# Determination of low-cycle fatigue parameters with rotating bending test

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**Abstract.** In the contribution theoretical approach for determination of low-cycle fatigue parameters using rotating bending test is presented. The stress ratio R = - 1 is induced on specimen loaded with pure bending moment exposed to rotation in respect of its longitudinal axis. Also the basic concept of special designed testing machine to performing tests is given.

## Introduction.

Fatigue life of structural components is becoming more and more important. Newly designed structures have to be cost effective yet safe in their predicted life time. Achieving those requirements is only possible with the properly known material fatigue parameters.

Concerning the design of cyclic loaded engineering structures and components, the prediction of their service life is of a great importance. The service life calculation of a cyclic loaded component is based on knowledge of the stresses or deformations in critical cross sections, usually calculated by means of finite element analysis (FEA). The main parameters influencing the fatigue life are the external loads and the material fatigue properties. Therefore, the appropriate fatigue properties of the material should be known for such analysis.

The strain-based approach for fatigue problems is widely used at present, especially in the regime of Low Cycle Fatigue (LCF) of machine parts and structures. The most common application of the strain-based approach is the fatigue of notched members as the strains can be calculated or measured more easily than the stress. Since fatigue damage is assessed directly in terms of local strain, this approach is also called the “local strain approach”. A reasonable expected fatigue life, can be determined by the known local strain-time history at a notch in a component and the unnotched strain-life fatigue properties of the material.

Because new materials are introduced in to the constructions, new fatigue tests are needed. These are also needed for proving known material properties. But as a matter of fact low cycle fatigue tests are preformed on the hydraulic test machines, which are expensive to operate, which means low cycle fatigue tests are also expensive. Because of that our aim is to introduce a new method for determining low cycle fatigue properties of materials with use of a rotating bending test.

Tests will be preformed on specially developed testing machine which will be providing needed parameters for evaluating material fatigue data using appropriate mathematical model.

Similar method has already been proposed and used for determination of low cycle parameters [1, 2, 3].

Main disadvantage of mentioned method is the loading procedure where the specimen is loaded with point force at cantilever free end that causes linearly distributed bending moment. The stress and therefore strain are related with diameter to the third power. That means special parabolic shaped specimens should be used to achieve constant stress and strain at surface in control section. Problem in mentioned procedure [1, 2, 3] was some how solved by use of conical shaped specimens. Such shaped specimens were providing region with permissible difference of stress and strain so it can be treated equal stress, but that region is small in comparison to specimen's dimensions.

The main characteristic of the proposed method is that it is based on specimen loading with pure bending moment. That is ensuring equal stress and strain at whole length on the cylindrical specimen. Use of cylindrical specimens means small overall diameter and simple machining process.

## Theoretical basics

Specimen is loaded with pure bending moment by use of special designed testing machine. Under applied load the specimen is deformed in the plane defined with un-deformed specimen longitudinal axis *z* and normal axis *y* about which main bending moment is applied. For the loading moments which are causing plastification of surface layer we can treat the problem as elastic plastic where low cycle fatigue takes place because of specimen rotation about longitudinal axis. Rotation causes points at specimen cross section to change stress state according to relative position. In the one full revolution observed point passes trough the section of minimum and maximum stress. If we assume symmetrical material behavior in tension and compression that means point is subjected to stress ratio R = -1. Same load ratio is used in axial tension low cycle fatigue tests and it means the same local stress conditions.
We can assume that strain is linearly distributed over specimen cross section regardless surface layer is plastically deformed as shown on Fig 1.

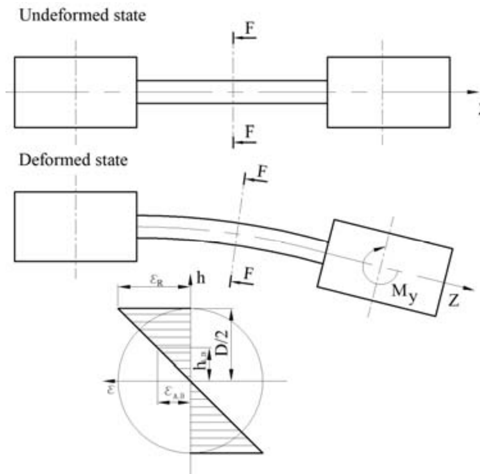


**Fig. 1:** Strain distribution

Because of this assumption we can calculate the stress for any point with known distance from deflection axis as we can easily calculate appropriate strain level. For the cyclic stress strain relation ship Ramberg-Osgood law is use, which is given in the form:

$$\varepsilon = \frac{\sigma}{E^*} + \left(\frac{\sigma}{K'}\right)^{\frac{1}{n'}} \quad (1)$$

In the equation (1) $E^*$ donates cyclic stress elastic modulus, which mostly can be used the same as the one obtained from tension test. $K'$ and $n'$ donates cyclic stress hardening coefficient and cyclic stress hardening exponent. Characteristic of low cycle fatigue loading regime is forming of hysteresis loops. Those are representing stress state of points, which are exposed to stress level above the yield limit. The change of stress during load cycle can be given with appropriate hysteresis loop (Fig. 2).

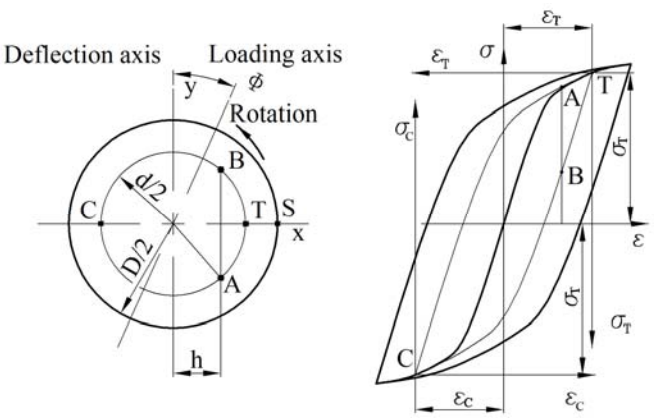


**Fig. 2:** Stress state

Maximal stress of any point during rotation is reached at *x* axis, and can be calculated using Eq. 1. For this propose, according to assumption of linear strain distribution, and known surface layer strain, we have to calculate appropriate strain of any point on *x* axis with following relation:

$$\varepsilon_T = \frac{\varepsilon_S}{D} \cdot d \tag{2}$$

Where *D* is specimen diameter and *d* is diameter of circle on which observed point is rotating during the test.

By use of Eq. 2 in the Eq. 1 we obtain equation for determination the stress in observed point T on *x* axis:

$$\varepsilon_S = \frac{D}{d}\left(\frac{\sigma_T}{E^*} + \left(\frac{\sigma_T}{K'}\right)^{\frac{1}{n'}}\right) \tag{3}$$

As Eq. 1 and consequently Eq. 3 can not be solved analytically for the stress solution with the known strain we must use iterative approach by use of appropriate methods. The most common method for solving this type of equation is Newton Raphson iterative approach.

In this way determined stress has same magnitude with negative sign for the observed point in the compression zone on the *x* axis in the point *C*. Points *C* and *T* are on the same circle on the cross section in respect to coordinate system origin. Stress strain relation of points on mentioned circle is changing according to the same hysteresis loop. For counter clock wise rotation as shown in Fig. 2, point *B* is changing stress from the highest value in tension to the highest value in compression. Point *A* is moving from the region of highest compression stress to region of the highest tension stress in point *T*. Hysteresis loop is given by use of Massing hypothesis as the cyclic stress strain curve scaled up with the scale factor 2 [4]. So we can define hysteresis loop with the following relation:

$$\varepsilon_H = \frac{\sigma_H}{E^*} + 2^{1-\frac{1}{n'}}\left(\frac{\sigma_H}{K'}\right)^{\frac{1}{n'}} \tag{4}$$

Eq. 4 is enabling us to calculate appropriate stress for any observed point on the cross section using appropriate hysteresis loop. For the points above the *x* axis (Fig. 3b) we can use the relation:

$$\varepsilon_{HB} = \frac{\sigma_{HB}}{E^*} + \left(\frac{\sigma_{HB}}{K'}\right)^{\frac{1}{n'}} \tag{5}$$

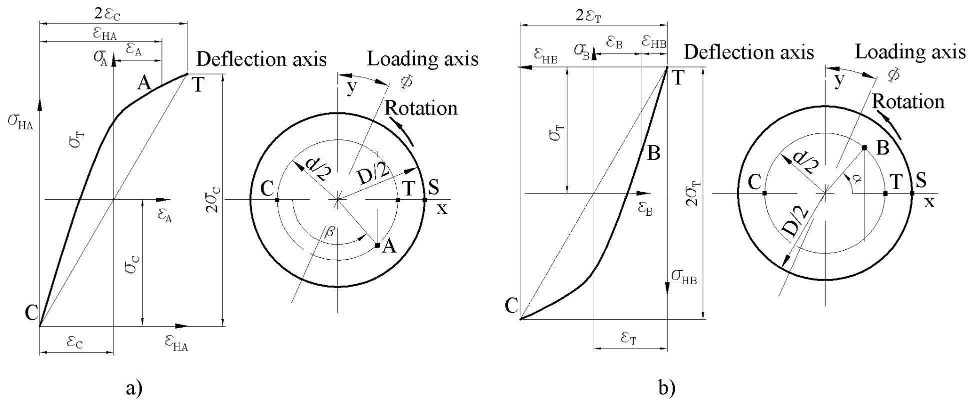


Fig. 3: Stress – strain state; a) tension bound, b) compression bound

Where the strain in observed point *B* is determined by use of Eq. 2 as:

$$\varepsilon_B = \frac{\varepsilon_S}{D} \cdot d\cos\alpha \tag{6}$$

respectively for the case where point B coincides with the point T as:

$$\varepsilon_B = \varepsilon_T = \frac{\varepsilon_S}{D} \cdot d \tag{7}$$

Appropriate strain value of point B in respect to hysteresis loop is defined by equation:

$$\varepsilon_{HB} = \varepsilon_T - \varepsilon_B \tag{8}$$

Using Eq. 6 and Eq. 7 in Eq. 8 we can obtain the following relation for the strain of point *B* on the hysteresis loop in respect position and specimen surface layer strain.

$$\varepsilon_{HB} = \frac{\varepsilon_S}{D} d(1 - \cos\alpha) \tag{9}$$

Similar relation is valid for the stress value in the observed point according to hysteresis loop:

$$\sigma_B = \sigma_T - \sigma_{HB} \tag{10}$$

Eq. 5, 9 and Eq. 10 are enabling us to calculate stress and strain for any chosen point, which position is defined by angle $\alpha$ on the circle with constant diameter $d$ on compression side of hysteresis loop.

The same relations are valid for the points on the cross section below $x$ axis (Fig. 3a). Those are changing stress strain state according to tension side of hysteresis loop and are defined on circle of the same diameter $d$ and angle $\beta$ in the same manner as $\alpha$ on compression side.

With the know stress and relative position on the cross section (Fig. 4) we can determine bending moments which are acting about $x$ and $y$ axis respectively. The bending moment acting about $y$ axis is the main load moment whereas the moment about $x$ axis is induced because of material hysteresis response means plastification of the specimen. But it must be noted that specimen is deformed only about $y$ axis.

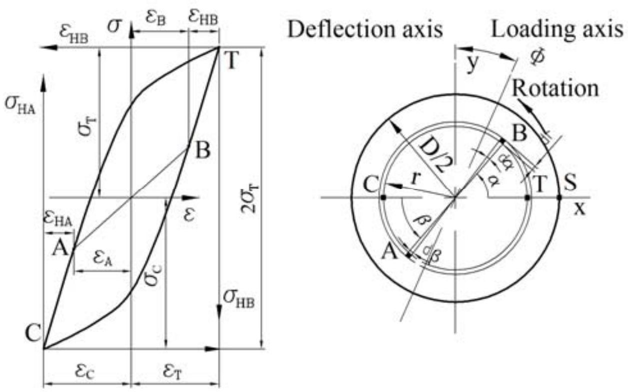


Fig. 4: Bending moments determination

For the moment about $y$ axis we can derive relation:

$$M_y = \int_0^{\pi}\int_0^{D/2} r^2 \mathrm{d}\alpha \mathrm{d}r \sigma_B \cos\alpha + \int_0^{\pi}\int_0^{D/2} r^2 \mathrm{d}\alpha \mathrm{d}r \sigma_A \cos\beta \tag{15}$$

In case of symmetrical chosen points about coordinate origin ($\alpha = \beta$) we get:

$$\sigma_A(r,\beta) = -\sigma_B(r,\alpha) \tag{16}$$

and

$$\cos\beta = -\cos\alpha \tag{17}$$

Using relations Eq. (16) and Eq. (17) in Eq. (15) we obtain:

$$M_y = 2\int_0^{\pi}\int_0^{D/2} r^2 \mathrm{d}\alpha \mathrm{d}r \sigma_B \cos\alpha \tag{18}$$

Same can be derived for the moment about $x$ axis

$$M_x = \int_0^{\pi} \int_0^{D/2} r^2 \mathrm{d}\alpha \mathrm{d}r \sigma_B \sin\alpha + \int_0^{\pi} \int_0^{D/2} r^2 \mathrm{d}\alpha \mathrm{d}r \sigma_A \sin\beta \tag{19}$$

Also in this case for the symmetrical chosen points we can use following simplifications beside Eq. (16):

$$\sin\beta = -\sin\alpha \tag{20}$$

So we can modify Eq. 18 to the form:

$$M_x = 2\int_0^{\pi} \int_0^{D/2} r^2 \mathrm{d}\alpha \mathrm{d}r \sigma_B \sin\alpha \tag{21}$$

In the rotating bending test required deformation work is applied to the specimen by the torque. Deformation work of specimen during the test is defined with hysteresis loop area bounded with the loop which formed for the points on the specimen surface. Specific work on the unit of volume for the observed specimen volume fraction can be expressed by the equation:

$$W = \sigma\varepsilon \tag{22}$$

In case of cylindrical specimen of diameter $D$ and length $l$ Eq. 22 can be written in the form:

$$W = 2\int_0^{D/2} \int_0^{\pi} \int_0^{l} \sigma\varepsilon r \mathrm{d}r \mathrm{d}\alpha \mathrm{d}l \tag{23}$$

For the rotation we can express the relation between torque and work by the equation:

$$\mathrm{d}W = M_t d\varphi \tag{24}$$

which gives us final relation form for determination of the torque as:

$$M_t = \frac{\int_0^{D/2} \int_0^{\pi} \int_0^{l} \sigma\varepsilon r \mathrm{d}r \mathrm{d}\alpha \mathrm{d}l}{\pi} \tag{25}$$

As it has been stated in the text before the Eq. 1 does not enabling us obtaining analytical solution of the stress. And therefore all stated equations are requiring iterative solving. Since it is impossible to integrate them and derive values of $K'$ and $n'$ respectively with known bending and torque moments and surface layer strain. For that propose the computer program has been developed which enabling us determination of the moments by known $\varepsilon_S$, $K'$ and $n'$. That way determined moments are enabling us to choose most appropriate values to the moments obtained in the test and so the determination of the material parameters $K'$ and $n'$.

For evaluation of parameters special computer routine is being developed which will enable us to determine low cycle fatigue parameters by means of Coffin Manson equation coefficients based on results obtained by rotating bending test.

## Test machine

Developed testing machine is shown schematically on the Fig. 5. It consists of three main parts: driving and loading assembly and foundation plate (14) on which the both pre-mentioned parts are attached.

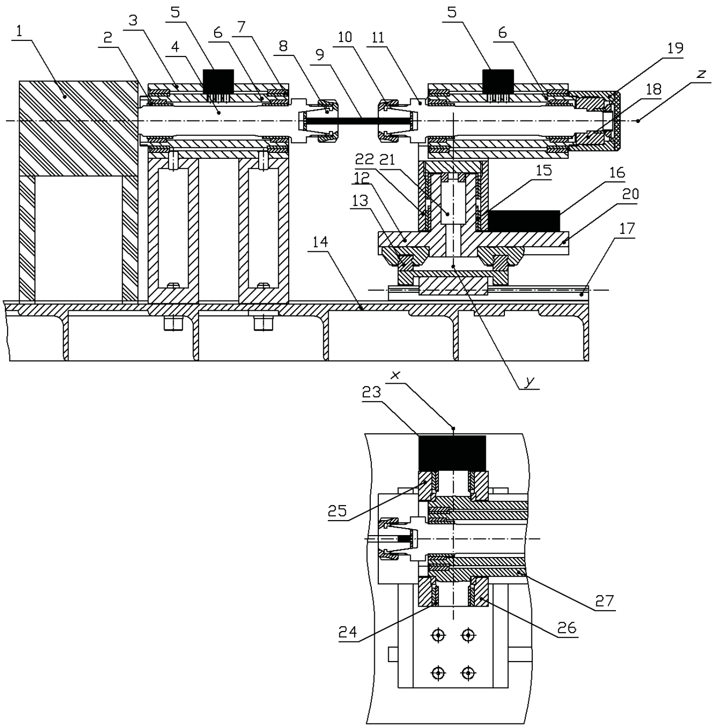


Fig. 5: Testing machine

Driving unit (1) made so it is enabling us to adjust output speed and so setting the appropriate rotation frequency of the specimen. The unit has integrated cycle counting device and torque-metering probe and it is also used for axial fixation of driving shaft (4). Both driving (4) and driven (11) shaft are compromising dual bearing system with intermediate housing. Such bearing system is enabling us to measure the torque, which is dissipated in the main bearings fitted on the shaft. Measurement is done with the probe (5). Torque measured with the probe fitted in driving unit is sum of both dissipated shaft bearing moments and moment required for specimen deformation work.

Specimen is clamped into the shafts by means of collet grip. The machine loading part consists of two perpendicular positioned pairs of linear guides. That way loading cross slide is free to move parallel with the specimen axis $z$ and traverse axis $x$. Loading is done by applying torque to the loading shaft housing holder (22) in respect to $y$ axis, which is inducing bending moment in the specimen. Measurement of the applied moment and deflection angle is done by the probe (16). Positions of longitudinal and cross slide are measured on the guides (13) and (17) respectively. Additional moment induced by rotation of the specimen under conditions of plastification is measured by loading probe (24). Probe (24) is also providing torque moment to the outer body. Compensation of the driven shaft (11) vertical deflection at the specimen end is compensated by the segment (22) which is also used as vertical force measuring. Measured force is feed back for the proper moment setting of the probe (24).

Correct method of failure condition determination will be developed during startup of prototype machine.

## Conclusion

Proposed method will be enabling determination of material low cycle fatigue parameters. The main advantage of the new method is use of small dimension cylindrical specimens. Because of mechanical driving unit operating costs in comparison to servo hydraulic systems will be reduced. Detailed procedure of evaluating equivalent number of cycles to specimen failure will be developed after startup of developed testing machine.

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