and L.Ahnfelt: Acta Orthop.Scand. 64 (1993) 497-506