Engineering Methods For Stress Intensity Factor Calculation And Their Practical Application

I.V. Orynyak

Institute for Problems of Strength, Tymiryazevska str 2, Kiev, Ukraine

or@ipp.kiev.ua

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Abstract. In this work we outline two engineering methods which are developed in the Institute for Problems of Strength. The point weight function method makes easy the accounting for the different laws of loading (stress gradient) of crack surfaces of the given body with crack. The second one is well-known crack compliance method which can be very useful in accounting for the change of the body's geometry. Using the known SIF values for infinite strip with the edge crack we obtained the ready to use formulas for SIF calculation in cracked cylinders and toroidal shells. The examples of practical application of these methods for nuclear pressure vessels and transit pipeline are given.

Introduction

Stress intensity factor (SIF) for a mode I crack, or K_I, is the key parameter for analysis of integrity and life time of structural elements with defects. Most often the SIF calculations are performed by FEM which requires a lot of time. In practical applications the crack dimensions as well as the loading conditions can hardly be unambiguously defined, thus multivariant calculations should be performed. In such cases the analytical formulas for SIF, which were constructed based on the analytical as well as numerical solutions, are the most useful for engineering applications. In spite that they are widely presented in the literature, they can not cover all the diversity of the real cases. Furthermore, the law of loading or the body geometry for the case of interest may be slightly differing from the basic one, the SIF solution for which is easily available and understandable. Is it worth to spend the time in searching the correct solution or we can use the basic solution? With this connection we can put some additional questions. What are the geometrical and loading parameters which predetermine the difference? Can we take them into account to construct the more accurate solution for the case of interest? As example, consider the pipe bend (toroidal shell). It is well known that the stress distribution patterns in it and in the straight pipe can differ significantly when the global bending moment is applied. Nevertheless, it is widely accepted that the SIF values for these two cases are the same if the cracked sections in both geometries are loaded by the same stress distribution.

Two types of structural elements are usually considered: thick walled and thin walled. For thick walled one the crack is considered to be the elliptic or part elliptic form. For an elliptic crack in an infinite body there is a strict analytical expression for SIF:

$$K_{10}(\theta) = \frac{\sigma_0\sqrt{\pi a}}{E(k)}\left[\frac{\sin^2\theta + \lambda^4\cos^2\theta}{\sin^2\theta + \lambda^2\cos^2\theta}\right]^{1/4} = \frac{\sigma_0\sqrt{\pi a}}{E(k)}\Pi^{1/4}(\theta) \tag{1}$$

where θ is the crack contour point angular coordinate, σ_0 are the uniform normal stresses acting on the crack surface, a is the crack semi depth, b is the crack semi length, $b \geq a$, $E(k)$ is the full elliptical integral of the second kind, where $k^2 = 1-(a/b)^2$. Usually the semi-elliptical cracks are considered. The SIF values for them were calculated numerically and for uniform loading approximated by the expressions of the following type [1]:

$$K_I = K_{10} \cdot f_{se} \cdot f_{op} \tag{2}$$

f_{se} is the correction on the semielliptical form, f_{op} is the correction on the proximity of the opposite body surface. These ready to use formulas of the above type are well established in the literature. The problems begin when the stress distributions are nonuniform, especially when the jumps of stresses on the crack surface take place. The example is the surface semielliptical crack in the reactor pressure vessels with cladding. To obtain the SIF value the boundary problem of mechanics of solids is usually solved by FEM. The point weight function method, which will be described below is a reasonable alternative to it.

The thin-walled elements are usually treated by the shell theory, thus the stress distribution has two components: the uniform one due to the resultant membrane force, N, and the linear one due to the resultant couple, M. Here, the practical significance have the long surface cracks, the length of which is much bigger than the wall thickness of shell, t. The starting point for a subsequent analysis is the solutions for the infinite strip with an edge crack, which are known (or should be known) for every structural integrity specialists:

$$K_{IN} = \sqrt{\pi a}\sigma_N Y_N(\alpha); \quad K_{IM} = \sqrt{\pi a}\sigma_M Y_M(\alpha) \tag{3}$$

where stresses $\sigma_N = N/t$ and $\sigma_M = 6M/t^2$, $\alpha = a/t$ is a relative crack depth, the dimensionless SIFs $Y_N(\alpha)$ and $Y_M(\alpha)$ are well known and tabulated in the various handbooks [2, 3]. The aim of the work is investigation of the influence of the body geometries (straight pipe, pipe bend) on the above SIF values. Especially we are interested when the influence of the change of the body geometry is significant and when it can be neglected.

Point Weight Function Method

The idea of PWFM. The weight function $W_{QQ'}$ is determined as the value of SIF in crack contour point Q' when the two unit concentrated forces, P are applied to the opposite crack surfaces in the same arbitrary point Q [4]. Thus, according to the above definition, the SIF value $K_{IQ'}$ in point Q' for any distributed symmetrical loading $q(S)$ can be found by relatively simple integration over the crack surface S:

$$K_{IQ'} = \iint_{(S)} W_{QQ'}(Q) q(Q) dS \tag{4}$$

where dS is the element of crack surface area. Thus availability of $W_{QQ'}$ excludes the necessity of the solution of the boundary problem. The idea of point weight function method consists in construction of an explicit expression for WF and is described in details in our works [5-7]. According to it the looking for WF for part- or elliptic crack for arbitrary 3D body, $W^S_{QQ'}$, is presented as a sum of an asymptotical, $W^A_{QQ'}$, and correction, $W^C_{QQ'}$, components:

$$W^S_{QQ'} = W^A_{QQ'} + W^C_{QQ'} \tag{5}$$

For an elliptic crack the value of $W^A_{QQ'}$ is chosen to be equal to the fundamental solution (WF) for the elliptic crack in the infinite body, the possible approximate expression for it was suggested in our work [7]:

$$W^A_{QQ'} = \frac{2\Pi^{1/4}(\theta)}{\sqrt{\pi a} l^2_{QQ'}} \left(1 - \frac{r^2(\varphi)}{R^2(\varphi)}\right)^{-0.5} \left(\int_\Gamma \frac{d\Gamma}{l^2_{QQ'}}\right)^{-1} \quad (6)$$

where r, φ are polar coordinates of the arbitrary point, $d\Gamma$ is the elementary crack contour length, $l_{QQ'}$ is the distance between points Q and Q'. For the quarter- and semi- elliptic crack, as it was shown in [5] it is needed to introduce the fictitious forces which are symmetrical to the given ones with respect to the free surface.

The correction function, $W^C_{QQ'}$, should be neglectfully small with respect to the asymptotical one, $W^A_{QQ'}$, when the point of the force application, Q, approaches to the crack contour. It is convenient to take $W^C_{QQ'}$ in the form Q:

$$W^C_{QQ'} = W^A_{QQ'}(1 - r(\varphi)/R(\varphi)) \cdot D(\theta, \lambda) \quad (7)$$

where $D(\theta)$ is the only unknown function in the PWFM.

Thus, WFs for elliptic, semi- and quarter- elliptic crack is approximated by the following expressions, respectively:

$$W^S_{QQ'} = W^A_{QQ'}(1 + D(\theta)(1 - r/R)) \quad (8)$$

$$W^S_{QQ'} = (W^A_{QQ'} + W^A_{Q_xQ'}) \cdot (1 + D(\theta)(1 - r/R)) \quad (9)$$

$$W^S_{QQ'} = (W^A_{QQ'} + W^A_{Q_xQ'} + W^A_{Q_yQ'} + W^A_{Q_{xy}Q'}) \cdot (1 + D(\theta)(1 - r/R)) \quad (10)$$

where Q is the point of force application; Q_x, Q_y, Q_{xy} are the points symmetrical to Q with respect to the axis x, y иand the origin of coordinates, respectively (see, Fig 1).

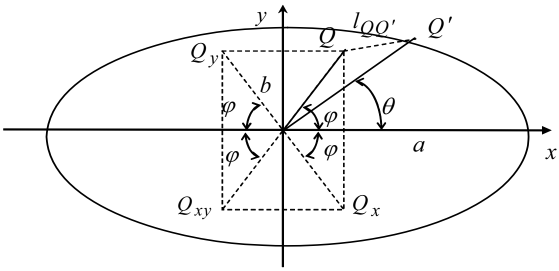


Fig 1. The scheme of the elliptic crack

The determination of the unknown function $D(\theta)$. For the given geometry of the body the function $D(\theta)$ can be determined given for some law of crack surface loading, $\sigma_r(x, y)$, the solution for SIF is known (so called reference solution K_{Ir}). In most cases in the literature the polynomial laws of the loading distribution are considered:

$$\sigma_{ij}(y,x) = q(Q) = c_{ij}(y/a)^i (x/b)^j \tag{11}$$

where c_{ij} are the known coefficients. Substitute Eq (11) and (5) into (4) and accounting the above expressions for asymptotical and correction components, we obtain the expressions for dimensionless SIFs, G_{ij}, (the influence coefficients):

$$G_{ij}(\theta,\lambda) = \frac{K_{1ij}(\theta,\lambda)E(k)}{c_{ij}\sqrt{\pi a}\Pi^{1/4}(\theta)} = I_{ij}^A(\theta,\lambda) + I_{ij}^C(\theta,\lambda)\cdot D(\theta) \tag{12}$$

where the following integrals are designated as dimensionless coefficients I_{ij}^A and I_{ij}^C:

$$I_{ij}^A(\theta,\lambda) = \frac{E(k)}{\sqrt{\pi a}}\Pi^{-0.5}(\theta)\iint_{(S)}(y/a)^i (x/b)^j W_{QQ'}^A dS \tag{13}$$

$$I_{ij}^C(\theta,\lambda) = \frac{E(k)}{\sqrt{\pi a}}\Pi^{-0.5}(\theta)\iint_{(S)}(y/a)^i (x/b)^j W_{QQ'}^C dS \tag{14}$$

Given the solution for G_{mn}, i.e. the SIF at $\sigma_r(x,y) = \sigma_{mn}$ (see, eq (11)), the twice application of (12) initially allows to find the function $D(\theta)$ and after to calculate the value of G_{ij} at any other law of loading $\sigma = \sigma_{ij}(x,y)$:

$$G_{ij}(\theta,\lambda) = I_{ij}^A(\theta,\lambda) + \left(G_{mn}(\theta,\lambda) - I_{mn}^A(\theta,\lambda)\right)\cdot I_{ij}^C(\theta,\lambda)/I_{mn}^C(\theta,\lambda) \tag{15}$$

Formula (15) presents the essence of practical application of PWFM. Note that $G_0 = f_{se}\cdot f_{op}$ (see eq 2) is usually taken as the reference solution. The error imposed by the method is in the most cases equal to 3-5% and is within the error of determination of the reference solution by FEM. Note that functions $I_{ij}(\theta,\lambda)$ do not depend upon absolute crack dimensions, they can be found in advance and can be presented by the analytical expressions [7].

The example of probability of fracture calculation based on Master Curve conception. Analysis of PTS (pressurized thermal shock event) in the reactor pressure vessels with a hypothetic crack requires a lot of calculations for different crack dimensions and various loading conditions. When considering the crack in the reactor with austenitic cladding with thickness, h, we need to account for the jump in the stress distribution at the cladding boundary. In this case we consider one additional law of loading:

$$\sigma_{jump} = \begin{cases} 1, & 0 \le y < h \\ 0, & h \le y \end{cases} \tag{16}$$

The polynomial laws (11) together with the “jump” law (15) can approximate any real stress distribution. Note, that the coefficients $I_{jump}(\theta,\lambda)$ can also be calculated in advance, and this is already realized in Ukrainian Recommended Practice for PTS analysis.

The PWFM can be efficiently used in calculation of fracture probability based on the Master Curve concept. This concept is widely used for the determination of the fracture toughness on the small specimens. In the Russian document on the reactor life time determination [8] the next logical step was made – if some methodology is applied for the specimen then this one can be applied for the structural element too. Thus the probability of fracture, P_{f_i}, of each elementary length of crack contour, ΔB_i, can be calculated by the following Weibull-type probability equation:

$$P_{f_i} = 1 - \exp\left(-\Delta B_i \big/ B_0 \times \left((K_{I_i} - K_{\min}) \big/ (K_0(T) - K_{\min}) \right)^b \right) \tag{17}$$

where $K_{\min} = 20MPa$, $B_0 = 25mm$, $b = 4$ and the fracture toughness curve with probability of fracture $P_f = 63.2\%$ is given by the following expression:

$$K_0 = 31 + 77 \cdot \exp(0{,}019(T - T_0 - \Delta T_b)) \tag{18}$$

where T_0 and ΔT_b are the material constants for the given condition of operation. Thus for the given crack in the real conditions of operation for each moment of PTS event the probability of fracture can be calculated. As an example, the relative (divided on the maximal value) fracture probability $\overline{P}_{f_i}$ distribution along the crack front for WWER-1000 is presented on the Fig2.

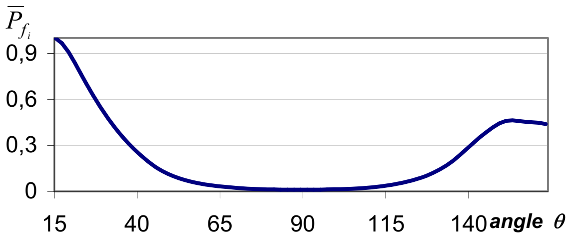


Fig 2. The distribution of the relative probability of fracture along the crack contour

Crack Compliance Method

The idea of CCM. This method is applicable only to 2-D problems, when the length of crack under consideration is long. The idea and practical implementation of the method belong to Cheng and Finnie [9]. According to it the crack can be considered as concentrated compliances (jumps in displacements, Δu, and rotation angle, $\Delta\theta$, take place in the section with a crack); while the rest part of a body can be treated by the beam-likes methods. Consider that the stress distribution in the cracked section in case of zero crack depth can be presented as $\sigma_q(y) = q \cdot \overline{\sigma}_q(y)$, where q is an intensity of this stress and $\overline{\sigma}_q(y)$ is the unitary stress distribution in y-direction (along the wall thickness). Then due to the above jumps in the cracked section the continuity (or boundary conditions) of the beam-like body would be formally violated if it were not for the additional bending moment M and longitudinal force N. As it was shown in our paper [10] for the thin-walled structures to be considered below the jumps in the displacement Δu can be neglected and the expressions for $\Delta\theta$ has the form [9]:

$$\Delta\theta = 6\pi \big/ E' (\beta_q q + \beta_M \sigma_M + \beta_N \sigma_N) \tag{19}$$

where β_i are the compliance coefficients for any type of loading designated by subscript "i" [9] and E' is the generalized Young modules:

$$\beta_i(\alpha) = \int_0^{\alpha} \alpha Y_M(\alpha) Y_i(\alpha) d\alpha \tag{20}$$

The coefficients $\beta_i(\alpha)$ can be easily integrated numerically and presented in the analytical form:

$$\beta_N = \alpha^2 \cdot (1-\alpha)^{-2}(0.628 - 1.45\alpha + 2.49\alpha^2 - 2.30\alpha^3 + 0.843\alpha^4) \tag{21}$$

$$\beta_M = \alpha^2 \cdot (1-\alpha)^{-2}(0.627 - 2.01\alpha + 3.68\alpha^2 - 3.57\alpha^3 + 1.35\alpha^4) \tag{22}$$

According to the principle of superposition the resulting SIF due to initial stress σ_q and additional stress σ_N and σ_M can be written:

$$K_I = K_{IN} + K_{IM} + K_{Iq} \tag{23}$$

Investigate the general form of presentation of the resulting K_I. First of all note, that as it was shown in [10] for the geometries to be considered below we can neglect the contribution from the longitudinal force N. Besides, all additional forces arise *only* due to the jump of the angle $\Delta\theta$ and should be proportional to it. Accounting for (19) and considering that additional σ_M is such that it reduces both the $\Delta\theta$ and SIF as compared with σ_q we have:

$$Z(\vec{G})\sigma_M = E'\Delta\theta/(6\pi) = (\beta_q q - \beta_M \sigma_M) \tag{24}$$

where $Z(\vec{G})$ is some function of the geometry of the body, $\vec{G}$. The determination of this function is a key problem in CCM. Expressing the σ_M from (24) and substituting into (23) we obtain the general equation for the looking for SIF:

$$K_I = K_{Iq}\left(1 - \omega(\alpha, \vec{G})\right) = K_{Iq}\left(1 - \Gamma(\alpha)/\left(Z(\vec{G}) + \beta_M(\alpha)\right)\right). \tag{25}$$

Here we introduce the SIF reduction function $\omega(\alpha, \vec{G})$ and designate $\Gamma(\alpha) = \beta_q Y_M / Y_q$ which is the known function of the relative crack depth. The unknown function $Z(\vec{G})$ can be found by some of the methods of the civil mechanics for the beam like structures. Authors prefer to use the method of initial parameters (MIP).

Let in some section $s = 0$ the vector of initial parameters of a curvilinear beam is known which describe all the geometrical and mechanical parameters $\vec{X}(s=0) = \vec{X}_0(u_0, w_0, \theta_0, N_0, Q_0, M_0)$, where w is the transverse displacement, Q is the transverse force. Then the value of the vector of state of the beam in any other point, s, according to the MIP can be written:

$$\vec{X}(s) = [A(s)]\vec{X}_0, \tag{26}$$

where the matrix $[A(s)]$ is known and at the point $\varphi = 0$ is unitary one. Then we break up the beam into zones at whose boundaries the concentrated forces (supports) are applied or a crack is placed. For these boundaries we write the conditions of equality of all six parameters with allowance for the above jumps. Consider the examples for the practically important geometries.

The allowance for a geometrical nonlinearity in a pressurized pipe with a long axial surface crack (ring with a radial crack). The system of the differential equations for the curvilinear beam

(Fig 3) we write in the geometrically nonlinear formulation, i.e. the expression for the curvature ρ^{-1} takes into account the deformation of the beam [10]:

$$\rho^{-1} = R_0^{-1} + M/(E'J) \tag{27}$$

where $J = t^3/12$. As it was shown in a many works [11] the accounting for second term in the right-hand of (27) is important only in a equation for the increase of the transverse force, thus:

$$dQ/d\varphi + NR_0/\rho = -P \cdot R_0; \quad dN/d\varphi - Q = 0 \quad dM/d\varphi = Q \cdot R_0 \tag{28}$$

$$d\theta/d\varphi = MR_0/E'J; \quad -du/d\varphi + w = 0 \quad dw/d\varphi + u = \theta \cdot R_0 \tag{29}$$

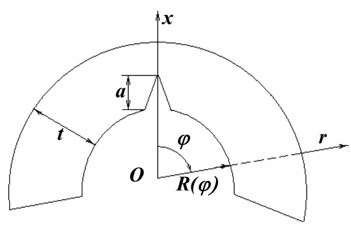


Fig 3 Ring with a radial crack

The solution of this system is found as the sum of the particular solution and to the general one of the homogeneous system. The first one is $\overline{N} = -PR_0$ and the latter system is reduced to the differential equation of the 6th order:

$$u^{VI} + (1+\chi^2)u^{IV} + u^{II} = 0 \tag{30}$$

where $\chi^2 = 1 - 12PR_0^3/E't^3 = 1 - \overline{p}$. In dependence of value χ^2 we have three different cases the general solutions for which (only for geometrical components) are presented in Table 1. To solve the equation (30) (determine the values of the initial parameters) we need to have 6 boundary conditions. They are $u_0 = \theta_0 = w_0 = 0$ and $u(\pi) = \theta(\pi) = 0$. The geometrical nonlinearity appears also in that large "particular" longitudinal force due to angular jump, $\Delta\theta$, in the section $\varphi = 0$ gives a relatively big projection on the radial axis, i.e. on the transverse force Q. Thus instead of $Q_0 = 0$ in this section we take $Q_0 = -N_0 \sin(\Delta\theta) \approx PR_0 \cdot \Delta\theta$. Omitting the details of calculation the resulting expression for $Z(\vec{G})$ can be presented in form [10]:

$$Z = R_0 \eta(\overline{p})/(9t) \tag{31}$$

For the linear case $\eta = 1$ the formula (31) gives a good correspondence with the literature data on SIF determination [10]. The dependence of $\eta(\overline{p})$ is shown on Fig 4. Thus, formula (31) allows us to state that with increasing pressure the dimensionless SIF decreases. Whether this effect has the practical significance is demonstrated on the Fig 5. Assuming that $E = 2 \cdot 10^5\ MPa$ and ν = 0.3, we plot graphs of the influence of the nominal stress σ_φ on the coefficient of the SIF reduction (Fig. 6) for some specific values of the R_0/t ratio (20 and 40) and relative crack depth (0.4 and 0.6). As it follows from the graphs, a noticeable reduction of the dimensionless SIF for typical pipes at a practically possible level of circumferential stresses is observed. Thus, for a pipe with a crack for $\alpha = 0.4$, $R_0/t = 20$, and $\sigma_\varphi = 200$ MPa, an additional decrease in the coefficient ω due to the action of the internal pressure is equal to $\Delta\omega = \omega(200) - \omega(0) = 0.06$. The influence of the pressure for deeper cracks is more noticeable, for example, for $\alpha = 0.6$, $\Delta\omega = \omega(200) - \omega(0) = 0.15$. The above values of ω are large enough to be accounted for in practical calculation of the SIF.

	$\chi^2 > 0 \quad \mu = \chi$	$\chi^2 = 0$	$\chi^2 < 0, \quad \mu = \sqrt{-\chi^2}$
θ	$\theta_0 + \frac{R_0 M_0}{E'J}\varphi + \frac{R_0^2 Q_0}{E'J}\left(\frac{1-\cos\mu\varphi}{\mu^2}\right) + -\frac{R_0^2 \overline{N}_0}{E'J}\left(\frac{\varphi}{\mu^2} - \frac{\sin\mu\varphi}{\mu^3}\right)$	$\theta_0 + \frac{M_0 R_0}{E'J}\varphi + \frac{Q_0 R_0^2}{E'J}\frac{\varphi^2}{2} - \frac{\overline{N}_0 R_0^2}{E'J}\frac{\varphi^3}{6}$	$\theta_0 + \frac{R_0 M_0}{E'J}\varphi + \frac{R_0^2 Q_0}{E'J}\left(\frac{ch\mu\varphi - 1}{\mu^2}\right) - \frac{R_0^2 \overline{N}_0}{E'J}\left(\frac{sh\mu\varphi}{\mu^3} - \frac{\varphi}{\mu^2}\right)$
u	$u_0\cos\varphi + w_0\sin\varphi + \theta_0 R_0(1-\cos\varphi) + \frac{R_0^2 M_0}{E'J}(\varphi - \sin\varphi) + \frac{R_0^3 Q_0}{E'J}\left(\frac{1}{\mu^2} - \frac{\cos\mu\varphi}{\mu^2(1-\mu^2)} + \frac{\cos\varphi}{1-\mu^2}\right) - \frac{R_0^3 \overline{N}_0}{E'J}\left(\frac{\varphi}{\mu^2} - \frac{\sin\mu\varphi}{\mu^3(1-\mu^2)} + \frac{\sin\varphi}{1-\mu^2}\right)$	$u_0\cos\varphi + w_0\sin\varphi + \theta_0 R_0(1-\cos\varphi) + \frac{R_0^2 M_0}{E'J}(\varphi - \sin\varphi) + \frac{R_0^3 Q_0}{E'J}\left(\cos\varphi - 1 + \frac{\varphi^2}{2}\right) - \frac{R_0^3 \overline{N}_0}{E'J}\left(\frac{\varphi^3}{6} - \varphi + \sin\varphi\right)$	$u_0\cos\varphi + w_0\sin\varphi + \theta_0 R_0(1-\cos\varphi) + \frac{R_0^2 M_0}{E'J}(\varphi - \sin\varphi) + \frac{R_0^3 Q_0}{E'J}\left(\frac{ch\mu\varphi}{\mu^2(1+\mu^2)} - \frac{1}{\mu^2} + \frac{\cos\varphi}{1+\mu^2}\right) - \frac{R_0^3 \overline{N}_0}{E'J}\left(\frac{sh\mu\varphi}{\mu^3(1+\mu^2)} - \frac{\varphi}{\mu^2} + \frac{\sin\varphi}{1+\mu^2}\right)$

Table 1. The MIP solution of the equation (30) for u and θ

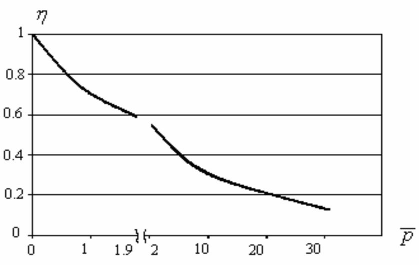
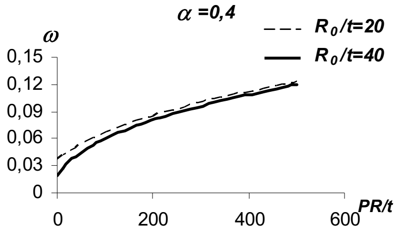


Fig 4. Dependence of the multiplier η on the dimensionless pressure $\overline{p}$

The analysis of SCC failure of the transit pipeline. The gas transit pipeline section was fractured due to stress corrosion cracking. The pipe diameter is 1420 mm, wall thickness 15.7 mm and the pressure, P, was 7.4 MPa. The crack depth, a, found during the failure investigation was 6.8 mm while its length, c, was 676mm. The mechanical characteristic of material are: yield stress $\sigma_Y = 500$ MPa; ultimate strength $\sigma_U = 630$ MPa. The two criteria approach is the most widely used method for the residual strength determination of the cracked structures. It envisages the determination of the reference stress σ_r (limit load) and SIF.



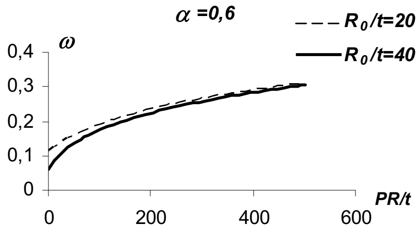


Fig 5. The influence of nominal circumferential stresses due to internal pressure on the SIF reduction coefficient for two dimensionless depths of the crack: (*a*) $\alpha = 0.4$, (*b*) $\alpha = 0.6$.

The value of σ_r depends upon relative crack depth $\alpha = a/t = 0.433$ and relative crack length $\lambda = 0.5\sqrt{c^2/Rt} \approx 3.2$. For such a long crack σ_r can be approximately calculated taking into account only the relative crack depth, namely $\sigma_r = (1-\alpha)^{-1} PR/t \approx 583$ MPa. The values of SIF are calculated a) as for the strip with an edge crack $K_I = 111.7 \ MPa\sqrt{m}$; b) by formula (25) with (31) without taking into account the geometrical nonlinearity ($\eta = 1$) $K_I = 109.333 \ MPa\sqrt{m}$; c) with accounting for the geometrical nonlinearity $K_I = 98.207 \ MPa\sqrt{m}$. The fracture toughness, K_{IC} $MPa\sqrt{m}$, of the pipe steel was determined by using the correlation of it with the impact toughness, KCV, J/cm^2:

$$K_{IC} = 7.36(KCV)^{0.63} \tag{32}$$

Accounting that $KCV \approx 151$ we obtain that $K_{IC} = 173 \ МПа\sqrt{м}$. The limit curve of the two criteria approach [12] is shown on Fig.6 where three points of fracture and corresponding loading beams according to three calculated values of SIF are shown. Evidently accounting for the geometrical nonlinearity is significant in practical applications.

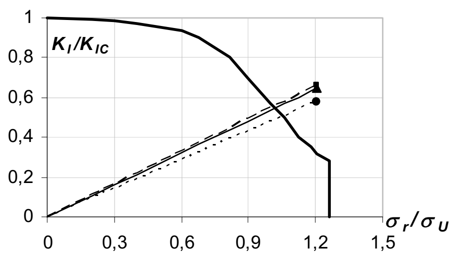


Fig 6. The limiting curve with calculated points of fracture according to different schemes of K_I calculation: ■ – as for strip with edge crack; ▲ – by formula (25) with (31) where $\eta = 1$, ● – with accounting for the geometrical nonlinearity

The full circumferential surface crack in pipe (Fig.7). Consider that in the defected section with zero crack depth the bending moment, M, and axial force, N are applied. The nature of these loading factors can be different. The differential equations for relevant geometrical and mechanical parameters are well known and have the form:

$$\theta = dw/dx,\quad M = d\theta/dx\, E'J,\quad Q = dM/dx\quad w = dQ/dx\, \frac{R^2}{Et} \tag{33}$$

which is reduced to the differential equation of the 4th order

$$w^{(IV)} + 4k^4 w = 0 \tag{33}$$

where $k = 3\left(1-\mu^2\right)/(R^2t^2)$

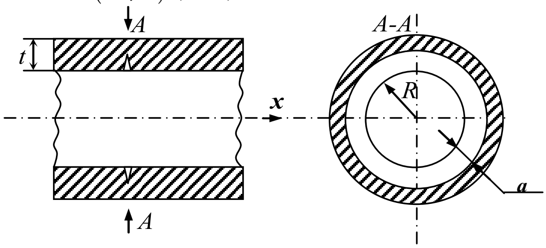


Fig 7. Pipe with a full circumferential surface crack

Equation (33) is supplemented by the boundary conditions:

$$Q_0 = 0,\ \theta_0 = \beta_L\left(\sigma_M + \sigma_M^{add}\right) + \beta_N\sigma_N \text{ and } \lim_{x\to\infty} w = 0\ , \tag{34}$$

where σ_M^{add} is additional bending stress. Omitting the usual algebraic operation the resulting expression for the geometric function $Z\left(\vec{G}\right)$ has the form:

$$Z\left(\vec{G}\right) = 1/(3\pi t k) \tag{35}$$

The dependence of dimensionless functions $\left(1-\omega(\alpha,\vec{G})\right)$ for cases when we initially (in the cracked section with crack of zero depth) had only the bending moment (it is designated as $\overline{M}$) as well as only longitudinal force (it is designated as $\overline{N}$) is shown on the Fig 8

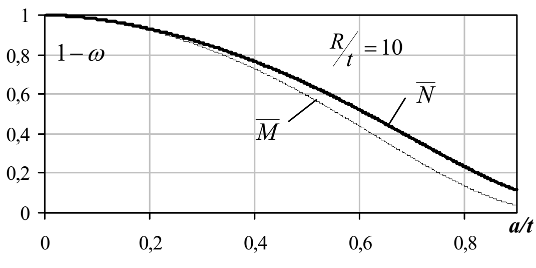


Fig 8. The dependence of the dimensionless SIF on the relative crack depth for $R/t = 10$ when $q = M$ and $q = N$.

The results obtained for dimensionless SIF for case of outer loading by the longitudinal force ($K_I = \sqrt{\pi a}\sigma_N Y_N \overline{K}$) are compared with numerical data given in [13]. The results testify about the good accuracy of the CCM.

a/t	$R/(R+t)=0.8$		$R/(R+t)=0.9$	
	results of [13]	CCM	results of [13]	CCM
0.1	1.138	1.146	1.158	1.158
0.2	1.198	1.223	1.253	1.268
0.3	1.286	1.311	1.392	1.407
0.4	1.397	1.416	1.568	1.579
0.5	1.529	1.559	1.779	1.8
0.6	1.688	1.714	2.025	2.037

Table 2.The comparison of the dimensionless SIF according to the CCM with results of [13]

The system of the periodical cracks in the ring loaded by the inner pressure. Formulas given in table 2 together with the procedure of MIP allow to easily organize the calculation for any number of cracks of different dimensions with a different spacing between them. Consider the simplest geometrically linear ($\eta = 1$) case of the symmetrical spacing of n cracks (where $n \geq 2$) of the same depth. The boundary conditions are following $Q_0 = 0$, and the value of the angle θ just in the right-hand of point $\varphi = 0$ is opposite to its value at the left-hand of the point $\varphi = 2\pi/n$. The ultimate formula for geometrical function $Z(\vec{G})$ is following:

$$Z(\vec{G}) = R_0/(3nt) \tag{36}$$

The results obtained were discussed in work [14]. It can be noted that for $n \geq 3$ the SIF values for pipe with a periodic system of cracks become smaller that this one for single crack.

Two symmetrical crack at the crown of pipe bend. The most important geometrical parameter which distinguish the stress patterns in pipe bend with such one in a straight pipe is the flexibility parameter $\lambda = R^2/Bt$, where R is radius of the bend section, B is the radius of bend, t is wall thickness. The difference in stress distribution reveals significantly when the pipe bend is loaded by the global bending moment and value of $\lambda > 1$. In this case the cross sections of the bend are subjected also by the transverse forces and they ovalize. The effect of ovalization of the pipe bend at bending is well understand and thoroughly described in the literature. From other hand, in the literature the view still prevails [15,16] that SIF in pipe bend is the same as in the straight pipe provided that the stress distribution through the wall thickness in each section is the same, i.e. the weight functions for both cases are the same.

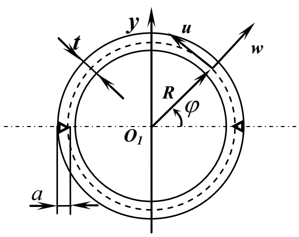


Fig.9. The cross section of pipe bend with two symmetrical cracks

Explore this statement about the difference between the bend and the straight pipe. It is expedient to make the analysis for two symmetrical (from the point of view of the simplicity) long axial surface crack situated at the crown, $\varphi = 0$ and $\varphi = \pi$, (Fig 9) of the bend (where the bending

stresses are maximal). The problem is reduced to the finding of the dependence of SIF with respect to λ. Then comparing the $SIF(\lambda)$ with $SIF(\lambda = 0)$ we can establish the influence of the flexibility parameter λ on the calculated SIFs.
The mathematical formulation of this task is very similar to that of the straight pipe. The difference is that homogeneous equation has a slightly different form caused by that axial u and radial w displacements lead to the appearance of the transverse force which ovalize the section. In general it can be presented in following formal writing [17]:

$$u^{VI} + (1+\chi^2)u^{IV} + u^{II} = CF(u^{II}, u) \quad (37)$$

where $C = 12\lambda^2(1-\mu^2)$ and the F is some operator which contains the functions u^{II}, u and their product with $\cos 2\varphi$. The scheme of solution is the same. We use both the analytical solution of the equation (37) as well as its numerical solution. The main results of both for the looking for SIF can be presented in the same form as for all above tasks:

$$Z(\vec{G}) = R/\left(6t\Psi^{add}(\lambda)\right) \quad (38)$$

where the function of pipe bend flexibility influence $\Psi^{add}(\lambda)$ calculated by analytical as well numeric procedure is shown on Fig 10. It can be concluded from it that increasing of λ decreases the values of SIFs. The values of SIF in pipe bend correspond to that in straight pipe which has the radius smaller in $\Psi^{add}(\lambda)$ times. To support the results obtained we cite the work [18], where for long axial crack in pipe bend the SIFs values were slightly smaller than those in a straight pipe of the same radius and thickness.

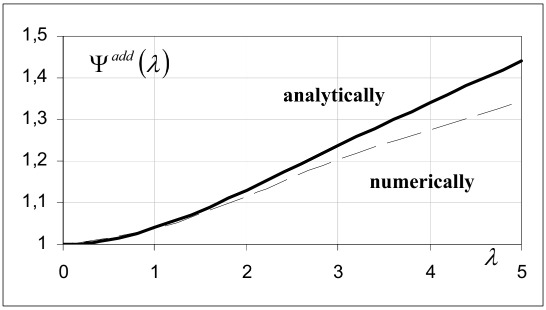


Fig 10. The analytically and numerically calculated dependences of $\Psi^{add}(\lambda)$.

Summary

Two engineering methods of SIF calculation are considered. The practical application of point weight function method for a high gradient stress distribution is reduced to the determination of an influence coefficients which can be calculated in advance and presented as the some known function along the crack front. The possibility of employing the Master Curve concept for the probability of fracture determination during the PTS event is demonstrated.
The application of crack compliance method for thin-walled 2-D structures with a 1-D crack allows establishing a general pattern of an analytical SIF presentation. The looking for SIF is written as a product of the SIF for an infinite strip with an edge crack and some correction function which is responsible for the particular geometry of the body under consideration.

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