Estimation of apparent fracture toughness of ceramic laminates based on generalized strain energy density factor

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Abstract. The paper presented deals with fracture behavior of ceramic laminates. Assumptions of the linear elastic fracture mechanics and small scale yielding are considered. In this frame the residual stresses in individual layers of Al_2O_3/5vol.%t-ZrO_2 (ATZ) and Al_2O_3/30vol.%m-ZrO_2 (AMZ) are determined. The procedure based on generalization of Sih's strain energy density factor to the case of a crack touching the interfaces between two dissimilar materials is used for determination of effective values of stress intensity factor at material interfaces. Important increase of the fracture toughness at AMZ/ATZ interface was predicted in comparison to the fracture toughness of individual material components. Predicted values were compared with data available in literature and mutual good agreement was found. Procedure suggested can be used for estimation of resistance to crack propagation of multilayered structures and its design. The procedure can contribute to enhancing of reliability and safety of the structural ceramic or generally of layered composites with strong interfaces.

Introduction

In last decades, remarkable advances have been achieved in improving the mechanical behaviour of ceramic material. Recently new strategies fundamentally different from the conventional approach (high homogeneity ceramics with very small flaws) have emerged aiming to achieve "flaw tolerant" materials by designing special microstructures that improve the toughness (or apparent toughness) of ceramics.

"Flaw tolerant" ceramic. One of the most attractive proposals for this latter approach consists of layered architectures that combine materials with different properties. As a result, laminates with mechanical behaviour superior to that of the individual constituents can be fabricated. In this regard, different fracture mechanics approaches have been attempted for the design of ceramic layered composites [1].

In particular, ceramic composites with a layered structure such as alumina-zirconia and mullite-alumina among others, have been reported to exhibit an increased apparent fracture toughness and energy absorption as well as noncatastrophic failure behaviour. For strongly bonded multilayers the elastic mismatch during sintering between adjacent layers, resulting from the difference in Young's moduli, thermal expansion coefficients, chemical reactions and/or phase transformations, generates residual stresses throughout the material. These residual stresses can be controlled in order to improve mechanical properties. Laminar ceramics designed with compressive stresses in the bulk may present a threshold strength below which catastrophic failure does not occur [2].

From mentioned perspectives, zirconia-containing laminar ceramics have been employed to develop compressive stresses in the internal layers by means of the tetragonal to monoclinic phase transformations that takes place when cooling down during sintering [2]. Under certain conditions, these compressive stresses may act as barrier to crack propagation. In other cases, crack bifurcation and/or deflection phenomena result in an increase of the material fracture toughness and energy absorption capability [3]. Procedure of preparation such composites and typical crack behavior can be find elsewhere [1,4-8].

Al_2O_3-ZrO_2 multilayred system

Paper presented deals with an estimation of apparent fracture toughness of a ceramic laminate or generally laminates made from materials with strong interfaces. As a typical example Al_2O_3-ZrO_2 laminate can be mentioned. Assumptions of the linear elastic fracture mechanics (LEFM) and small scale yielding are considered. For next considerations about estimation of apparent fracture toughness the Al_2O_3-ZrO_2 ceramic laminate can be used as an example.

Material characteristics. Material characteristics and geometry of considered composite body were taken from references [2,9] to make comparison with published data. The composite studied was composed from nine layers of Al_2O_3/5vol.%t-ZrO2 (alumina with tetragonal zirconia, noted as ATZ) and Al_2O_3/30vol.%m-ZrO2 (alumina with monoclinic zirconia, noted as AMZ), see Fig. 1. The particle size of individual material components was about 0.3 μm [2].

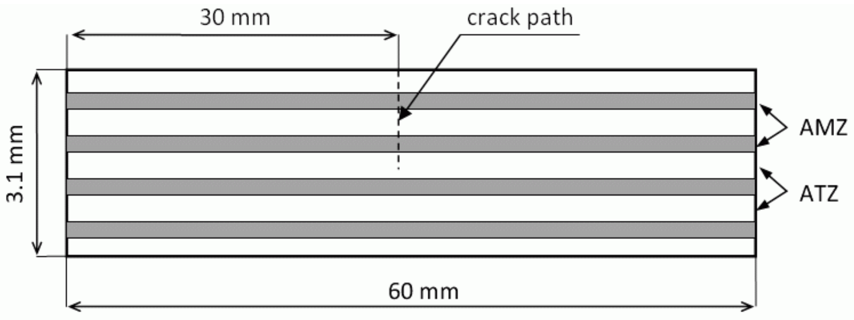

Fig. 1. Considered Al_2O_3-ZrO_2 ceramic laminate

The thickness of ATZ layers was considered as t_{ATZ} = 0.52 mm and thickness of AMZ t_{AMZ} = 0.1 mm. All material properties used for simulations are summarized in Table 1.

Table 1. Material properties of Al_2O_3-ZrO_2 laminate [2,9]

Property	Units	ATZ	AMZ
Young's modulus E	GPa	390	280
Poisson's ratio ν	-	0.22	0.22
Coefficient of thermal expansion α_t	$10^{-6}\cdot K^{-1}$	9.82	8.02
Fracture toughness K_{IC}	$MPa\sqrt{m}$	3.2	2.6
Layer thickness t	mm	0.52	0.1

Determination of residual stresses. The studied type of laminate is prepared by sintering and mainly due to different coefficients of thermal expansion of used ceramics, layers contain rather high compressive and tensile residual stresses, which significantly influence the fracture behaviour of the laminate. Residual stresses that develop during the sintering process were determined by numerical calculations in commercial code ANSYS 11.0 as 2D calculation with approximation of plane strain conditions. As residual stress free temperature was considered sintering temperature

1250°C. Than was the composite specimen subjected to cooling to room temperature (20°C). Resultant residual stresses in individual layers of composite are shown in Fig. 2.

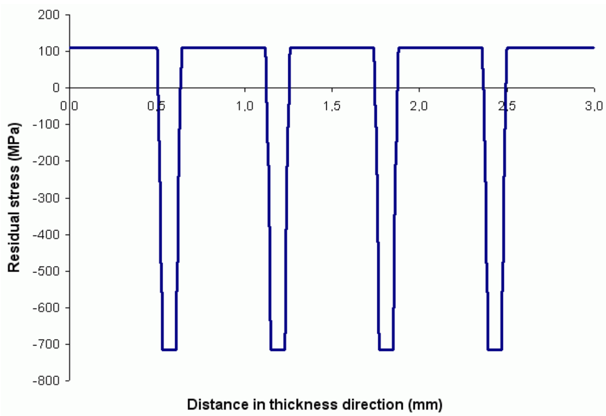

Fig. 2. Resultant residual stresses through the layers of composite

In ATZ layers the tensile residual stresses of value 110 MPa were found out and in AMZ layers compressive stresses of value -715 MPa were numerically determined. These results are in good agreement with early mentioned works [2,9].

In next investigations, the presence of crack in laminate was assumed. The edge crack growing from surface through the first four layers of laminate (to the middle of specimen) was considered, see Fig. 1. Values of stress intensity factor K_{Ires} were determined for different crack length. This procedure is usually called K-calibration. The specimen was loaded by thermal residual stresses only. Results from calculations are shown in Fig. 3.

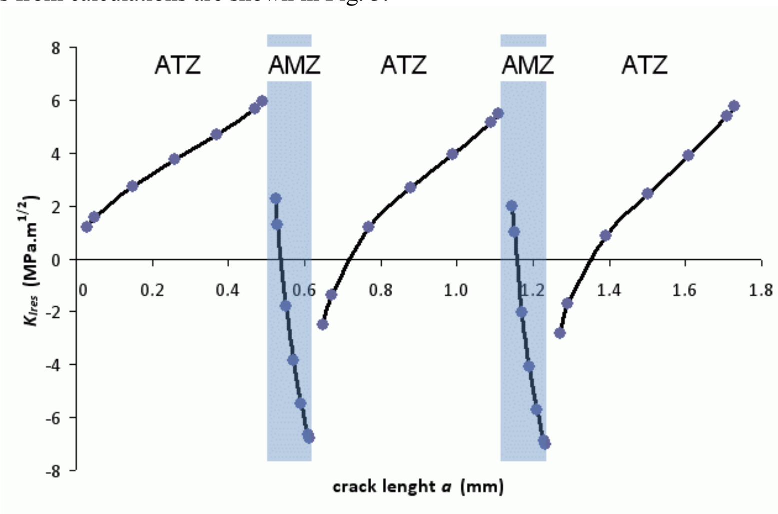

Fig. 3. Values of stress intensity factor K_{Ires} caused by thermal residual stresses calculated along crack path leading through the thickness of the specimen from surface to the middle

Positive values of K_{Ires} are in tensile loaded layers of ATZ. In ATZ cracks propagate easy due to existence of tensile residual stresses there. Negative values of K_{Ires} in Fig. 3 are physically impermissible, but right interpretation of this fact here is, that crack is closed and crack faces are loaded by pressure keeping crack closed. For crack propagation an additional remote tensile loading is requested.

Values of apparent fracture toughness can be calculated from following expression

$$K_{apt,c} = K_{Ic} - K_{Ires}, \tag{1}$$

where K_{Ic} is the fracture toughness of each individual layer, see Table 1. Apparent fracture toughness of multilayer laminate is shown in Fig. 4.

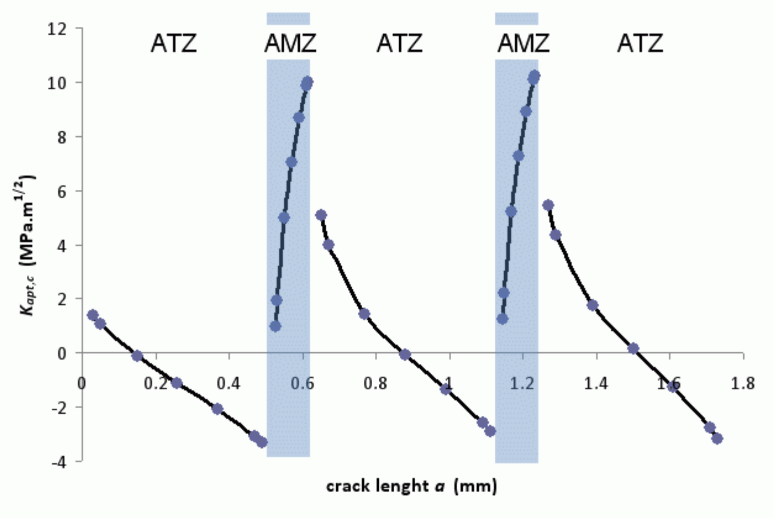

Fig. 4. Apparent fracture toughness $K_{apt,c}$ considering residual stresses in the laminate

Negative values of $K_{apt,c}$ represent intervals, where loading of layers due to residual tensile stresses exceed fracture toughness K_{Ic} of individual layers and crack propagate without additional external loading. The negative values of $K_{apt,c}$ have no other physical meaning. Positive values of $K_{apt,c}$ represent fracture toughness increased by the compressive residuals stresses and presence of interfaces between two dissimilar materials.

Apart from intervals without apparent fracture toughness in Fig. 4, there are material interfaces, where fracture toughness cannot be determined by classical approaches of LEFM due to different stress singularity of a crack touching the interface. In this case the stress singularity exponent lies generally in the interval $0 < p \neq 1/2 < 1$. The apparent fracture toughness cannot be calculated directly due to different stress singularity exponents and a special approach taking into account this fact should be applied for estimation of apparent fracture toughness $K_{apt,c}$ at interfaces.

Crack touching the interface

The knowledge of $K_{apt,c}$ at interfaces allows determine fracture toughness for all considered crack length and estimate the apparent fracture toughness of all laminate, because extreme (minimal or maximal) values of $K_{apt,c}$ can be found close to the interfaces. Thus $K_{apt,c}$ values at interfaces play key role in fracture behaviour of such laminates and its determination is important for description of such behavior.

For estimation of $K_{apt,c}$ values the procedure based on generalized strain energy density factor can be used [10].

Strain energy density concept. The strain energy density concept in fracture mechanics was originally proposed for a crack in homogeneous material (i.e. for power of singularity p = 1/2). On the basis of strain energy density Sih [11,12] introduced strain energy density factor S, which is independent on distance r ahead of crack tip:

$$S = a_{11}K_I^{\ 2} + 2a_{12}K_I K_{II} + a_{22}K_{II}^{\ 2}, \tag{2}$$

where K_I, K_{II} are stress intensity factors corresponding to mode I, II of loading and a_{11}, a_{12} and a_{22} are known functions. Note that S can be related to K_I when only mode I of loading is considered

$$S = K_I^2 \frac{1-2\nu}{4\mu}, \tag{3}$$

where μ is shear modulus of material.

Generalization of strain energy density factor S for V-notches, i.e. for sharp stress concentrator with stress singularity different from ½, was suggested in works [13,14]. In the works [10,15] was suggested generalization of S for crack touching the interface between two materials. The generalized strain energy density factor is defined as

$$\Sigma(r,\theta) = r^{1-2p_1} A_{11}H_1^{\ 2} + r^{1-p_1-p_2} 2A_{12}H_1H_2 + r^{1-2p_2} A_{22}H_2^{\ 2}, \tag{4}$$

where H_1, H_2 [MPa.m^p] are generalized stress intensity factors and A_{11}, A_{12} and A_{22} are known functions, see [15] for details. Distance r ahead of crack tip corresponds to mechanism of failure. On the base of idea of equality $S(K_I) = \Sigma\,(H_I)$, is possible to find relation between H_I and K_I, for details see e.g. [10]

$$K_{I,eff} = \left(\frac{1-2\nu}{(1-p)^2\left(4(1-2\nu)+(g_R-p)^2\right)} \right)^{-\frac{1}{2}} r_c^{\frac{1}{2}-p} H_I\,. \tag{5}$$

The critical distance r_c in equation (5) can be estimated from relation (see e.g. [13,16])

$$r_c = \frac{K_{Ic}^2}{2\pi\sigma_f^2}\,. \tag{6}$$

σ_f is ultimate strength of material where the crack will propagate. The influence of critical distance r_c on values of $K_{I,eff}$ is very week and is sufficient to determine only order of magnitude of the critical distance. Finite element calculations were performed to determine corresponding H_I values at interfaces and relations (5) and (6) were used for estimation of $K_{I,eff}$. Constants included in equation (5) for individual interfaces are written in Table 2, see e.g. [10] for details.

Table 2. Constants of analytical solution used in equation (6)

type of interface	p (-)	ν (-)	g_R (-)	r_c (mm)
ATZ/AMZ	0.53609	0.22	0.54513	1.5E-05
AMZ/ATZ	0.46452	0.22	0.47242	7.0E-06

$K_{apt,c}$ values are than given by expression

$$K_{apt,c} = K_{Ic} - K_{I,eff} \,. \tag{7}$$

Resultant values of $K_{apt,c}$ obtained for interfaces are introduced in Table 3.

Table 3. Calculated apparent fracture toughness at interfaces

Crack length a (mm)	$K_{I,eff}\left(\mathrm{MPa}\sqrt{\mathrm{m}}\right)$	$K_{apt,c}^{\mathrm{interface}}\left(\mathrm{MPa}\sqrt{\mathrm{m}}\right)$	[1] $K_{apt,c}^{\mathrm{interface}}\left(\mathrm{MPa}\sqrt{\mathrm{m}}\right)$
0.52 (ATZ/AMZ interface)	2.48	0.12	-
0.62 (AMZ/ATZ)	-4.78	7.98	7.1
1.14 (ATZ/AMZ)	2.22	0.38	-
1.24 (AMZ/ATZ)	-5.08	8.28	8.1

[1] Values published in reference [2] predicted analytically through the weight function analysis for room temperature

Calculated results and results obtained by analytical solution through weight function analysis introduced in [2] for very similar geometry and conditions are evidently in good agreement.

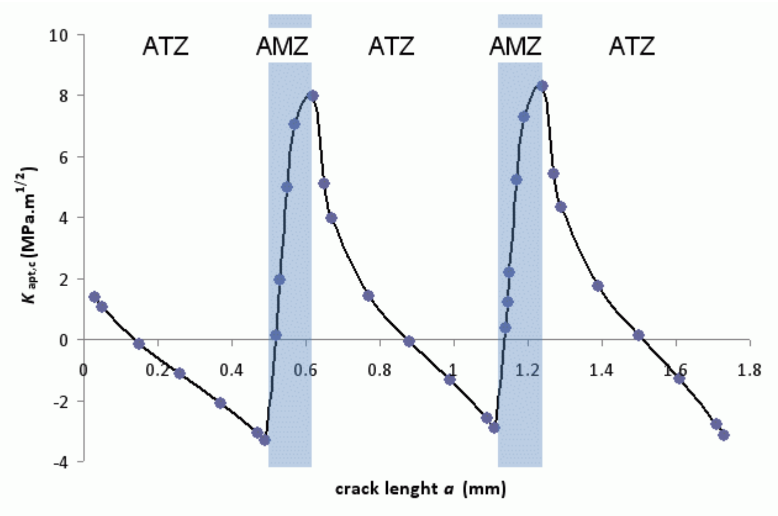

Fig. 5. Apparent fracture toughness of Al_2O_3-ZrO_2 laminate

For similar material configuration and conditions maximal value of $K_{apt,c}$ of AMZ/ATZ interface 8.6 $\mathrm{MPa}\sqrt{\mathrm{m}}$ was found in literature, see [1] for details.

In Fig. 5 is shown dependence of apparent fracture toughness $K_{apt,c}$ of Al_2O_3-ZrO_2 laminate on crack length considering residual stresses. In ATZ layers are due to high tensile stresses regions, where K_{Ires} exceed fracture toughness of ATZ and therefore the apparent fracture toughness in these regions is out of practical meaning. Extreme values can be found at material interfaces, which determine fracture properties of studied laminate. Fracture toughness of AMZ/ATZ interface is significantly higher in comparison to fracture toughness of individual layers. In ATZ layer propagating crack can bifurcate in slim AMZ layer or can stay arrested in front of AMZ/ATZ interface. These mechanisms increase apparent fracture toughness of studied laminate. The values of apparent fracture toughness in studied case are quite independent on absolute crack length in comparison to cracks in homogeneous materials, because the opening of crack faces is strongly influenced by presence of compressive residual stresses in AMZ layers. $K_{apt,c}$ values are influenced mainly by residual stresses and distance of the crack tip from interfaces.

Conclusions

Paper presented is focused on "flaw tolerant" ceramic laminate. In the frame of the linear elastic fracture mechanics residual stresses in layers of Al_2O_3-ZrO_2 composite were numerically determined. So-called *K*-calibration was done for a crack growing through the thickness of laminate specimen. The concept of generalized strain energy density factor was used for estimation of effective values of stress intensity factor at material interfaces. Then the apparent fracture toughness (with influence of residual stresses and interfaces) was determined. The interface values of the fracture toughness are crucial for estimation of fracture behavior of ceramic laminate. Important increase of fracture toughness at AMZ/ATZ interface was predicted in comparison to toughness of individual material components. Predicted values were compared with data available in literature and mutual good agreement was found.

Procedure suggested can be used for estimation of resistance to crack propagation of multilayered structures and its design. The procedure can contribute to enhancing of reliability and safety of structural ceramic or generally of the layered composites with strong interfaces.

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