Crack initiation at the inclusions under static loading

Sylovanyuk V.P.[1,a], Yukhym R.Ya.[1,b]

[1] H.V.Karpenko Physico- Mechanical Institute, 79601, Lviv, Ukraine

[a] sylovan@ipm.lviv.ua, [b] romashka@ipm.lviv.ua

Keywords: crack, inclusion, fracture, plastic zone, elastic-plastic deformation, integral equation.

INTRODUCTION. Fracture from the positions of modern concepts of material strength is considered as a process of crack initiation and propagation in the stress concentration regions, where there occur the irreversible changes in the material as a result of deformation beyond the elasticity limit. In this connection it is not enough to have only elastic solutions for study of the phenomenon of local fracture near defects. It is necessary to construct the elastic-plastic models of materials deformation at the inclusion .In literature practically no works are available that model the irreversible processes of deformation and fracture of such materials. The exception is the models of plastic yield on thin absolutely rigid inclusions [1-3]. In this work from the positions of the deformation strength criterion and δ_c-concept [4], the peculiarities of elastic-plastic deformation and fracture in the vicinity of elastic inclusions are investigated.

PROBLEM FORMULATION. Consider an infinite elastic-plastic body, that contains the foreign elastic inclusion with a smooth oblate surface Σ symmetric with respect to the middle plane $y=0$. Consider that on the material interface Σ the conditions of perfect mechanical contact take place. A body is subjected to monotonous growing tensile stresses p, symmetric with respect to plane $y=0$.

Stress concentration growth is observed during the increase of the external loading p in the vicinity of the inclusion $(x=\pm a)$. At a certain level of loading $p=\sigma_T/k$ (k – stress intensity factor) the stresses will attain the value σ_T – material yield strength. With further increase of external loading the area of non-elastic deformations (plastic zone) is formed at the inclusion , surrounded by the elastic-deformed material (Fig 1, a).

Cut out mentally from the body the inclusion and plastic zone that adjoins the body. Taking into account the small thickness of the inclusion and plastic zone as well as the surface smoothness replace their action on basic material by some stresses on the cavity surface , namely:

1) in the contact zone of inclusion materials and a matrix give the stresses in accordance with the model of the compliant inclusion [4]

$$\sigma_y(x)=\frac{u_y^*(x)}{d(x)}E_i,\sigma_{xy}=0; \tag{1}$$

Here u_y^* - displacement of inclusion surface; E_i - the Young's modulus of inclusion material; $2d(x)$ - inclusion thickness;

2) assume stresses at the interface of the plastic zone and elastic stress-deformed material to be equal to:

$$\sigma_y = \sigma_0,\ \sigma_{xy} = 0. \qquad (2)$$

Here $\sigma_0 = \sigma_T$ – in case of perfect elastic-plastic material, and $\sigma_0 = (\sigma_T + \sigma_в)/2$ – for hardened material; $\sigma_в$ – yield strength .

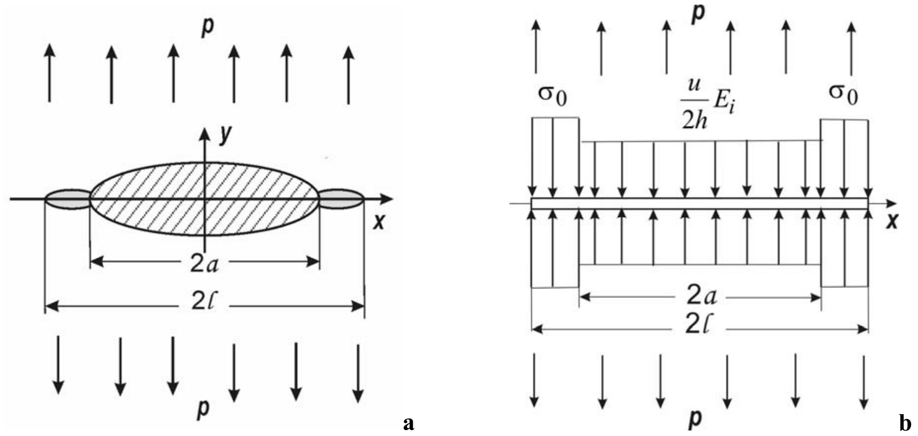

Fig. 1. The inclusions with plastic zone (a) and their calculation scheme (b).

Move the boundary conditions from the surface to axis Ox because of the small thickness of a formed cavity. As a result we get the boundary-value problem of the theory of elasticity about tension at infinity of an infinite body with a cut $2l$, on which the stresses are set: in the interval ($0 \le x \le a$) – by relation (1), in the interval ($a \le x \le l$) – by relation (2) (Fig.1, b). The size of the plastic deformation area $(l - a)$ in direction of axis x will be established during calculation of the boundary-value problem from the condition in finiteness of stresses at the ends of a cut $2l$.

SINGULAR INTEGRO-DIFFERENTIAL EQUATION AND ITS SOLUTION. The solution of this boundary-value problem for a cut is reduced to singular integro-differential equation with respect to the unknown displacement u_y of cut edges $[-l, l]$

$$\frac{1}{2\pi c}\int_{-l}^{l}\frac{u'_y(t)}{t-x}dt - \frac{u_{y(x)}}{d(x)}\lambda \cdot H(a-x) = \frac{1}{E}\left(-p + p\lambda \cdot H(a-x) + \sigma_0 \cdot H(x-a)\right), \qquad (3)$$

where $H(x)$ is the Heavyside function, $\lambda = E_i/E$; $c = 1$ – for the plane stress state and $c = 1 - \nu^2$ – for plane deformation. Here it is taken into account that $u_y^* \approx u_y + u_y^0$; u_y^0 is the displacement of the surface points $\sum$ in a homogeneous (with no inclusion) body under external stresses action.

The solution of this equation can be obtained numerically or by approximate analytical methods.

If to assume the surface of inclusion with axes $2a$ and $2c$ ($a > c$) to be elliptic and the stress state in the inclusion to remain homogeneous with formation of plastic zone in its vicinity, equation (3) has the exact solution

$$u_y(x) = \frac{c}{E}\Big\{2\pi p(1-\lambda k)\sqrt{l^2-x^2} + (\sigma_0 - \lambda k p)\cdot\big[(x-a)\Gamma(l,x,a) - \\ -(x+a)\Gamma(l,x,-a) - 4\sqrt{l^2-x^2}\arccos a/l\big]\Big\}, \qquad 0 \le |x| \le l, \tag{4}$$

here $k = \dfrac{1+2\beta}{1+2\beta\lambda}$; $\beta = \dfrac{a}{c}$; $\Gamma(l,x,a) = \ln\dfrac{l^2 - xa - \sqrt{(l^2-x^2)(l^2-a^2)}}{l^2 - xa + \sqrt{(l^2-x^2)(l^2-a^2)}}$.

Determine the size of the plastic area in direction of axis x from the condition of limitedness and continuity stresses at points $x = \pm l$

$$l - a = a\left(\sec\frac{\pi p(1-\lambda k)}{2(\sigma_0 - \lambda k p)} - 1\right). \tag{5}$$

It is obvious that the size of the plastic area in the direction of axis x depends on inclusion rigidity (λ), its geometry (a, β) and intensity of external loading (p). The character of this dependence on the mentioned factors is graphically presented in Fig. 2 .

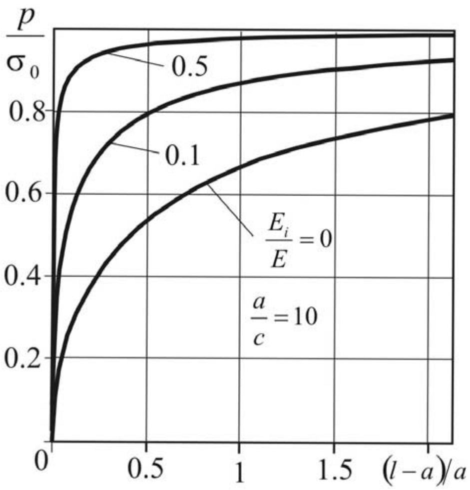

Fig. 2. Dependence of the plastic zone ($\dfrac{l-a}{a}$) size on external loading.

CRITERION OF MATERIAL FRACTURE IN THE VICINITY OF INCLUSION. Let us assume that deformability and strength of the inclusion, its adhesion with basic material is sufficient for initial fracture to occur in the matrix in the vicinity of the inclusion

To establish the limiting loading p_*, when the crack is formed in the vicinity of the inclusion, we will proceed from the deformation criterion of strength according to which local fracture takes place under condition that maximum tensile deformation reaches the limiting value of ε_c:

$$\varepsilon = \varepsilon_c . \tag{6}$$

Determine the deformation ε of the plastic nucleus material at the inclusion using the following reasons. Deformation in the inclusion in accordance with the model dependence (1) is calculated by the relation

$$\varepsilon^* = u_y^*(x) / d(x).$$

In particular, at points at the inclusion tips ($x \to a$) this relation is transformed as:

$$\varepsilon^* \approx \delta / (2\rho), \tag{7}$$

where $\delta = 2u_y(a)$; ρ it is the curvature radius of the inclusion tip.

Obtain the relation from the condition of deformations equality in the inclusion and in the plastic zone at a point $x = a$ ($\varepsilon^* = \varepsilon$),

$$\varepsilon = -\frac{4ac}{E\rho}(\sigma_0 - \lambda kp)\ln\cos\frac{\pi p(1-\lambda k)}{2(\sigma_0 - \lambda kp)}. \tag{8}$$

Deformation in the vicinity of the inclusion at the low external loading ($p << \sigma_0$) can be determined by the dependence

$$\varepsilon = \frac{p^2 \pi ac(1-\lambda k)}{2E(\sigma_0 - \lambda kp)\rho} = \frac{K_I^2(1-\lambda k)c}{2E(\sigma_0 - \lambda kp)\rho}, \tag{9}$$

where $K_I = p\sqrt{\pi a}$ is stress intensity factor for a crack of length $2a$.

Deformation in the cavity vicinity is obtained by putting in relations (8), (9) $\lambda = 0$.

Then

$$\varepsilon = -\frac{4ac\sigma_0}{E\rho}\ln\cos\frac{\pi p}{2\sigma_0} \tag{10}$$

for the case of large plastic areas , and

$$\varepsilon = \frac{p^2 \pi ac}{2E\sigma_0 \rho} = \frac{cK_I^2}{2E\sigma_0 \rho} \qquad (11)$$

for the case of localize plasticity in the vicinity of cavities .

Bearing in mind that during fracture ($\varepsilon = \varepsilon_c$, $K_I = K_{IC}$) in the conditions of plane deformation, using dependence (11), determine the curvature radii of cavities that in fracture will behave as cracks,

$$\rho \le \frac{K_{IC}^2 \left(1 - \nu^2\right)}{2E\sigma_0 \varepsilon_c}. \qquad (12)$$

In case of the large plastic areas criterion relation (6) looks like

$$\frac{\delta_c}{2\rho} = \varepsilon_c, \qquad (13)$$

δ_c is critical crack tip opening displacement.

The curvature radius of a cut that behaves in fracture as a crack in a rather plastic material, is determined by the inequality

$$\rho \le \frac{\delta_c}{2\varepsilon_c}. \qquad (14)$$

Intensity of the external loading $p = p_c$, that causes crack initiation in the vicinity of the inclusion on the basis of equations (6), (8) is evaluated by solving the equation

$$-\frac{4ac}{E}(\sigma_0 - \lambda k p_c) \ln \cos \frac{\pi p_c (1 - \lambda k)}{2(\sigma_0 - \lambda k p)} = \varepsilon_c \rho. \qquad (15)$$

In Fig. 3 results of numerical solution of equation (15) is presented. It can bee seen the cracks appear first of all (at the lower levels of loading) near inclusions of a plane shape , low rigidity, that corresponds to experimental observations in steels and cast-irons near the non-metal inclusions [6], in the composite systems with dispersion particles and in other structurally heterogeneous materials [7].

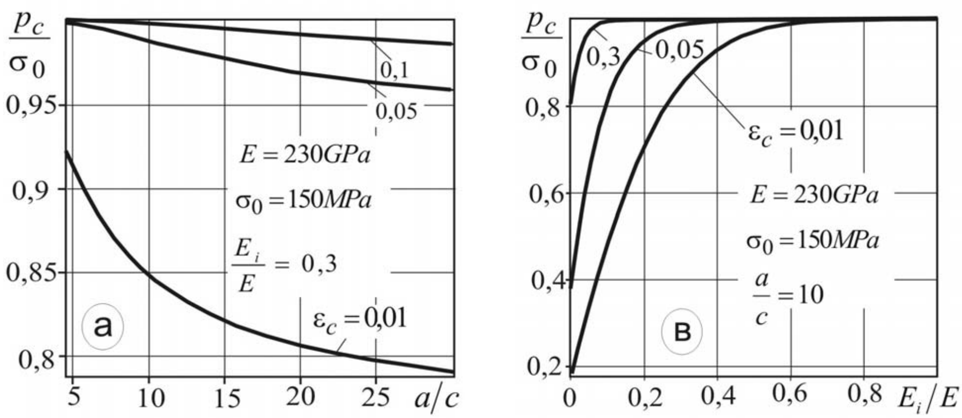

Fig. 3. Dependence of the maximum loading on inclusion geometry (a) and relation of the modules of elasticity of matrix and inclusion materials (b) on different values of limiting deformation of materials ε_c.

Establish the size of inclusion that at the given level of external loading p do not initiate a crack in its vicinity on the basis of relation (15)

$$a < \frac{\varepsilon_c \rho E}{2c(\sigma_0 - \lambda k p)} \ln \cos \frac{\pi p(1-\lambda k)}{2(\sigma_0 - \lambda k p)}. \tag{16}$$

Important feature of equation (5),(8)-(16) in practice is that they, obtained for elliptic inclusions, can be used for establishment of limiting parameters of local fracture in the vicinity of thin non-elliptic inclusions. As one can see, the basic parameters, that influence the formation of the process zone in the vicinity of elliptic inclusion are: a typical size $2a$, curvature radius at the tip ρ, parameter of rigidity λ and a certain coefficient $k = (1+2\beta)/(1+2\lambda\beta)$ which is an elastic coefficient of stress concentration near the elliptic inclusion [5]. It is possible to say that non-elliptic inclusion with the rigidity parameter λ, typical size $2a$ and stress intensity factor k will generate local fracture in its vicinity according to equation (15). The results of numerous researches confirm this fact. It results from them that the inclusion geometry, except of the local geometry at the tip, slightly influences the distribution of stresses in the vicinity of the thin inclusion.

CONCLUSIONS. In accordance with the chart considered in the paper the value of elastic-plastic deformation in the vicinity of the inclusion depends on such parameters as: the level of applied loading (p), typical size (a), defect geometry at its tip (ρ), mechanical properties of materials ($E, E_i, \sigma_T, \sigma_в$).

Crack initiation in the vicinity of the inclusion of a certain form and nature is determined significantly by deformation ability of the matrix material (ε_c). With the increase of ε_c the crack growth resistance also increases.

Dangerous from the point view of local fracture is the laminated inclusion material of a large size.

There are critical sizes of the inclusion that are not able to initiate a crack in its vicinity. This size is established in the paper for an isolated defect. When concentration of inclusions is considerable , their interaction must be taken into account, as it can cause substantial corrections in the process of material local fracture.

References

[1] Vytvytski P.M. and Kryven' V.A. *Antiplane elastic-plastic deformation of a body with rigid inclusions* // Reports of AS Ukr. SSR. Ser. A. – 1979. – №2. – P. 104 – 107. (in Ukrainian).

[2] Berezhnytskyii L.T. and Kundrat LM.M. *On plastic strips at the tip of a rigid linear inclusion* // Problems of Strength. – 1982. – №11. – P. 66 – 69. (in Russian).

[3] Sylovaniuk V.P. *Development of plastic strips on a rigid disc-like inclusion* // Reports of AS Ukr.SSR. Ser. A. – 1985. – №10. – P.35 – 38. (in Russian).

[4] Panasyuk V.V. *Mechanics of quasi-brittle fracture of materials*. – Kiev: Naukova Dumka Publ.House, 1991. – 416 p. (in Russian).

[5] Panasyuk V.V., Stadnyk M.M. and Sylovaniuk V.P. *Stress concentration in 3-D bodies with thin inclusions*.– Kiev, Naykova Dumka Publ.House:, 1986. – 216 p. (in Russian).

[6] Volchok I.P. *Fracture resistance of steels and cast iron*. –Moscow: Metallurgy Publ.House, 1993. – 192 p. (in Russian).

[7] Brotman L .and Crook R. *Fracture and fatigue . Structural materials*. Vol.5.– Moscow: Mir Publ. House. –1978. – 485 p. (in Russian).