# Automatic Diagnostic System for Fatigue Damage with Statistical *F* test

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**Abstract.** In general, fatigue damage is calculated from *S-N* curve for materials or structures. However this method contains unreliability for stress and cycles which are measured or estimated. Even if on-line monitoring system is used to obtain stress and cycles in operation, *S-N* curve itself contains much scatter. In addition, in usual on-line monitoring each sensor is monitored independently, which is easily affected by change of circumstances. Therefore the estimated residual life still contains unreliability. This novel statistical diagnostic method described in this paper is a low cost simple system. The method employs system identification using a response surface with several measured points and the damage is automatically diagnosed by testing the change of the identified system by statistical *F* test. Therefore *S-N* curve is not needed to estimate fatigue damage. In this paper, the experiments were carried out to confirm whether this novel method is effective to estimate fatigue damage. As a result, it became clear that this novel method can be applied to diagnose fatigue damage.

## Introduction

Components are frequently subjected to repeated loads and therefore fatigue is a principal failure mechanism of the components and a major concern in engineering design and maintenance. Safe and accurate methods to predict fatigue life or damage are required in order to assess the reliability of such components. In general, fatigue life is calculated from *S-N* curve for materials or structures e.g. [1], [2]. However this method contains unreliability for the stresses and the cycles. If design stresses and cycles are used to estimate the fatigue life of a component, the stresses and the cycles are estimation containing safety factor, therefore it is difficult to determine the accurate life of the component. Even if stresses and cycles are obtained from operating condition, the life still remains unreliability. Because components are normally exposed to irregular variation of loads. Several methods are proposed to count the cycles and stresses under random loads [3]-[5] and to estimate *S-N* curve used for irregular stress histories [6]-[8]. However these are estimation to obtain the fatigue life and also contain unreliability. In addition, *S-N* curve itself contains much scatter (around factor of 10 for number of cycles) [1].

On the other hand, structural health monitoring systems are equipped with many sensors and are used to evaluate the damage state of structural components under operating condition. In order to prevent serious failure of the components, the structural health monitoring system has recently become a technology of interest [9]-[12]. Therefore many damage diagnosis methods for structural health monitoring system have been proposed (e.g. [13], [14]). For most of the diagnosis methods, the damage was measured by identifying the relationship between damage and sensor output and the damage was identified from the actual sensor measurement. However, these methods required the modeling of each component or a vast amount of learning data to identify the relationship between the sensor output and the damage condition of the component. Moreover, the output of the sensor was easily affected by various factors such as change of circumstances and operation, and electromagnetic noise, as the change of output from each sensor was monitored independently. Consequently, it was quite difficult to diagnose damage in structures efficiently.

This novel statistical diagnostic method described in this paper is a low cost simple system [15]. The method employs system identification using a response surface with several measured points and the damage is automatically diagnosed by testing the change of the identified system by statistical *F* test. Therefore *S*-*N* curve is not needed to estimate the fatigue damage. In this paper, the result of experiments to confirm whether this novel method is effective to estimate fatigue damage is described.

## Damage Diagnosis Method Using Statistical Similarity Test of Response Surfaces

Damage diagnosis in the present statistical method is carried out with the relationship between sensors using response surface and the change of the response surface using statistical tests. The damage diagnosis method with statistical tests is schematically shown in Fig. 1. The method can be divided into two parts; learning mode and monitoring mode. In the learning mode, reference data are obtained from sensors at components at the beginning of operation as non damaged state and a response surface is calculated. The response surface is called as reference response surface hereafter in this paper. In the monitoring mode, data are also obtained from sensors at components under operation and a response surface is calculated, which is called as measured response surface hereafter. After that, the measured response surface is compared with the reference response surface with statistical test of similarity. If damage occurs at the component, the equivalence between the measured response surface and the reference response surface can not be identified. As a result, it can be judged that some damage exists at the component. Therefore, in this method information about the damage state of the components is not previously needed to identify whether the damage occurs.

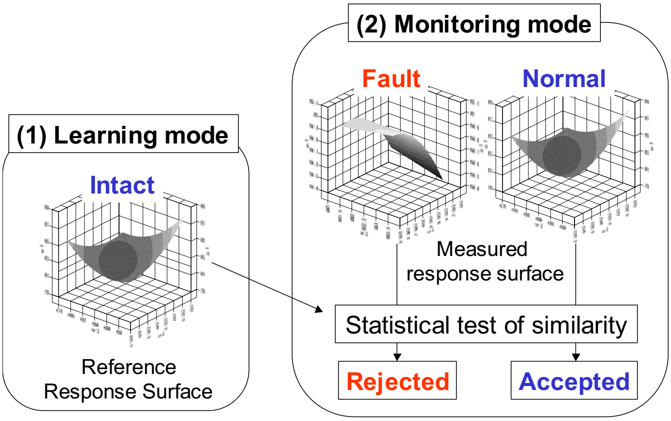


Fig. 1 Schematic diagnosis method with statistical test

**Response surface methodology.** Response surface methodology is employed for process optimisation in a quality-engineering field. In the response surface methodology, if the relationship between a response $y$ and predictors $x_i$ ($i = 1,2,\ldots,k$) can not be clearly determined, the approximate relationship is obtained using multiple-regression model. Generally, a response surface is represented by

$$y = f(x_1, x_2, \cdots, x_k) + \varepsilon \,. \tag{1}$$

where $\varepsilon$ is regression error. Any kinds of function can be used for *f*(), but in general polynomials are used for the response surface.

In this paper, quadratic polynomials of predictors described below are used for the approximation of the reference surface.

$$y = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \beta_{ii} x_i^{\,2} + \sum_{i=1}^{k-1} \sum_{j=i}^{k} \beta_{ij} x_i x_j \tag{2}$$

where $\beta$ is the regression coefficient. In Eq. (2), the number of predictors including squares or interactions of the predictors $x_i^2$ and $x_i x_j$ becomes $1+k+k+k(k-1)/2$, determined as $p$. Hence, Eq. (2) can be written as the following linear regression model.

$$y = \beta_0 + \sum_{i=1}^{p-1} \beta_i x_i \tag{3}$$

From $n$ times observations, Eq. (3) can be expressed as the following matrix form.

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$$
$$\mathbf{y} = \begin{Bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{Bmatrix}, \mathbf{X} = \begin{bmatrix} 1 & x_{11} & \cdots & x_{1p-1} \\ 1 & x_{21} & \cdots & x_{2p-1} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & x_{n1} & \cdots & x_{np-1} \end{bmatrix}, \boldsymbol{\beta} = \begin{Bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_{p-1} \end{Bmatrix}, \boldsymbol{\varepsilon} = \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{Bmatrix} \tag{4}$$

A least squares estimator $b$ of regression coefficient vector $\beta$ becomes

$$\mathbf{b} = (\mathbf{X}^{\mathrm{T}}\mathbf{X})^{-1}\mathbf{X}^{\mathrm{T}}\mathbf{y} \,. \tag{5}$$

Hence, Square Sum of Error (SSE) can be represented as

$$SSE = \mathbf{y}^{\mathrm{T}}\mathbf{y} - \mathbf{b}^{\mathrm{T}}\mathbf{X}^{\mathrm{T}}\mathbf{y} \,. \tag{6}$$

**Similarity test of response surfaces using *F*-test.** In the proposed method, damage diagnosis at a component can be carried out using an equivalence test between the reference response surface and a measured response surface. The equivalent test of the two response surfaces is conducted by the statistical *F*-test, which is generally used for testing the similarity of two distributions.

Reference response surface and measured response surface can be written as

$$\begin{aligned} \mathbf{y}_1 &= \mathbf{X}_1\boldsymbol{\beta}_1 + \boldsymbol{\varepsilon}_1 \\ \mathbf{y}_2 &= \mathbf{X}_2\boldsymbol{\beta}_2 + \boldsymbol{\varepsilon}_2 \,. \end{aligned} \tag{7}$$

where the number of experiments for regression are $n_1$ and $n_2$, respectively. A statistical hypothesis, $H_0$, where the two response surfaces are equivalent can be defined as

$$H_0 : \boldsymbol{\beta}_1 = \boldsymbol{\beta}_2 . \tag{8}$$

Assume that each error term ($\varepsilon$) is independent and has the same distribution in both experiments. In this case, the $F$-statistic value $F_0$ can be defined as follows.

$$F_0 = \frac{SSE_0 - (SSE_1 + SSE_2)}{SSE_1 + SSE_2} \times \frac{n-2p}{p} \tag{9}$$

where $SSE_1$ and $SSE_2$ are SSE (see Eq. (6)) for the reference response surface and the measured response surface, respectively. $SSE_0$ is SSE of the response surface with all data (i.e. data of the response surface and the measured response surface) and $n$ is the sum of $n_1$ and $n_2$. When the reference response surface and the measured response surface are equivalent, The $F$-statistic value $F_0$ follows a $F(p, n-2p)$ distribution. Hence, the critical region for hypothesis $H_0$ is determined with significance level $\alpha$ as

$$F_{p,n-2p}\left(1-\frac{\alpha}{2}\right) < F_0 < F_{p,n-2p}\left(\frac{\alpha}{2}\right). \tag{10}$$

If $F_0$ is larger than $F_{p,n-2p}(\alpha/2)$, the similarity of the response surfaces is rejected. Hence, the critical region of the similarity test of the response surfaces is determined only from the significant level and the model of the response surface and the information about damage state is not needed.

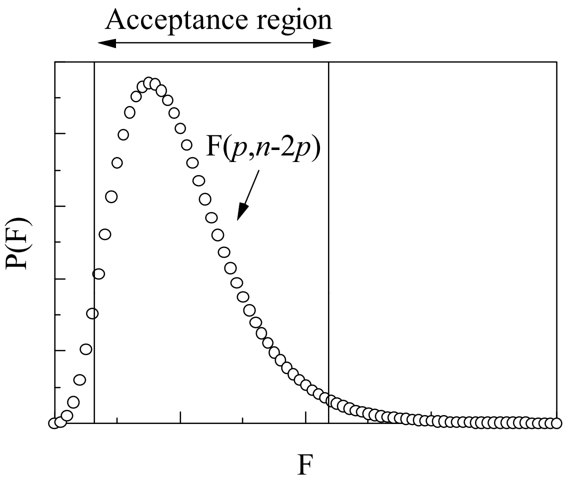


Fig. 2 $F_0$ distribution for equivalence between reference response surface and measured response surface and acceptance region of hypothesis $H_0$

## Diagnosis of Fatigue Crack in welded component

In order to examine the effectiveness of the present diagnosis methods (called as "SI-F method" hereafter) to fatigue damage, fatigue testing was carried out using an welded component shown in Fig. 3. Three-point bending testing was carried out with the span of 4000 mm. Locations of strain gauges and AE sensors to monitor the fatigue damage are schematically shown in **Fig. 4**. Based on reference [2], the welded joints monitored with strain gauges are located as the design fatigue lives become the same cycles.

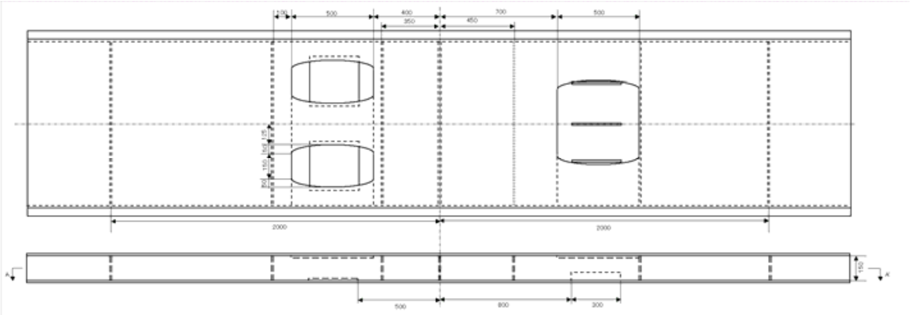
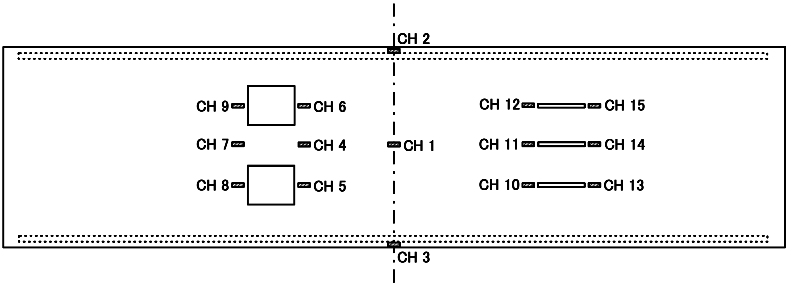
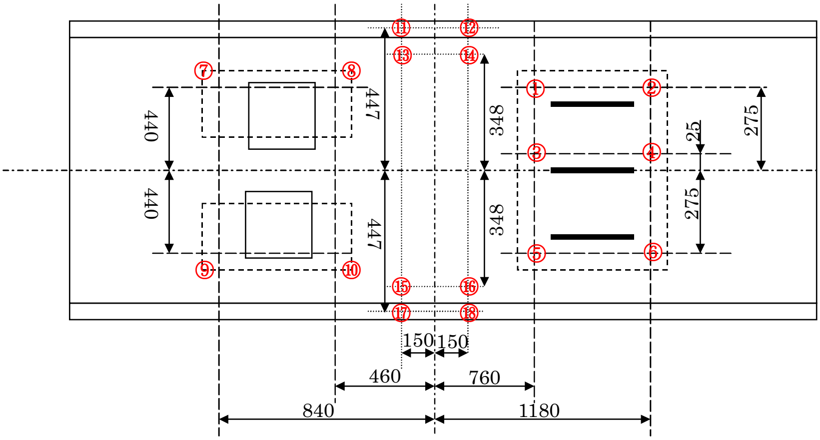


Fig. 3 Welded component for fatigue test



(a) Location of strain gauges



(b) Location of AE sensor
Fig. 4 Locations of sensors to monitor fatigue damage

**Experimental Result.** Fatigue testing condition was shown in **Table 1**. Visual inspection was carried out every 75,000cycles. Until 355,000 cycles, fatigue crack was not observed by the visual inspection.

Fatigue crack of 5 mm length was firstly detected by visual inspection on 371,500 cycles at the location of Ch. 3 in Fig. 4(a). The result of AE sensors No. 15 to 18 in Fig. 4(b), is shown in Fig. 5. It might be seen that the fatigue crack was detected at around 354,000 cycles, due to the change of the number of AE over 60 dB.

Table 1 Fatigue Testing Conditions

| | |
|---|---|
| Frequency | 0.8 Hz |
| Loading conditions | 155 kN ± 150 kN until 355,000cycles<br>165 kN ± 160 kN after 355,000cycles |

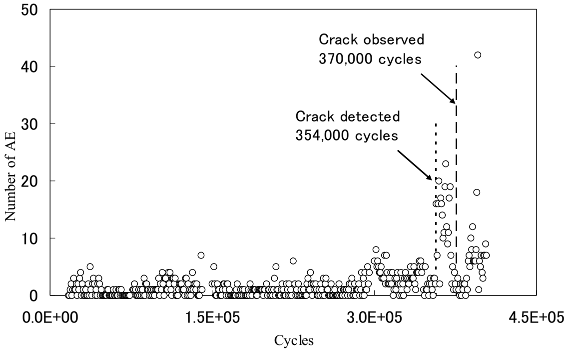


Fig. 5 Results of AE sensor (Number of AE over 60dB)

## Detection of Fatigue Crack Using SI-F method

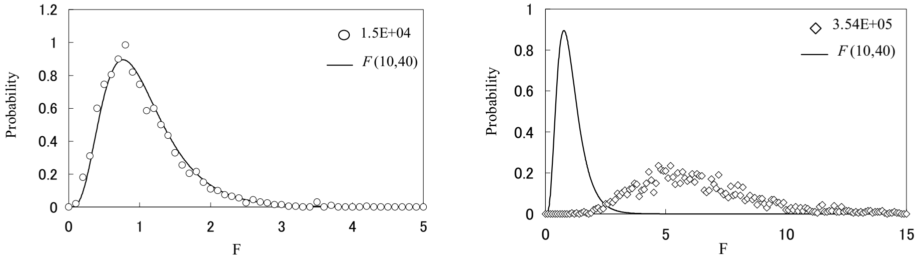
**Optimisation of Sensors used for SI-F method with statistical *t*-test.** The number of strain gauges used in the present study is 15. If all sensors are used to derive a response surface, the number of predictors in Eq. (3) becomes 106, which lead to time consuming process. Therefore, it is needed that the effective and strong correlated sensors for each response are selected to calculate the response surface in all sensors. In order to derive the optimum regression model for the response surface, the statistical *t*-test has been performed. The statistical t-value can be defined as

$$t = \frac{b_i}{\sqrt{\frac{SSE}{n-p} C_{ii}}} . \tag{11}$$

where $b_i$ is the least-squares estimator of $\beta$ and $C_{ii}$ is diagonal section of variance-covariance matrix. From Eq. (11), the term which has the worst *t*-statistical value when the term derives worth regression result is deleted, until the degree of accuracy for the regression becomes maximum. The degree of accuracy is determined as follows

$$R^2{}_{adj} = 1 - \frac{SSE/(n-p)}{Sy/(n-1)} . \tag{12}$$

In the present study, 4 sensors were selected to calculate response surface. F-distributions for Ch.3 as a response at 15,000 cycles and 354,000 cycles are shown in Fig. 6 (a) and (b), respectively. It is clear that at 15,000 cycles $F_0$ distribution is equivalent to theoretical $F(10,40)$ distribution and therefore fatigue damage does not exit. Whilst, at 354,000 cycles the $F_0$ distribution is completely different to the theoretical $F(10,40)$ distribution, which indicates that fatigue damage occurred at around Ch.3. If Ch. 4 is selected as a response, fatigue damage is not detected at 354,000 cycles as shown in Fig. 7. Average value of $F_0$ with 100 points for Ch.3 as a response with time is shown in Fig. 8. It is clear that the average of $F_0$ changes from 330,000 cycles, which is faster than the cycles where crack was detected with visual inspection. Compared Fig. 8 and Fig. 5, the change of the average value of $F_0$ at 330,000 cycles indicates the sign of the occurrence of fatigue crack.



(a) 15,000 cycles (b) 354,000 cycles

Fig. 6 Results of $F$-tests for Ch.3 as a response

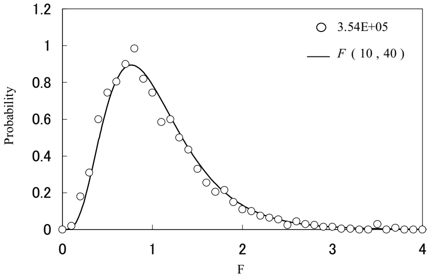


Fig. 7 Results of $F$-tests for Ch.4 as a response at 354,000 cycles

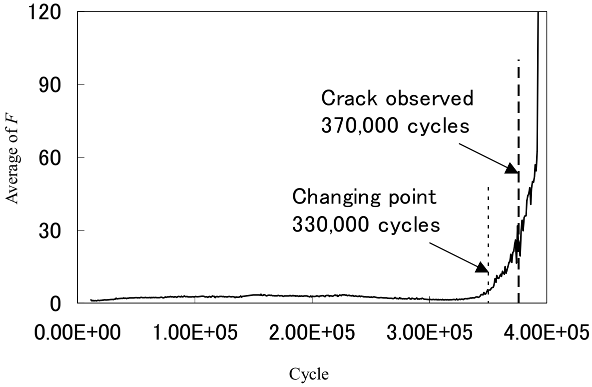


Fig. 8 Average of $F$ value for Ch. 3 as a response with time

## Summary

In this paper, SI-F method was proposed as a diagnostic system for fatigue damage. In order to examine the effectiveness, fatigue test was carried out using welded components. The results are summarised below.

1) The SI-F method can be applied to diagnose fatigue damage, as the method detected fatigue damage faster than visual inspection.
2) Compared to results of AE sensor, the SI-F method detected the fatigue damage faster than AE sensor, which indicates that the SI-F method can detect the early stage of fatigue damage.

In future, the following tasks remain to apply the SI-F method to actual components.

1) Relationship between the locations of sensor and the degree of accuracy for the diagnosis. In addition, the amount of area for one sensor to be diagnose should be specified.
2) The influence of noise and random load should be also clarified.

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