Fatigue Crack Growth - As a Macroscopic Damage Process

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Abstract. The paper proposes a new model for the description of the damage process takes place during fatigue crack propagation. This is characterizable by failure of atomic bonds (or by the fracture of elementary tensile specimens at the crack tip), and by decreasing of load capacity because of the increases the crack length. From this it follows, that the model makes it possible to form a direct analytical relationship between the loading parameter (K, S, J or COD) and fatigue crack growth rate. The evaluation of the experimental results and literature sources proves the validity of the proposed model.

1. Introduction

Despite the fact, that the earlier paper related to the fatigue phenomena has been published more that 150 years ago and even in our days more than 10 papers/days appear, mostly of the failures are connected with fatigue crack growth, which is finished by break in two different parts of the constructional elements. Description of the fatigue problem at engineering point of view has always practical sense [1-10]. It is obvious that the description of the damage process takes part during the fatigue crack growth has an important role in reliability assessment of structural elements have crack like defects and loaded cyclically. Considering this requirement the following problems are erasing: How can be defined the lower and upper limits of the damage, and what kind of the damage accumulation process can be taken into account for description of the damage process?

2. Description of the damage process during fatigue crack growth

Let us denote the damage process by the d, which takes place during the extension of the fatigue crack. It is obvious that d = 1 at the moment of total failure, i.e. at the number of cyclic loading which cause the break of the structural element into two parts. The critical length of the crack is denoted by a_{crit} and the crack vicinity circumstances can be reflected by any parameters (K_{fc}, J_{fc}, S_{fc}) of fracture mechanics. This definition is the upper limit of the damage, which can be accepted from engineering point of view. The lower bound of the damage process in engineering approach is connected with the propagate-able length of the main crack, i.e. with a_{th}. The damage process (d) can be explained in the range of $a = a_{crit} - a_{th}$. Supposing that in this range of the crack length the L_0 number of elementary tensile specimens can be defined (see in Fig.1).

If the crack is extended than the "elementary tensile specimens are broken" and their "the still living number" will be decreased. The situation at a given crack length (a) are illustrated in Fig. 2.

The number of the living elementary tensile specimens depends on the number of loading cycle, i.e. L*(N). Denoting the ratio of the living L*(N), and the original (L_0) elementary tensile specimens by L(N), i.e. $L(N)= L^*(N)/L_0$ the temporary value of damage can be defined. Supposing that in a given cycle (N), the ratio of the elementary specimens which will be decreased is proportional with the existing one, it can be expressed on the following way:

$$-dL = L(N)\, m(N)\, dN \tag{1}$$

Where the sign of negative express the fact, that the ratio of the living tensile specimens decreases, and the m(N)is a function which has the following features: $0 < m(N) \leq 1$, i.e. its value shows the temporary value of damage.

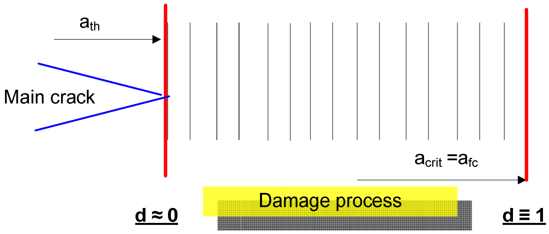

Fig. 1. Description of the damage process during fatigue crack extension

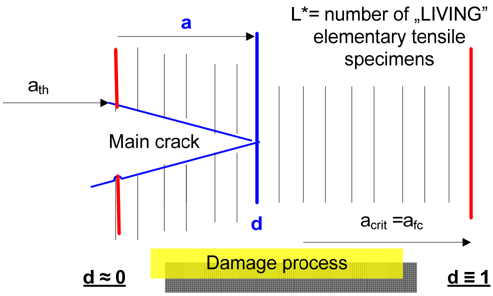

Fig. 2. Illustration of the damage process during the fatigue crack extension

The living ratio of the elementary tensile specimens can be calculated from the equation (1), i.e.

$$L(N) = \exp\left[-\int_{1/4}^{N} m(N)dN\right] \tag{2}$$

The damage value is

$$d = 1 - L(N) = 1 - \exp\left[-\int_{1/4}^{N} m(N)dN\right] \tag{3}$$

Considering the following boundary conditions:

- If N=1/4 (i.e. the first loading cycle has the maximum value), than $a = a_{th}$, and $d \approx 0$, and (4)
- If $N = N_{crit}$ (i.e. at the moment of fracture), than $a = a_{fc}$, and $d \equiv 0$, (5)

the most simple function which fulfil the equation of (3) is a power type one, i.e.

$$c(da/dN)^b \tag{6}$$

Combining the relationships of (3) and (6), the damage value at a given number of cycles (N) crack length (a), crack propagation rate (da/dN)

$$d = 1 - \exp\left[-c\left(\frac{da}{dN} \right)^{b} \right]. \tag{7}$$

These conditions can be accepted because of the crack propagation rate at the a_{th} value is approximately zero, i.e. $da/dN \approx 0$, at the a_{fc} the crack propagation rate is some order higher, i.e. it can be regarded as infinity in comparing it with the value at $a_{th.}$ The relationship (7) shows that the discrete values of the crack propagation rate follows the Weibull-distribution having two parameters.

The damage process takes place during the stable crack propagation can also be described by the changing of the load capacity. The simplest expression is also a power type one in the following form:

$$d = \left[\frac{K_{\max} - K_{th}}{K_{fc} - K_{th}} \right]^{nK} = \left[\frac{\Delta K - \Delta K_{th}}{\Delta K_{fc} - \Delta K_{th}} \right]^{n\Delta K} = \left[\frac{S_{\max} - S_{th}}{S_{fc} - S_{th}} \right]^{nS} = \left[\frac{J_{\max} - J_{th}}{J_{fc} - J_{th}} \right]^{nJ} \tag{8}$$

depending on the fracture mechanics parameter used for description of the crack vicinity area. The exponents of nK, nΔK, nS or nJ refer only to the description procedure of the crack vicinity area. From the above mentioned cases selecting the stress intensity amplitude type description and combining the expressions of (7) and (8), the fatigue crack growth rate can be described by the following expression:

$$\frac{da}{dN} = \left\{ -\frac{1}{c} \ln\left[1 - \left(\frac{\Delta K - \Delta K_{th}}{\Delta K_{fc} - \Delta K_{th}} \right)^{n} \right] \right\}^{\frac{1}{b}} \tag{9}$$

The expression of (9) contains 5 parameters. Two of them, the ΔK_{th} and ΔK_{fc} can be determined easily. For determination of the others the following procedure can be used:

a. Division of the $\Delta K_{fc} - \Delta K_{th}$ range for any parts (i= 1, …, n)

b. Reading the coordinates of $(\Delta K \text{ vs. } da/dN)_i$.

c. Calculating the b and c parameters of the equation of (7) by the procedures using for calculation of Weibull-distribution.

d. In order to calculating the exponent of n, the expression of (8) can be supplemented by a multiplier in the following way:

$$d = C_0 \left[\frac{\Delta K - \Delta K_{th}}{\Delta K_{fc} - \Delta K_{th}} \right]^{n} \tag{10}$$

The value of C_0 in principle has to be equal to the unit. The parameters b and C_0can be determined by using the linear regression.

The parameters for 100 different cases is summarised in Table 1. including steels with very different strengths, cast irons, maraging steels, rail steels, austenitic steels, tool steels, welded joints, weld metal, Ti-alloys, Mg-alloys, Al-alloys, etc. Taking into account the calculated values which are shown in the Table 1., it can be conclusions, as follows:

- The values C_0 of changes in the range of $0.973 \leq C_0 \leq 1.128$, the mean value of them is $\overline{C}_0$ = 1,029 and the value of standard deviation is S = $\pm$ 0.035. The value of the coefficient of variation is v= 100 S/$\overline{C}_0$ =2.7 %.
- The values of correlation coefficients (r) during determination of the parameters c and b changes in the range of 94.10 % $\leq$ r $\leq$ 99.42 %. The mean value $\bar{r}$ = 97.02 %, the standard deviation S = $\pm$1,27% and the coefficient of variation is v = 1,31 %.
- The correlation index at the calculation of the parameters n changes in the range of 96.64 % $\leq$ r $\leq$ 99.99 %, the mean value of them $\bar{r}$ =99.25 %, the standard deviation S = $\pm$0.69 % and the coefficient of variation is v = 0.69 %.
- The value of the exponent b in the equation (6) changes in the range of $0.292 \leq b \leq 0.79$.
- The value of the exponent in the equations (9) or 10) changes in the range of $0.35 \leq n \leq 0.88$.

3. Conclusion

Considering the aim of this paper, the following conclusions may be drawn:

1. A definition has been suggested for description damage process in the range of critical - threshold region during fatigue crack growth. The short crack problem has not been considered in this approach.
2. A damage accumulation model m(N) has been suggested.
3. The damage process can be characterised by
    a. Crack extension process
    b. Decreasing of load capacity
4. On the basis of the characterisation of the above mentioned damage process a fatigue crack growth rate law has been suggested.
5. The model has been proved for 100 different kinds of materials.

References

[1] Romvári, P. – Tóth, L., Analiz zakonomernoszti rasprostranenija usztalosztnykh treshhin v metallah. Problemy Prochnosti, Problemy Prochnosty, 1980/12. p.12-18.

[2] Jarema, S.Ja., Metodologija opredelenija kharakteristik soprotivlenija razvitiju treshhin (treshhinostijkosti) materialov pri ciklicheskom nagruzhenii. Fiz. Khim. Mech. Mat. 1984/4 p.100-110.

[3] Tóth, L., A fáradt repedés terjedési sebességének leírása a károsodási folyamat jellemzésével. Gép, 1981/7. p.257-262.

[4] Romvári P., Tóth L.: K voprosu povresdaemoszti pri rasprostranenii ustalostnykh treshhin. Mekhanicheskaja usztaloszt' metallov. Kiev. Naukova Dumka, 1983. p. 278-284.

[5] Tóth L.: Schädigungsprozess und Ermüdungsrißausbreitung. VI. Symp. "Verfor-mung und Bruch", Magdeburg, 1982. szeptember 7-9. Magdeburg, 1982. Teil 2. p. 1.

[6] Romvári P., Tóth L.: A Damage Accumulation Model for Description of Fatigue Crack Propagation. In "Basic Mechanisms in Fatigue of Metal", Editors: Lukás, P. Polák, J. Academia. Prága, 1988. p. 297-304.

[7] Romvári P., Tóth L., Kocsisné Baán M.: Az anyag károsodását jellemző paraméterek a fáradt repedés terjedése során. Gép, (33), 1981/9. p. 337-340.

[8] Tóth L.: Az anyagok fáradásos repedés terjedésével szembeni ellenállása. Gép, (41), 1989/4. p. 121-125.

[9] Tóth L., Kocsisné Baán M.: Fatigue Crack Growth and the Damage Factors of Materials. VIII. Congress on Material Testing, Budapest, 1982. Szeptember 28-Október 1. OMIKK-TECHNOINFORM, Budapest, 1982. p. 379-385. (magyar nyelven is)

[10] Tóth L.: Reliability Assessment of Cracked Structural Elements under Cyclic Loading. 41 p., in: "Handbook of Fatigue Crack Propagation in Metallic Structures" edited by A. Carpinteri, ELSEVIER (1994).

Table 1. The parameters of the fatigue crack growth circumstances characterized by relationship of (9) for 100 different kinds of materials

N^0	c	b	ΔK_{th} [MPa√m]	ΔK_{fc} [MPa√m]	n	C_0	Type of Material
1	101.77	0.459	5.30	40.0	0.575	1.008	St38
2	202.43	0.523	5.40	44.8	0.608	1.065	H 60
3	316.84	0.568	5.85	63.2	0.486	1.076	H 75/3
4	129.00	0.506	5.15	71.0	0.566	1.046	HSLA
5	144.63	0.506	5.40	71.0	0.523	1.091	NAXTRA 70
6	56.94	0.429	3.08	62.0	0.457	1.056	300 M
7	46.74	0.396	2.95	33.5	0.457	0.991	300 M
8	30.13	0.379	5.05	70.1	0.485	1.060	300 M
9	535.23	0.548	2.28	9.8	0.636	1.021	300 M
10	393.30	0.552	2.30	18.6	0.610	1.048	300 M
11	327.47	0.527	2.40	22.0	0.536	1.071	300 M
12	59.98	0.427	3.68	56.0	0.514	1.101	300 M
13	355.66	0.540	2.30	18.3	0.581	1.027	300 M
14	162.57	0.470	2.38	24.8	0.488	1.019	300 M

N^0	c	b	ΔK_{th} [MPa√m]	ΔK_{fc} [MPa√m]	n	C_0	Type of Material
15	40.06	0.398	3.03	64.0	0.438	1.012	300 M
16	24.05	0.357	3.00	73.0	0.491	1.012	300 M
17	55.03	0.430	3.58	55.5	0.554	1.080	300 M
18	854.00	0.616	2.32	18.4	0.661	1.109	300 M
19	665.16	0.592	2.46	21.0	0.546	1.032	300 M
20	217.36	0.504	2.45	25.0	0.534	1.063	300 M
21	57.19	0.448	3.01	63.0	0.533	1.049	300 M
22	27.68	0.384	5.10	75.0	0.505	1.043	300 M
23	106.98	0.455	1.04	23.0	0.522	1.106	Al-alloy
24	16.98	0.353	2.85	27.1	0.580	1.022	Al-alloy
25	114.84	0.498	2.32	20.0	0.611	1.128	Al-alloy
26	76.19	0.449	1.38	18.0	0.669	1.111	Al-alloy
27	109.50	0.443	1.20	7.80	0.713	1.010	Al-alloy
28	32.38	0.351	7.60	53.0	0.434	1.027	Rail steel
29	34.61	0.353	7.55	57.0	0.441	1.085	Rail steel
30	27.65	0.357	7.80	66.0	0.438	0.986	Rail steel
31	28.33	0.363	7.80	67.0	0.434	0.999	Rail steel
32	17.97	0.335	9.50	76.0	0.524	1.012	Welded joint
33	18.29	0.332	10.40	80.0	0.495	1.008	Welded joint
34	43.84	0.419	4.30	72.0	0.589	1.023	Welded joint s
35	36.46	0.458	7.30	77.0	0.623	0.978	Welded joint
36	122.88	0.485	5.90	46.0	0.524	0.973	St 52
37	90.87	0.457	4.80	52.0	0.535	0.992	Weld material
38	123.30	0.507	9.10	51.0	0.623	1.017	HAZt
39	185.21	0.508	6.90	43.0	0.596	1.030	HAZ
40	71.18	0.414	7.55	41.0	0.559	1.015	HAZ
41	110.22	0.477	5.30	42.0	0.578	0.974	Welded joint
42	14.36	0.304	3.80	33.2	0.476	1.000	Al-alloy
43	11.56	0.291	3.80	47.1	0.445	1.028	Al-alloy
44	50.70	0.473	3.70	26.0	0.877	1.001	40 CrSi
45	19.60	0.409	4.30	55.8	0.880	0.977	40 CrSi
46	19.60	0.409	4.30	68.2	0.707	0.974	40 CrSi
47	19.60	0.409	5.60	87.0	0.664	1.006	40 CrSi

N^0	c	b	ΔK_{th} [MPa√m]	ΔK_{fc} [MPa√m]	n	C_0	Type of Material
48	21.97	0.516	6.80	51.0	0.614	1.090	Ti-6Al-4V
49	7.44	0.396	9.30	112.0	0.600	1.003	Ti-6Al-4V
50	7.31	0.379	0.11	10.0	0.350	1.009	Ti-6Al-4V *
51	6.60	0.351	0.11	8.6	0.426	1.004	Ti-6Al-4V **
52	100.56	0.622	0.07	3.0	0.463	1.002	300 M *
53	831.08	0.790	0.11	2.3	0.469	1.017	300 M *
54	113.95	0.635	0.07	3.1	0.468	0.999	300 M *
55	43.82	0.578	0.11	6.9	0.422	1.040	HP 9-4-0.3 *
56	250.92	0.734	0.11	6.5	0.387	1.055	HP 9-4-0.3 *
57	89.49	0.562	0.08	6.4	0.389	1.051	HP 9-4-0.3 *
58	50.09	0.574	0.11	5.9	0.393	1.003	HP 9-4-0.3 **
59	283.26	0.721	0.09	6.1	0.356	1.069	HP 9-4-0.3 **
60	151.41	0.612	0.05	3.4	0.392	1.058	HP 9-4-0.3 **
61	11.92	0.445	0.05	4.4	0.440	1.005	Al-7075-T73 *
62	26.54	0.495	0.07	4.5	0.392	1.023	Al-7075-T73 *
63	13.34	0.429	0.04	4.0	0.402	1.012	Al-7075-T73 **
64	11.60	0.399	0.05	2.8	0.421	1.026	Al-7075-T73 **
65	15.41	0.437	0.05	4.8	0.389	1.029	Al-7075-T73 **
66	161.06	0.702	0.03	0.5	0.693	1.038	Al-2219-T85**
67	42.95	0.405	4.69	69.1	0.483	1.005	HSLA
68	922.86	0.614	2.96	26.0	0.567	0.997	HSLA
69	50.06	0.411	3.96	67.7	0.480	1.026	HSLA
70	56.72	0.441	4.28	69.1	0.521	0.991	HSLA
71	36.62	0.388	3.96	75.0	0.471	1.053	HSLA
72	60.51	0.444	5.42	69.1	0.549	1.004	HSLA
73	44.93	0.434	6.03	75.8	0.563	1.010	HSLA
74	46.46	0.432	4.80	74.1	0.549	1.019	HSLA
75	43.81	0.419	5.05	72.4	0.512	1.003	HSLA
76	10.42	0.336	1.70	13.2	0.635	1.013	Mg- alloy
77	10.42	0.336	1.80	16.2	0.656	1.017	Mg- alloy
78	10.42	0.336	2.05	19.0	0.654	1.035	Mg- alloy
79	12.25	0.337	2.25	19.5	0.567	1.027	Mg- alloy
80	12.25	0.337	2.05	21.7	0.434	1.012	Mg- alloy

N^0	c	b	ΔK_{th} [MPa√m]	ΔK_{fc} [MPa√m]	n	C_0	Type of Material
81	73.4	0.502	4.50	97.9	0.612	1.008	NK (tool steel)
82	110.9	0.521	4.50	62.0	0.611	0.998	Cr 13 (tool st.)
84	267.08	0.363	9.50	31.0	0.709	1.003	Al- alloy
85	277.75	0.366	8.00	53.0	0.547	0.999	Al- alloy
86	515.15	0.365	2.40	61.0	0.382	1.065	HT 80
87	313.37	0.338	2.49	85.0	0.835	1.005	HT 80
88	780.08	0.396	7.79	88.0	0.354	0.998	HT 80
89	265.80	0.326	2.30	98.0	0.302	1.004	HT 80
90	702.78	0.560	17.00	92.8	1.056	0.995	Ti- alloy (BT3-1)
91	702.78	0.560	13.0	57.0	1.010	1.009	Ti- alloy BT-25)
92	250.85	0.603	7.50	88.0	0.709	1.005	18/8
93	250.31	0.600	5.10	75.0	0.660	1.002	18/8
94	792.43	0.595	2.70	12.5	1.227	0.999	Cast iron
95	364.59	0.556	5.30	24.5	0.749	1.003	Cast iron
96	144.20	0.508	9.50	36.0	0.700	1.005	Cast iron
97	701.81	0.828	14.90	54.6	0.545	1.001	Cast iron
98	693.50	0.802	16.20	40.3	0.659	0.999	Cast iron
99	196.11	0.674	13.30	42.5	0.811	1.002	Cast iron
100	1385.4	0.885	13.00	33.1	1.054	1.001	Cast iron

Notes: * instead of ΔK the $\mathbf{\Delta S}$ - kN/m (stain energy density),

** $\alpha\Delta \mathbf{S}$-kN/m, where α =const. characterises the loading type