Fatigue Fracture of Materials in the Region of Solids Cyclic Contact

Volodymyr Panasyuk and Olexandra Datsyshyn

Karpenko Physico-Mechanical Institute of the National Academy of Sciences of Ukraine

panasyuk@ipm.lviv.ua

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Introduction

In the last ten years the problem of fatigue fracture of materials in the vicinity of cyclic contact of deformed solids has become a separate section of fracture mechanics of cracked bodies. This problem is tightly related with ensuring the reliability and the durability of such engineering systems which elements operate under conditions of rolling contact, fretting-fatigue, pulsating contact, frictional fatigue etc. In particular these are such systems as "wheel–rail" in railway transport, different rolling mills in metallurgy, bearings in engineering, supports of drill bits in gas and oil producing industry and other. Application of the methods and concepts of fracture mechanics for solving this problem, as a separate direction of fracture mechanics has intensively started to develop in the late twentieth century [1-13]. In Ukraine new important results on this problem have been obtained [1, 2, 5, 6, 10]. In particular, the corresponding calculation models and the algorithms of calculation of fatigue crack propagation paths depending on the loading conditions of bodies and the character of friction forces between them have been formulated. As an example of such calculations for a system "wheel–rail" in the frames of the concepts of fracture mechanics the period of pitting formation or crumbling of the material on the rail surface has been established [1, 2]. A general calculation model for the durability estimation for such a system of contact bodies has been formulated.

Calculation model

The calculation model (when contacting bodies are considered as two-dimensional) for the analysis of crack propagation in a half-plane under action of contact pressure $p(x, \lambda, t)$, where t is the time of pressure action, is shown in Fig. 1. If the friction (slipping) occurs between bodies under contact, the tangential traction q will act on the contact surface, that are determined by the Coulomb's law, namely:

$$q = f \cdot p(x, \lambda, t), \tag{1}$$

where f is the friction coefficient. Introduce a system of coordinates xOy, as it is shown in Fig. 1 and consider that the beginning of this system is at point O in which a mouth of one of the cracks is located on the half-plane boundary. The maximum value of the normal contact pressure $p = p_0$ is at the distance $x_0 = \lambda a$ from point O where $2a$ is the width of the contact region. This allows to perform a quantitative analysis of such a crack growth under the unidirectional cyclic contact of rolling bodies.

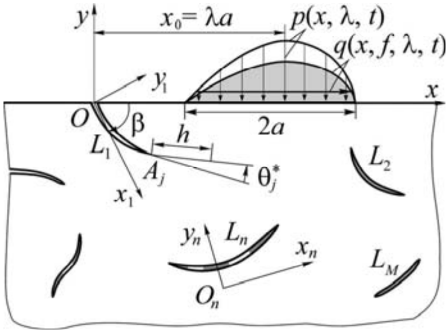


Fig. 1. A scheme of the calculation model.

Durability N_* (a number of rolling cycles up to pitting appearance) of this system will be evaluated by

$$N_* = N_i + N_p = N_i + N_{p\tau} + N_{p\sigma}. \tag{2}$$

Here N_i is the period of the initial macrocrack initiation which is evaluated using the additional physical conditions, and period N_p is the life time of the system (a semi-plane) with an initial macrocrack which is considered to be a rectilinear of a length $l_{0\tau}$. In this case consider also that N_p consists of two parts: $N_{p\tau}$ is the period of this crack growth from the value $l_{0\tau}$ to $l_{c\tau} \approx l_{0\sigma} = l_0$ by mode II fracture, i.e. when coefficients ΔK_{II} are dominant and $N_{p\tau}$ is the period of the crack growth from $l_{0\tau}$ to $l_{c\tau}$, when in this section the crack growth path (up to reaching the value $l_{c\tau}$) the stress intensity factors ΔK_{I} are dominant (mode I fracture). To calculate the periods $N_{p\tau}$ and $N_{p\sigma}$ we use:

$$N_{p\tau} = \int_{l_{0\tau}}^{l_{c\tau}} v^{-1}[\Delta K_{\text{II}\theta}(l,...)]\,dl\,, \qquad N_{p\sigma} = \int_{l_{0\sigma}}^{l_{c\sigma}} v^{-1}[\Delta K_{\text{I}\theta}(l,...)]\,dl\,, \tag{3}$$

where $v(\Delta K_{\text{I}\theta}...)$, $v(\Delta K_{\text{II}\theta}...)$ are macrocrack growth rate depending on the stress intensity factor range ($\Delta K = K_{\max} - K_{\min}$); $K_{\text{I}\theta}, K_{\text{II}\theta}$ are the stress intensity factors of circular and shear stress at the crack tip.

Consideration of N_p as a sum of two components ($N_{p\tau} + N_{p\sigma}$) as well as a procedure of their calculation is the important complement (generalization) of the currently available similar concepts of life time assessment of the contacting bodies system. In the known approaches the conditions of the initial macrocrack start and not kinetics of its growth were mainly evaluated. Still, fact that the initial crack starts during cyclic loading does not mean that the durability of the contacting bodies system has exhausted. That is why the important component of the theory is the consideration of the crack kinetics (its growth path) under cyclic contact of bodies. In this case note, that the initial period of this kinetics, according to the experiments, is related with the durable crack propagation by mode II fracture.

To realize the proposed calculation model it is necessary, first of all, to have curves of the macrocrack growth rate in the material in dependence on the coefficients ΔK_{I} and ΔK_{II}, that is to know functions $v = F_{\text{I}}(\Delta K_{\text{I}})$ and $v = F_{\text{II}}(\Delta K_{\text{II}})$ [3, 4].

Besides it is necessary to establish the conditions for evaluation of the calculation model parameters, namely: $l_{0\tau}$, $l_{0\sigma}$, $l_{c\tau}$, $l_{c\sigma}$. For this purpose the following conditions are assumed. Propagation of a curvilinear or arbitrarily oriented rectilinear macrocrack (angle β in Fig. 1) in each rolling cycle occurs in such a direction (angle θ^* in Fig. 2) and at such location of a counterbody ($\lambda = \lambda^*$) at which the stress intensity factors $K_{\mathrm{I}\theta}(p, f, \lambda, \beta, \theta)$ and $|K_{\mathrm{II}\theta}(p, f, \lambda, \beta, \theta)|$ attain the maximum values of θ and λ, i.e.:

$$\max K_{\mathrm{I}\theta}(p,l,f,\lambda,\beta,\theta) = K_{\mathrm{I}\theta}(p,l,f,\lambda^*,\beta,\theta^*), \tag{4}$$

$$\max K_{\mathrm{II}\theta}(p,l,f,\lambda,\beta,\theta) = \left|K_{\mathrm{II}\theta}(p,l,f,\lambda^*,\beta,\theta^*)\right|, \tag{5}$$

where

$$\left.\frac{\partial K_{\mathrm{I}\theta}(p,l,f,\lambda,\beta,\theta)}{\partial\theta}\right|_{\theta=\theta^*}, \qquad \left.\frac{\partial K_{\mathrm{I}\theta}(p,l,f,\lambda,\beta,\theta)}{\partial\lambda}\right|_{\lambda=\lambda^*}. \tag{6}$$

$$\left.\frac{\partial K_{\mathrm{II}\theta}(p,l,f,\lambda,\beta,\theta)}{\partial\theta}\right|_{\theta=\theta^*}, \qquad \left.\frac{\partial K_{\mathrm{II}\theta}(p,l,f,\lambda,\beta,\theta)}{\partial\lambda}\right|_{\lambda=\lambda^*}. \tag{7}$$

Utilizing Eqs. (4)–(7) and curves $v = F_{\mathrm{I}}(\Delta K_{\mathrm{I}})$, $v = F_{\mathrm{II}}(\Delta K_{\mathrm{II}})$ to evaluate parameters $l_{0\tau}$, $l_{0\sigma}$, $l_{c\tau}$, $l_{c\sigma}$ we get such equations:

$$\Delta K_{\mathrm{II}\theta} = \Delta K_{\mathrm{II}} = K_{\mathrm{II}\max}(p,l_{0\tau},f_c,f,\lambda_1^*,\beta,0) - K_{\mathrm{II}\min}(p,l_{0\tau},f_c,f,\lambda_2^*,\beta,0) = \Delta K_{\mathrm{II}th}, \tag{8}$$

where $\Delta K_{\mathrm{II}th} = \Delta K_{\mathrm{II}}$ (10^{-10} m/cycle), f_c is the friction coefficient between the crack faces.

$$\Delta K_{\mathrm{I}\theta} = K_{\mathrm{I}\theta\max}(p,l_{0\sigma},r,f,\lambda_1^*,\beta,\theta_1^*) - K_{\mathrm{I}\theta\min}(p,l_{0\sigma},r,f,\lambda_2^*,\beta,\theta_1^*) = \Delta K_{\mathrm{I}th}, \tag{9}$$

where $\Delta K_{\mathrm{I}th} = \Delta K_{\mathrm{I}}$ (10^{-9} m/cycle), r characterises the pressure on the crack faces ($0 \le r \le 1$).

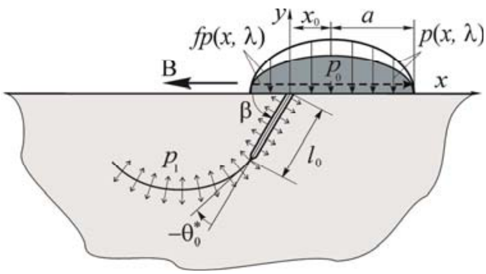
$$K_{\mathrm{I}\theta\max}(p, l_{c\sigma}, r, f, \lambda_1^*, \beta, \theta_1^*) = K_{\mathrm{I}fc}, \tag{10}$$

where $K_{\mathrm{I}fc}$ is the critical fatigue crack growth resistance under mode I fracture.

Construction of the crack growth paths and durability estimation of rails

By using Eqs. (2)–(10) and fatigue crack growth curves $F_{\mathrm{I}}(\Delta K_{\mathrm{I}})$ and $F_{\mathrm{II}}(\Delta K_{\mathrm{II}})$ for RSB12 [4] and 75ХГСТ [3] steels, and also the stepwise construction of the crack growth paths, the values of $l_{0\tau}$, $l_{c\tau}$, $l_{0\sigma}$, $l_{c\sigma}$, $N_{p\tau}$, $N_{p\sigma}$ and N_* were calculated and presented in the Table. The fatigue initially rectilinear crack propagation paths in the rail were constructed versus the coefficient r that characterizes the pressure (under boundary lubrication conditions) of the lubricant penetrating into a edge crack and creating additional pressure on its faces (Fig. 2).

The growth paths of the initial crack $l_0 = 0.43\,a$ versus r (r = 1.0, 0.1, 0.0) are shown (dashed lines) in Fig. 3. It is seen from this figure that pitting formation is possible when between the bodies under contact there is a lubricant (medium that reduced the friction coefficient and penetrated inside the initial crack). The idea that a pitting is formed when there is the lubricant between bodies in contact has been suggested for the first time by S. Way [14]. In this case it is proved theoretically. It is clear from Fig. 3 that when r = 0, i.e. there is no lubricating medium, penetrating into the crack, the propagation path of the initial crack is directed into the body. This means that a body can fail, however no pitting will be formed.



$$p(x) = (p_0/a)\sqrt{a^2-(x-x_0)^2}\ ;$$
$$p_1 = rp_0\sqrt{1-(x_0/a)^2}\ ,$$
$$0 \le r \le 1,\ \lambda = x_0/a$$

Fig. 2. A calculation model. **B** is the direction of the counterbody motion; f is the friction coefficient.

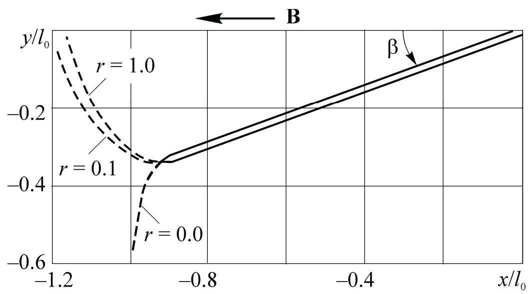


Fig. 3. Crack growth paths versus parameter r of the pressure intensity on its faces; β = 20º, $f = 0.1$, $\varepsilon = l_0/a = 0.43$.

The contact life time of the rail pearlitic steels (RSB12, 75ХГСТ, 900A) under long-term cyclic loading was evaluated experimentally in [7, 12, 13]. Formation of pitting on the body surface was taken as the durability criterion. The experimental data for these steels demonstrated the following:

1. $N_* \approx 10^6 \div 10^7$cycles [7, 13], $N_i \approx 10^3 \div 10^4$ cycles ([12], BS11; [7], 900A); $N_* \approx 1.5\times10^6$ cycles ([7], 900A, p_0 = 1100 MPa);

2. The average dimensions of pits are the following: depth – 0.5÷5 mm, length 1÷12mm.

These experimental data agree well with theoretical results obtained by the proposed calculation model (see Table). By using the results of the calculations within the frames of the proposed model it has been found that $N_* = (1.2 \div 1.3)\times 10^6$ cycles, and the pit dimensions: depth ≈ 2mm, length = 7 mm.

f_c	The crack length at different stages of its growth [mm]				Durability $N \times 10^{-6}$ cycle		
	$l_{0\tau}$	$l_{c\tau}$	$l_{0\sigma}$	$l_{c\sigma}$	$N_{p\tau}$	$N_{p\sigma}$	N_*
0.00	0.76	2.73	2.72	4.62	0.37	0.43	0.80
0.10	0.97	2.73	2.72	4.62	0.82	0.43	1.25
0.15	1.28	2.74	2.72	4.62	0.90	0.43	1.33
0.20	1.79	2.76	2.72	4.62	0.83	0.43	1.26

Table. Calculation data under conditions $\beta = 30°$, $r = 0.1$, $f = 0.1$, $p_0 = 1100$ MPa, $a = 7$ mm; f_c – the coefficient of friction between the crack faces.

So, the proposed concept of the life time calculation of the system of two bodies subjected to long-term cyclic contact allows the realization of the life time prediction of this system, however it is necessary to have experimental curves of fatigue crack growth resistance $F_{\mathrm{I}}(\Delta K_{\mathrm{I}})$ and $F_{\mathrm{II}}(\Delta K_{\mathrm{II}})$ of materials of contacting bodies. In this respect a very important task of fracture mechanics is to construct such curves for structural materials.

Propagation paths of the system of edge parallel cracks

Let us analyse the propagation paths of the system M of the edge cracks that comes out on the half-plane (Fig. 4). Assume that the growth of cracks is controlled by parameter $K_{\mathrm{I}\theta}$, that determines the intensity of normal circular stresses at the crack tips. The location of the contact region is fixed with respect to the first crack by parameter $\lambda = x_0/a$. Assume also that crack mouths are equidistant and the distance between them is specified by parameter $\delta = b/a$. In every rolling cycle, with the contact loading motion along the half-plane boundary (when λ is changing) parameter $K_{\mathrm{I}\theta i}$ $(i = 1,\ldots, M)$ for every crack varies, taking at particular $\lambda = \lambda_i^*$ and $\theta = \theta_i^*$ its maximum value $K_{\mathrm{I}\theta i}$. The crack is assumed to grow only at $\lambda = \lambda_i^*$ in the direction, specified by angle θ_i^* according to the σ_θ-criterion, under condition that the $K_{\mathrm{I}\theta i}$ value is higher than the threshold fatigue crack growth, i.e. K_{Ith}, for a given material. At such locations of the counterbody, when the edges contact arises on one of the cracks, assume that the crack presence does not affect the stress state in the body.

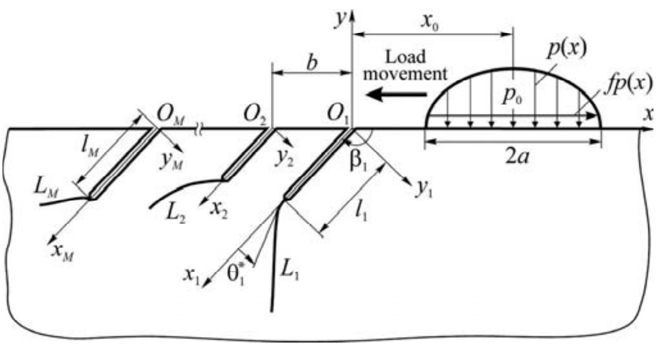


Fig. 4. A general scheme of the problem.

Using the proposed calculation model [1, 2] the crack increments (the h_i steps of the paths construction, Fig. 1) are constructed proportional to the movement rates of their tips. At each stage of the path construction, solve a system of SIE of the primary problem of the elasticity theory for a half-plane with the edge curvilinear cracks (each time for the new crack lengths).Reduce the SIE system to the system of linear algebraic equations by the method of mechanical quadratures.

The crack growth rate is calculated by the Paris formulae:

$$v_i = C(K^*_{\mathrm{I}\theta i})^n,\ i = 1, 2, \ldots, M.$$

Calculations were done for rail steel 75ХГСТ with the lamellar pearlite structure, and fatigue crack growth resistance characteristics $\mathrm{C} = 3.09\times10^{-12}\ \mathrm{MPa}^{-n}\ \mathrm{m}^{1-n/2}$, $n = 3.48$.

Propagation of the system of two and tree cracks of the equal and different length was considered. The initial inclination angle of the cracks to the tangential traction direction was $\beta = 5\pi/6$ and was chosen with the account of experimental data [4, 15, 16]. Calculations were done for the friction coefficients: f = 0.05; 0.10; 0.30 that correspond to different service conditions of the couple wheel-rail (dry and wet weather, lubrication).

As a result of calculations, performed for two parallel cracks at small friction coefficient in the rolling bodies contact [17], i.e., the value $f \le 0.1$, it has been found that two initially equal parallel cracks ($l_1 = l_2 = l$) with a rather large ($\delta = b/a > 1.0$) distance (b, Fig. 4) are propagating parallel to the contact boundary in the direction of the counterbody motion (Fig. 5 a). It coincides with one crack growth tendency (Fig. 5 b) [6]. When the distance between the initial cracks is smaller ($\delta \le 1.0$) the first crack turns at once deep into the material (Fig. 5 b, c), while the second grows parallel to the contact surface.

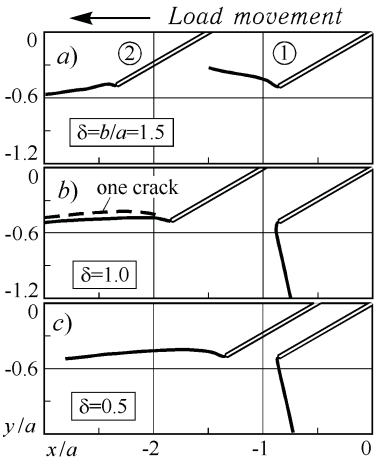


Fig. 5. Propagation paths of two equal parallel cracks depending on distance δ between them; $f = 0.1$; $\varepsilon = l/a = 1.0$.

The numerical analysis of two cracks with different initial lengths at small friction coefficients is carried out [17, 18]. Obtained results (Figs. 6 and 7) show that if the first crack is shorter than the second, it grows predominantly towards the second one and under a certain number of loading cycles can join with it. In this case for each fixed length of the second crack, at the given length of the first one, there is a certain critical distance δ^* between them. For $\delta > \delta^*$ the first crack propagates towards the second one, and at $\delta < \delta^*$ – deep into the material. If the length of the first crack increases the value of δ^* also increases. Note, that among the considered values of parameters

$f, \varepsilon_1, \varepsilon_2, \delta$, there exist such values when the first crack approaches the second one closely, causing the danger of cracking (curve 1 in Fig 6b). When the first crack is short ($\varepsilon_1 < 0.5$) the friction coefficient f decrease results in high probability of crumbling (Fig. 7).

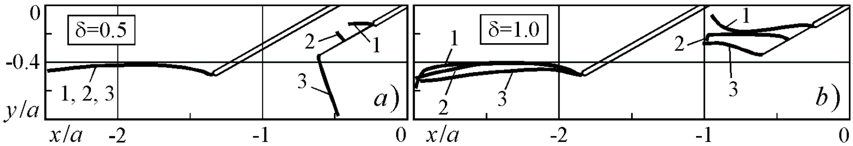


Fig. 6. Propagation paths of two non-equal parallel cracks depending on the first crack length; $f = 0.1$; $\varepsilon_2 = l_2/a = 1.0$;
$1 - \varepsilon_1 = l_1/a = 0.3$; $2 - \varepsilon_1 = 0/5$; $3 - \varepsilon_1 = 0.7$.

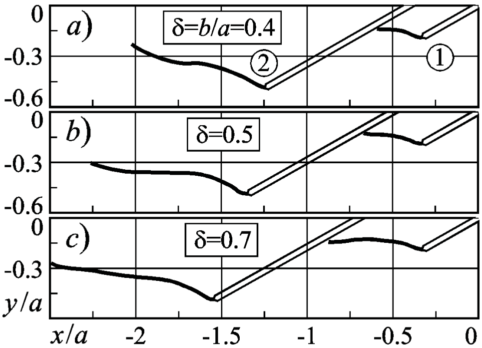


Fig. 7. Propagation paths of two non-equal cracks depending on the distance between them; $f = 0.05$; $\varepsilon_1 = l_1/a = 0.4$; $\varepsilon_2 = l_2/a = 1.0$.

The system of three edge parallel cracks is also considered. The propagation paths for this system are shows in Fig. 8. It has been established, that for the cracks both of the equal (Fig. 8a) and different (Fig. 8b) lengths, the cracks are growing as follows: the boundary crack propagates intensively, while the middle one propagates very slowly. In this case the SIF values along the propagation path are much lower then for the system of two cracks. For the first crack they are unstable, especially, when it approaches the neighbour crack tip.

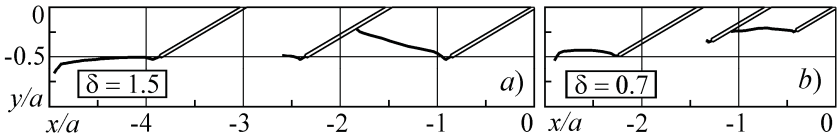


Fig. 8. Propagation paths of three equal (a) and non-equal (b) cracks:
$a - \varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 1.0$; $b - \varepsilon_1 = 0.5$; $\varepsilon_2 = 0.7$; $\varepsilon_3 = 1.0$.

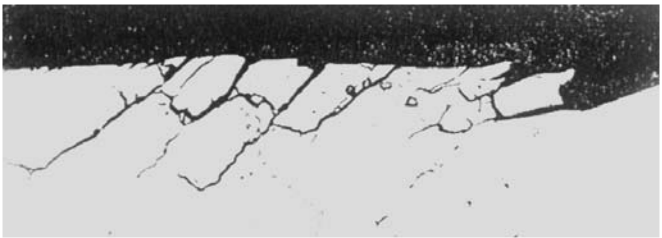


Fig. 9. Crumbling of the roller surface from the rail steel under contact fatigue testing [19].

Generally speaking, at smaller friction coefficients in the region of bodies contact the edge parallel cracks propagation established on the basis of the proposed model, shows a tendency to their joining and crumbling of the contact surface. This tendency is more pronounced for the system of non-equal cracks when the first crack is shorter during the counterbody motion. The obtained numerical results agree well with experimental data [19] (Fig. 9).

References

[1] O.P. Datsyshyn: Mat. Sci. № 6 (2005), p. 5

[2] V.Panasyuk, O.Datsyshyn, A.Glazov: Mashynoznavstvo (in Ukrainian) № 4 (2007), p. 3

[3] O.Romaniv, Ye.Shyr, V.Simin'kovich: Soviet Material Sci. № 2 (1983), p.111.

[4] P.E. Bold, M.W.Brown, R.J.Allen: Wear. Vol. 144 (1991), p. 307

[5] O.P.Datsyshyn, V.M.Kadyra: Int. J. Fatigue Vol. 28/4 (2006), p. 375

[6] O.P.Datsyshyn, V.V. Panasyuk: Wear vol. 251 (2001), p. 1347

[7] G.Donzella, M.Faccoli, A.Ghidini, A.Mazzu, R.Roberti: Eng.Fract. Mech. Is. 2 (2005), p. 287

[8] Engineering Fracture Mechanics Vol. 72, Is. 2 (2005), p. 163

[9] Fatigue & Fracture of Engineering Materials and Structures Vol. 26, № 10 (2003), p. 861

[10] V.V.Panasyuk, O.P.Datsyshyn, H.P.Marchenko: Eng. Fract. Mech. Vol. 52 (1995), p. 179

[11] Trybology 2001: scientific achievements, industrial applications, future challenges, edited by F.Franek, W.J.Bartz, A.Pauschitz, Vienna, The Austrian Tribology Society (2001), p 444

[12] W.R.Tyfour, J.H. Beynon, A.Kapoor: Wear Vol. 197 (1996), p. 255

[13] U.Zerbst, K.Madler, H.Hintze: Eng. Fract. Mech. Vol. 72, Is. 2 (2005), p. 163

[14] S.Way: J. Appl. Mech., Trans. ASME 2 (1935), p. A49

[15] K.J.Miller: London: Inst. Mech. Eng. (2001), p. 24

[16] Y.Murakami, C.Sakae, S.Hamada, in: Engineering Against Fatigue, edited by Beynon J. H., Brown M. W., Lindley et al. Rotterdam, Balkema Publ. (1999).

[17] V.V.Panasyuk, O.P.Datsyshyn, A.B.Levus, in: *Evolution of a system of edge cracks in the region of rolling bodies cyclic contact*, edited by A.Neimitz and al., Vol. I-III. Sheffield, UK, EMAS Publishing (2002).

[18] V.Panasyuk, O.P.Datsyshyn, H.P.Marchenko: Material Sci. № 1 (2001), p.1

[19] A.Mašin: Strojirenstvi . vol. 35, № 8 (1985), s. 447