Damage analysis of fibre-matrix system

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Abstract. Conditions of damage initiation in fibre composite materials are analysed in the paper. The main objective of the presentation is to quantify the influence of fibre-matrix material mismatch on failure critical stress under which a crack is initiated in the vicinity of a free fibre end. The considerations are derived from the knowledge of the stress state caused by the existence of the bi-material interface. The step change of material properties in the interface leads to singular peak of stresses, which may be responsible for crack initiation and consequently for failure of the structure. The singular stress state near the interface is analysed by means of generalized linear elastic fracture mechanics and conditions of crack initiation are assessed. On the basis of the results the question of interfacial debonding between the fibre and matrix is discussed.

Introduction

Failure in structures frequently starts at stress concentrators. Typically, in fibre composites failure initiation of the composite may be induced by singular stress distributions caused by material and geometry discontinuities at the ends of fibres. For example singular stress appearing at fibre ends causes crack initiation, crack propagation and final failure under cyclic loading [1].

In the present paper two special configurations of fibre in matrix are analysed. The first one is formed by a fibre end touching a free surface (see fig.1a, b) and the second one corresponds to a pulled out fibre (fig.1c). In both cases the stress distribution around the points of material and geometric discontinuities (corner C) has a singular character with respect to the distance r, see fig.1, and can be described as general singular stress concentrators. In both cases the corresponding singular stress distribution can be modelled by means of a bi-material notch. The basic feature of the solution is in this case the existence of singular stress distribution with a type of singularity different from those of a crack in homogeneous material. In this case the stress and displacement distribution depends on the distance r from the singular point as $\sigma_{ij} \approx r^{-p}$ and $u_i \approx r^{1-p}$, where $0 < p \neq 1/2 < 1$ is the stress singularity exponent.

The fact that the stress singularity exponent $p \neq 1/2$ complicates the applications of LEFM to problems of general singular stress concentrators stability. In this case a general approach to the assessment of the fracture mechanics behaviour of general stress singular concentrators has to be applied. To evaluate the influence of fibre ends on the mechanical strength of fibre composites, two problems have to be solved. First, the singular stress distribution around fibres ends has to be quantified and calculated and second, the way of assessing the stability of the general singular stress concentrator has to be formulated.

In the following an in-plane fibre end (fig.1a) and a pulled out fibre end (fig.1c) perpendicular to the free surface are considered only. In the previous papers, see e.g. [2], the singular stress distribution around the free end of a fibre has been analysed and the generalized stress intensity factors (GSIFs) have been evaluated by a combination of analytical and numerical methods for various geometrical and material configurations. On the other hand, little has so far been done to describe the behaviour of fibre ends from the point of fracture mechanics.

The objective of the presentation is to express the conditions under which a crack is initiated in the vicinity of a free fibre end and to estimate the presumed crack initiation direction. The basic idea follows an approach of generalized LEFM formulated in [3], [4].

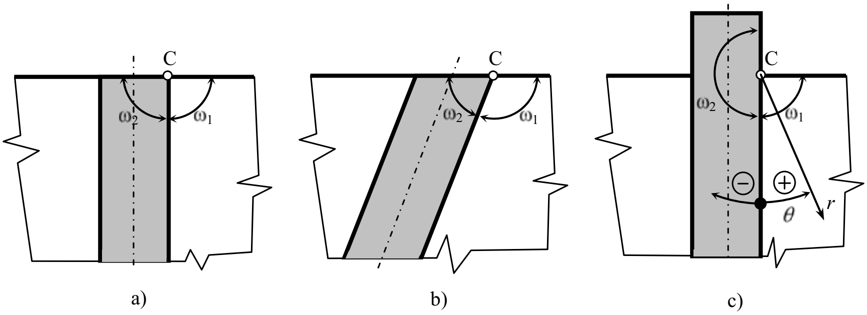

Fig. 1 Possible configurations of the fibre-matrix intersection at a surface considered in the paper. Polar coordinates (r, θ) with sign convention of the direction of crack initiation.

Stress distribution

The singular stress concentrators shown at Fig.1 can be modelled as bi-material notches. In the vicinity of the stress concentrator, i.e. for $r \to 0$, the plane strain approximation corresponding to real axisymmetric conditions was used.

The singular stress distribution in the vicinity of the notch tip can be generally written in the following form, e.g. [4]:

$$\sigma_{mij} = \sum_{k=1}^{2} \frac{H_k}{\sqrt{2\pi}} r^{-p_k} F_{ijkm}(p_k, \theta) \tag{1}$$

where

$$F_{rrkm} = (2-p_k)(-a_{mk}\sin((2-p_k)\theta) - b_{mk}\cos((2-p_k)\theta) + 3c_{mk}\sin(-p_k\theta) + 3d_{mk}\cos(-p_k\theta))$$
$$F_{\theta\theta km} = (p_k^2 - 3p_k + 2)(a_{mk}\sin((2-p_k)\theta) + b_{mk}\cos((2-p_k)\theta) + c_{mk}\sin(-p_k\theta) + d_{mk}\cos(-p_k\theta))$$
$$F_{r\theta km} = (2-p_k)(-a_{mk}\cos((2-p_k)\theta) + b_{mk}\sin((2-p_k)\theta) + c_{mk}\cos(-p_k\theta) - d_{mk}\sin(-p_k\theta))$$

In the formula (1): $i, j = r, \theta$, the subscript $m = \{F, M, I\}$ determines one of the materials (fibre, matrix) eventually their interface, where the stresses are ascertained. The subscript k =1 or 2 and corresponds to the number of the singular term. H_k is the generalized stress intensity factor - which has to be estimated from the numerical solution of the studied geometry with given materials and boundary conditions - and p_k is a stress singularity exponent. Both H_k and p_k correspond to a particular singular term k. Further, a_{mk}, b_{mk}, c_{mk}, and d_{mk} in the shape functions F_{ijkm} are known constants resulting from the boundary conditions. For the given geometry the number of the singular terms and the values of the stress singularity exponents depend on the combination of materials. The fibre ending in the plane of the surface (fig.1a)) implies 1 singular term with the stress singularity exponent p_1. In the case of the pulled out fibre there are two exponents p_1 and p_2 describing two singular terms. For details see Dundurs' parallelograms in [4], [5].

Crack initiation at fibre-matrix intersections at a surface

On the basis of the known stress distribution around the fibre end the conditions of failure initiation can be formulated and analysed. To assess the problem of crack initiation in the vicinity of fibre ends, the direction of a potential crack initiated at the point C, see fig.1, has to be determined first. Then a criterion for crack initiation and propagation in the selected direction can be applied.

Direction of crack initiation. Generally, the stress distribution around the fibre end represents an inherently combined mode of normal and shear loading. The combined loading mode is induced even for one singularity because the shape function F_{ijkm} in each singular term in the relation includes both sine and cosine functions. The polar coordinate system in the bi-material notch tip with the sign convention of a polar angle is shown in fig. 1c. Note that the direction $\theta = 0$ is oriented in the bi-material interface and the positive angle goes anticlockwise.

The direction of presumed crack initiation can be estimated on the basis of finding extremes of particular controlling magnitudes. In the present case the condition of the maximum tangential stress (MTS) is applied. Corresponding to the basic assumptions of linear elastic fracture mechanics the crack propagation direction coincides with the direction θ_{0m} of the local maximum of tangential stress component $\sigma_{\theta\theta}$. It is shown in [4] that the calculated value θ_{0m} slightly depends on the distance r where the condition is applied. In order to eliminate this dependence r, the mean value of $\sigma_{\theta\theta}$ is evaluated and used. The tangential stress is monitored in both materials. To find its maximum the following two conditions have to be satisfied:

$$\left(\frac{\partial \overline{\sigma_{m\theta\theta}}}{\partial \theta}\right)_{\theta_{0m}} = 0, \quad \left(\frac{\partial^2 \overline{\sigma_{m\theta\theta}}}{\partial \theta^2}\right)_{\theta_{0m}} < 0 \tag{2}$$

where the mean value of $\sigma_{\theta\theta}$ is as follows:

$$\overline{\sigma_{m\theta\theta}}(\theta) = \frac{1}{d}\int_0^d \sigma_{m\theta\theta}(r,\theta)\mathrm{d}r = \frac{H_1}{\sqrt{2\pi}}\frac{d^{-p_1}}{(1-p_1)}F_{\theta\theta 1m} + \frac{H_2}{\sqrt{2\pi}}\frac{d^{-p_2}}{(1-p_2)}F_{\theta\theta 2m} \tag{3}$$

and for its first derivation it follows:

$$\frac{d^{-p_1}}{1-p_1}\frac{\partial F_{\theta\theta 1m}}{\partial \theta} + \Gamma_{21}\frac{d^{-p_2}}{1-p_2}\frac{\partial F_{\theta\theta 2m}}{\partial \theta} = 0 \tag{4}$$

where for $k = 1, 2$:

$$\frac{\partial F_{\theta\theta km}}{\partial \theta} = (p_k^{\,2} - 3p_k + 2)\left[(2-p_k)\ \ a_{mk}\cos((2-p_k)\theta) - b_{mk}\sin((2-p_k)\theta\ \ - p_k\ \ c_{mk}\cos(p_k\theta) + d_{mk}\sin(p_k\theta)\ \ \right]$$

The mean value is calculated across the distance d from the stress concentrator. Parameter d has to be specified according to the mechanism of failure and is related to the microstructure of the material components.

It is obvious that by inserting the ratio of the values $\Gamma_{21}=H_2/H_1$ (obtained from the numerical solution) into the relation (4) we obtain a simple equation for the value of the direction θ_{0m}. Note that directions θ_{0m} do not depend on the absolute values of GSIFs H_1 and H_2, but on their ratio Γ_{21} only. As a result the possible directions of the crack initiation are estimated in both materials (fibre and matrix) and their interface and they have to be tested for possible crack initiation. In Fig. 2 there are results of the numerical analyses of the mean value of the tangential stress around the free fibre ending in the plane of surface. The maximum of the mean value of tangential stress occurs in fibre in the direction θ_{0F}. Nevertheless there is also the maximal value in the matrix coinciding with the value in the material interface, and it is in the direction $\theta_{0M} \equiv \theta_{0I} = 0$. To decide if really the assumed crack initiates and propagates in this direction, a stability criterion has to be applied.

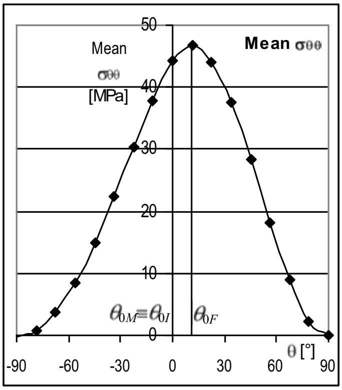

Fig. 2 Numerical analyses of the mean value of the tangential stress around the free fibre ending in the plane of surface.

Stability criterion. All of the calculated directions of the possible crack initiation directions θ_{0m} have to be tested to see whether or not the crack is really initiated in this direction. For the assessment of the critical places the criteria of stability of classical linear elastic fracture mechanics are modified considering the particularities of general singular stress concentrators. In the case of brittle fracture the stability criterion is expressed in terms of GSIFs as:

$$H_1(\sigma_{\mathrm{appl}}) < H_{1C}(K_{IC}) \quad . \tag{5}$$

The generalised fracture toughness H_{IC} is derived as a function of fracture toughness K_{IC} of the material in which the crack is initiated (matrix, fibre or the interface). The generalised fracture toughness value H_{1C} is derived on the basis of the equality of the values of a controlling magnitude in cases of crack propagation in a homogeneous material and in the case of crack initiation in the general singular stress concentrator, see [4]. In the present contribution the comparison of the mean values of the maximum tangential stresses in the case of crack initiation near the fibre end and corresponding crack propagation in a homogeneous material is applied and the following relation for the determination of H_{1C} is used

$$H_{1C} = \frac{2K_{IC}}{\dfrac{d^{\frac{1}{2}-p_1}}{1-p_1} F_{\theta\theta 1m}(\theta_{0m}) + \Gamma_{21} \dfrac{d^{\frac{1}{2}-p_2}}{1-p_2} F_{\theta\theta 2m}(\theta_{0m})} \tag{6}$$

Note that for the case of $\theta_{0M} \equiv \theta_{0I} = 0$ corresponding to a crack initiation and propagation along the interface the previous relation simplifies to:

$$H_{1C}(\theta_{0m} = 0) = \frac{2K_{IC}}{d^{\frac{1}{2}-p_1}(2-p_1)(b_{m1}+d_{m1}) + \Gamma_{21} d^{\frac{1}{2}-p_2}(2-p_2)(b_{m2}+d_{m2})} \tag{7}$$

Where b_{mk}, d_{mk} are known constants from the shape functions F, see eq. (1), see [4] and [7] for details. The advantage of this method is the fact that we do not need to measure no new material characteristic, but the approach is based on known values of fracture toughness of particular materials or the interface [8], [9].

Thus the three critical values of H-factor: $H_{1C}(\theta_{0F})$, $H_{1C}(\theta_{0M})$, $H_{1C}(\theta_{0I})$ are gained for the possible crack initiation into fibre, matrix or the interface respectively. For the final evaluation of the stability of the stress concentrator it is necessary to calculate the critical applied stress σ_{crit} from the ratio of the calculated GSIF H_1 and its critical values H_{1C}. This value is ascertained as a minimum of the three studied cases:

$$\sigma_{\text{crit}} = \min\left\{\sigma_{\text{appl}}\frac{H_{1C}(\theta_{0F})}{H_1(\sigma_{\text{appl}})}, \sigma_{\text{appl}}\frac{H_{1C}(\theta_{0M})}{H_1(\sigma_{\text{appl}})}, \sigma_{\text{appl}}\frac{H_{1C}(\theta_{0I})}{H_1(\sigma_{\text{appl}})}\right\}. \tag{8}$$

The critical value σ_{crit} represents the magnitude of the applied stress σ_{appl} under which crack initiates in the stress concentrator tip. Stability of the concentrator is then ensured by the following condition:

$$\sigma_{\text{crit}} < \sigma_{\text{appl}}. \tag{9}$$

When the condition (9) is violated the damage of the fibre-matrix system starts by one of the following ways:

(i) If the minimal critical applied stress is connected with the direction θ_{0m} oriented into fibre ($m = F$), the rupture of the fibre probably occurs, (Fig. 3a)).

(ii) In the case of minimal σ_{crit} calculated from $H_{1C}(\theta_{0I})$ gained from the fracture toughness of the interface, debonding of the fibre and matrix interface can be presumed, and the further crack growth along the interface as well, (Fig. 3b)).

(iii) When the fracture toughness of the matrix leads to the lowest critical stress in the matrix, the matrix breaking away from the fibre while keeping the connection of the interface is presupposed, (Fig. 3c)).

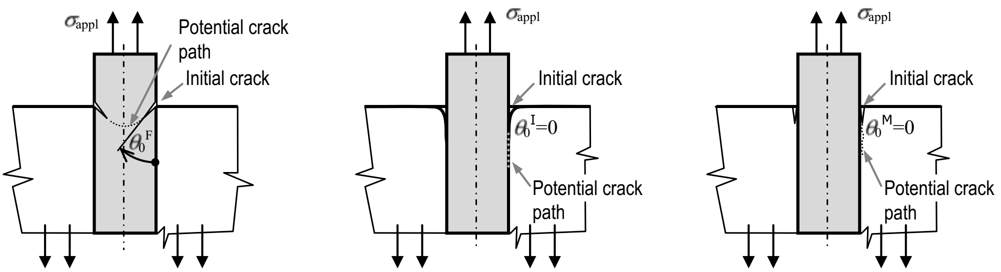

Fig. 3 Damage eventualities of fibre-matrix system in the case of pulled-out fibre (e.g. while crack bridging) a) rupture of fibre, b) interfacial crack, c) crack in the matrix (crack direction parallel to the interface)

Conclusion

The fibre ends or their intersections with the matrix at a free surface represent dangerous stress concentrators, and in many cases the failure of the composite can be initiated in these points. The procedure for the assessment of singular stress concentrations caused by fibre ends at a free surface was presented. The fibre end has been modelled as a general singular stress concentrator with a stress singularity exponent different from 1/2. The criterion for a crack initiation direction and for the stability conditions estimation was suggested on the basis of the mean value of tangential stress.

Although from the distribution of the mean value of tangential stress it usually follows only one possible direction of the crack initiation, also the maxima of $\sigma_{\theta\theta}$ in the other material and in the materials interface should be tested for the crack initiation for given boundary conditions. The direction of assumed crack initiation following from the global maximum is regularly oriented into the material with higher Young`s modulus, i.e. into the fibre. On the other hand, in the matrix, there is the maximum of $\sigma_{\theta\theta}$ as well. It usually is coincident with the direction of the interface. Knowing the three critical values of GSIF and consequently the critical applied stresses in all the three directions, one can decide among the damage eventualities. The crack initiation is assumed in the direction and in the way which corresponds to minimal value of the critical applied stress. The failure probability caused by fibre ends can be generally minimized by a proper choice of materials.

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