About Unambiguity and Reliability of the Solution for Plastic Zone Magnitude around Crack Tip in Isotropic Strain Hardening Material

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Abstract. In this article, a review on the results of research of dependence of plastic zone magnitude around crack tip r_p on *strain hardening exponent n* is being carried out. For determining a plastic zone magnitude r_p are used the analytical methods and the Dugdale´s model as well. Isotropic and non linear strain hardening of a material is assumed. The strain hardening exponent n is changed among the discreet values $n = 3, 5, 7, 10, 25$ and ∞. An analytical solution is being given by means of gamma function *Γ(x)* and hypergeometric function ${}_2F_1(\alpha,\beta;\gamma;z)$. Afterwards, a hypergeometric function is expanded into a series. In further analysis we can take only first member, or first two members, or maybe three and more members of the series. How does it influence on an accuracy and reliability of a solution for plastic zone magnitude r_p?

Introduction

The investigations have been performed on the thin infinite plate in which a straight plane crack of a length $2a$ is incorporated. The plane stress is assumed, i.e. $\sigma_{xx} = \sigma_{xx}(x,y)$, $\sigma_{yy} = \sigma_{yy}(x,y)$ and $\sigma_{xy} = \sigma_{xy}(x,y)$. There are two axes of symmetry in the plate, x and y. The shear stresses $\sigma_{xy}(x,y)$ at the axes of symmetry equal zero, i.e. $\sigma_{xy}(x,0)=0$ and $\sigma_{xy}(0,y)=0$. This statement will have as a consequence that the normal stresses at those axes will be, at the same time, the principal stresses: $\sigma_{xx}(x,0)$, $\sigma_{yy}(x,0)$, $\sigma_{xx}(0,y)$ and $\sigma_{yy}(0,y)$. If a response of a structure on the given external load is elastic, then in the crack tip, $x = a$, a singularity in a stress distribution will occure. Assuming that the plate material is ductile the small plastic zones around the crack tips will appear. The appearance of the plastic zones around the crack tips will cancel the mentioned singularity.

In the article, the Dugdale strip yield model is being used for determining the plastic zone magnitude around the crack tip r_p, [1,2,3,4,5]. Although that model considerably simplifies the real physical picture of material behavior around the crack tips, it was shown very efficacious for solving many engineering problems of Elastic Plastic Fracture Mechanics (EPFM). The essence of the Dugdale strip yield model consists of a fact that this model, instead of the physical elastic crack, length of $2a$ and with a stress singularity within its tips, observes a *fictitious elastic crack*, the length of $2b = 2(a + r_p)$. The normal stress $\sigma_{yy}(b,0)$ at the tip of that fictitious elastic crack has a final magnitude. The physical blunt crack and the plastic zone around its tip make the sharp fictitious elastic crack. A non-singularity stress condition within the tip of fictitious elastic crack is possible to write analytically in the following way

$$K(a+r_p) = K_{ext}(a+r_p) + K_{coh}(a+r_p) = 0 . \qquad (1)$$

The singularity within the tip of a fictitious elastic crack $x = b = a + r_p$, of the external load of a plate is canceled with the singularity of the cohesive stresses within the plastic zone.

Modeling of the cohesive stresses in isotropic strain hardening material

As the exact analytical solution of distribution of the cohesive stresses around the crack tip is unknown, one of the possible approaches to the problem is following. It is possible to determine the distribution of the cohesive stresses, for example, by the finite element method and then that distribution is approximated with an analytical expression, i.e. with some function, for example with an exponential function, or with a logarithmic function, or with a hyperbolic function and so on. The same approach was used in the paper [7] and it was shown very well. The authors M. Hoffman and T. Seeger have proposed in their article [2] the next analytical expression

$$p(x) = \sigma_0 \cdot \left[r_\mathrm{p} / (x-a) \right]^{1/(n+1)}, \tag{2}$$

for the distribution of the cohesive stresses. The quantity $p(x)$, in that expression, is a function of two parameters, i.e. *the magnitude of the plastic zone* around the crack tip r_p and *the strain hardening exponent n.* In the article [7] it has been shown that this expression approximates excellently the distribution of the cohesive stresses obtained by means of the finite element method. The same expression, the authors X. G. Chen, X. R. Wu and M. G. Yan have used in their paper [1].

Determination of the stress intensity factor from the cohesive stresses by means of the Green's functions

The stress intensity factor at the tip of fictitious elastic crack $K_\mathrm{coh}(b)$ can be determined using the method of the Green's functions, knowing the distribution of the cohesive stresses $p(x)$

$$K_\mathrm{coh}(b) = \int_a^b p(x) \cdot m(x,b)\,\mathrm{d}x = \int_a^b \sigma_0 \cdot \left[r_\mathrm{p} / (x-a) \right]^{1/(n+1)} \cdot 2\sqrt{b/\pi} \cdot \left(b^2 - x^2 \right)^{-1/2} \mathrm{d}x, \tag{3}$$

It is suitable, before integration, to transform the expression (3), introducing a new independent variable ξ which is with a previous one connected by the relation, according to [1]

$$\xi = 1 + (a-x)/r_\mathrm{p}. \tag{4}$$

From the expression (4) it is obtained $x = a + r_\mathrm{p} \cdot (1-\xi) = b - r_\mathrm{p} \cdot \xi$, so it is $\mathrm{d}x = -r_\mathrm{p} \cdot \mathrm{d}\xi$. It is easily noticed that at the tip of physical crack it is $x = a$, and consequently, $\xi = 1$. Similarly, at the tip of fictitious elastic crack it is $x = b = a + r_\mathrm{p}$ and therefore $\xi = 0$. The stress intensity factor $K_\mathrm{coh}(b)$, according to the equation (3), now it is possible to express through the new independent variable ξ and it looks like [6]

$$K_\mathrm{coh}(b) = \sqrt{2r_\mathrm{p}/\pi} \cdot \sigma_0 \int_0^1 \frac{1}{(1-\xi)^{1/(n+1)}} \cdot \frac{1}{\left\{ \xi \left[1 - (r_\mathrm{p}/2b) \cdot \xi \right] \right\}^{1/2}} \mathrm{d}\xi. \tag{5}$$

To the solution of the above integral it is possible to approach in two ways:

- not introducing any assumptions, or restrictions, i.e. fully exactly,
- by introducing the assumptions about small plastic zone around the crack tip. In this case, it is possible to take $r_\mathrm{p}/2b \approx 0$.

Analytical solution obtained using commercial package „Mathematica“. The exact solution of above integral (5) has been obtained using the software *„Mathematica“*, [8], so we obtain

$$K_{\mathrm{coh}}(b)=\sqrt{2r_{\mathrm{p}}/\pi}\cdot\sigma_0\cdot\sqrt{\pi}\cdot\left[\Gamma\left(\frac{n}{n+1}\right)\Big/\Gamma\left(\frac{3}{2}-\frac{1}{n+1}\right)\right]\cdot{}_2F_1\left(\frac{1}{2},\frac{1}{2},\frac{3}{2}-\frac{1}{n+1},\frac{r_{\mathrm{p}}}{2b}\right). \tag{6}$$

At the expression (6), $\Gamma(x)$ stands for the gamma function, or the Euler's integral of second order, while ${}_2F_1\left(\frac{1}{2},\frac{1}{2},\frac{3}{2}-\frac{1}{n+1},\frac{r_{\mathrm{p}}}{2b}\right)$ denotes the hypergeometric function. That function, within the commercial code *„Mathematica“*, is denoted as ${}_2F_1(a,b;c;z)$. As it is usually within the fracture mechanics, the mark a stands for the length of physical crack, while the mark b denotes the length of a fictitious elastic crack. In order to avoid confusion, we will the hypergeometric function ${}_2F_1$ denote in the following way: ${}_2F_1(\alpha,\beta;\gamma;z)$. It is easily to notice that the argument $\left(3/2-1/(n+1)\right)$ is same as $\left(1/2+n/(n+1)\right)$, so it is valid $\Gamma\left(3/2-1/(n+1)\right)=\Gamma\left(1/2+n/(n+1)\right)$. It is possible to expand the hypergeometric function ${}_2F_1(\alpha,\beta;\gamma;z)$ into the series. The solution which gives commercial package *„Mathematica“* for the definition domain $\{z,0,3\}$ looks like

$$\begin{aligned}{}_2F_1(\alpha,\,\beta;\,\gamma;\,z)=1+\frac{\alpha\beta z}{\gamma}+\frac{\alpha(1+\alpha)\cdot\beta(1+\beta)\cdot z^2}{2\gamma(1+\gamma)}+\\+\frac{\alpha(1+\alpha)(2+\alpha)\cdot\beta(1+\beta)(2+\beta)\cdot z^3}{6\gamma(1+\gamma)(2+\gamma)}+O[z]^4\,.\end{aligned} \tag{7}$$

Now, it is clearly seen, that is possible to get the different values of the stress intensity factor $K_{\mathrm{coh}}(b)$, according to expression (6), depending if we take from the series expansion (7) only first member, or only first two members, or the first three members, or rather more members. That will sure influence at the final result. But, the question is how and in what measure? We have investigated the problem in this article.

If we restrict on forming the small plastic zone around the crack tip, then the ratio $r_{\mathrm{p}}/2b$ in the integral (5) can be taken approximately zero, so that the integral in that case takes the form

$$K_{\mathrm{coh}}(b)=\sqrt{2r_{\mathrm{p}}/\pi}\cdot\sigma_0\cdot\int_0^1\frac{1}{(1-\xi)^{1/(n+1)}}\cdot\frac{1}{\sqrt{\xi}}\mathrm{d}\xi\,. \tag{8}$$

That integral is possible to solve fully exactly, analytical. The solution is explained in detail in the article [6]. Only the final result is being quoted here

$$K_{\mathrm{coh}}(b)=\sqrt{\frac{2}{\pi}\cdot r_{\mathrm{p}}}\cdot\sigma_0\cdot\Gamma\left(\frac{1}{2}\right)\cdot\Gamma\left(\frac{n}{n+1}\right)\Big/\Gamma\left(\frac{1}{2}+\frac{n}{n+1}\right)=\sqrt{\frac{2}{\pi}\cdot r_{\mathrm{p}}}\cdot\sigma_0\cdot B\left(\frac{n}{n+1},\,\frac{1}{2}\right). \tag{9}$$

If this solution is compared with the one we got with the commercial software *„Mathematica“*, (6), then it is seen, that the solution (9) will be identical to the solution (6), under a condition that from the series expansion (7) we take only first member.

Magnitude of the plastic zone around the crack tip in isotropic strain hardening material and unambiguity of the solution

Analytical solution by the assumption about small plastic zone around the crack tip, i.e. taking in a consideration only first member of the series expansion, according to (7). The stress intensity factor within the tip of the fictitious elastic crack from the external loads is equal

$$K_{\text{ext}}(a+r_{\text{p}}) = \sigma_{\infty} \cdot \sqrt{\pi \cdot b}\,. \tag{10}$$

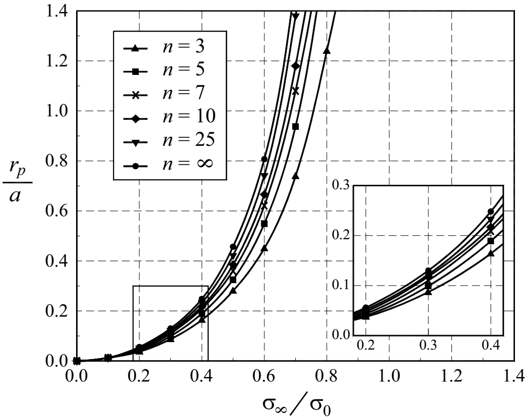

Figure 1. Dependence of plastic zone magnitude around the crack tip r_{p}/a on a monotonously increasing external load of a plate σ_{∞}/σ_0 , for the different values of a strain hardening exponent *n*

By equating the right sides of the expressions (10) and (9), in accordance with the condition (1) and after insignificant transformations, the analytical expression for calculating of a magnitude of plastic zone r_{p} around the crack tip is obtained

$$r_{\text{p}} = \frac{\pi}{2}\left(\frac{\sigma_{\infty}}{\sigma_0}\right)^2 \cdot \frac{\left[\Gamma\left(\frac{1}{2}+\frac{n}{n+1}\right)\Big/\Gamma\left(\frac{n}{n+1}\right)\right]^2}{1-\frac{\pi}{2}\left(\frac{\sigma_{\infty}}{\sigma_0}\right)^2 \cdot \left[\Gamma\left(\frac{1}{2}+\frac{n}{n+1}\right)\Big/\Gamma\left(\frac{n}{n+1}\right)\right]^2} \cdot a\,. \tag{11}$$

On a base of the analytical expression (11), the magnitude of the plastic zone around the crack tip r_{p} was calculated, in dependence on a monotonously increasing external load of a plate σ_{∞} and for six different values of *the strain hardening exponent* $n = 3, 5, 7, 10, 25$ and ∞. The diagram is in a non dimensional form, $r_{\text{p}}/a = f\left(\sigma_{\infty}/\sigma_0, n\right)$, shown at the Fig. 1.

Unambiguity and reliability of the solution taking into consideration first two members of the series expansion, according to (7). If we take the first two members of the series expansion (7) and insert that result into the expression (6), the final analytical expression for the stress intensity factor caused by the cohesive stresses is obtained

$$K_{\text{coh}}(b) = \sqrt{\frac{2}{\pi} \cdot r_{\text{p}}} \cdot \sigma_0 \cdot \sqrt{\pi} \cdot \left[\Gamma\left(\frac{n}{n+1}\right)\Big/\Gamma\left(\frac{1}{2}+\frac{n}{n+1}\right)\right] \cdot \left(1+\frac{1}{4}\cdot\frac{n+1}{3n+1}\cdot\frac{r_{\text{p}}}{b}\right). \tag{12}$$

If we introduce the marks

$$p=\sqrt{\pi}\cdot\Gamma\left(\frac{n}{n+1}\right)\Big/\Gamma\left(\frac{1}{2}+\frac{n}{n+1}\right)\qquad\text{and}\qquad q=\frac{1}{4}\sqrt{\pi}\cdot\frac{n+1}{3n+1}\cdot\Gamma\left(\frac{n}{n+1}\right)\Big/\Gamma\left(\frac{1}{2}+\frac{n}{n+1}\right)\quad(13)$$

the expression (12) is possible to write down compactly

$$K_{\mathrm{coh}}(b)=\sqrt{\frac{2}{\pi}\cdot r_{\mathrm{p}}}\cdot\sigma_0\cdot\left(p+q\cdot\frac{r_{\mathrm{p}}}{b}\right).\qquad(14)$$

In accordance with the condition (1), we equate the right sides of the expressions (10) and (14)

$$\sigma_\infty\cdot\sqrt{\pi\cdot b}=\sqrt{\frac{2}{\pi}\cdot r_{\mathrm{p}}}\cdot\sigma_0\cdot\left(p+q\cdot\frac{r_{\mathrm{p}}}{b}\right).\qquad(15)$$

By sorting of the above expression, by taking into a consideration that is $b=a+r_{\mathrm{p}}$ and introducing a new mark $m=\pi^2\cdot\sigma_\infty^2/\sigma_0^2$, it is obtained a *cubic equation* for determining the plastic zone magnitude r_{p} around the crack tip. That equation in an arranged form looks like

$$\left[m-2(p+q)^2\right]\cdot r_{\mathrm{p}}^3+a\cdot\left[3m-4p\cdot(p+q)\right]\cdot r_{\mathrm{p}}^2+a^2\cdot\left(3m-2p^2\right)\cdot r_{\mathrm{p}}+ma^3=0.\qquad(16)$$

It is possible to solve this equation using software package „*Mathematica*“, or any other mathematical tool. If that equation is solved for the discreet values of the parameters, for example $a=10$ mm, $\sigma_\infty/\sigma_0=0.5$, n = 5, $\Gamma(n/(n+1))=1.13216$, $\Gamma(1/2+n/(n+1))=0.89338$, the roots of the cubic equation are obtained

$$r_{\mathrm{p}\cdot 1}=-9.16661-0.112682\mathrm{i}\ [\mathrm{mm}],\qquad r_{\mathrm{p}\cdot 2}=-9.16661+0.112682\mathrm{i}\ [\mathrm{mm}],\qquad(17)$$
$$r_{\mathrm{p}\cdot 3}=3.05705\ [\mathrm{mm}].$$

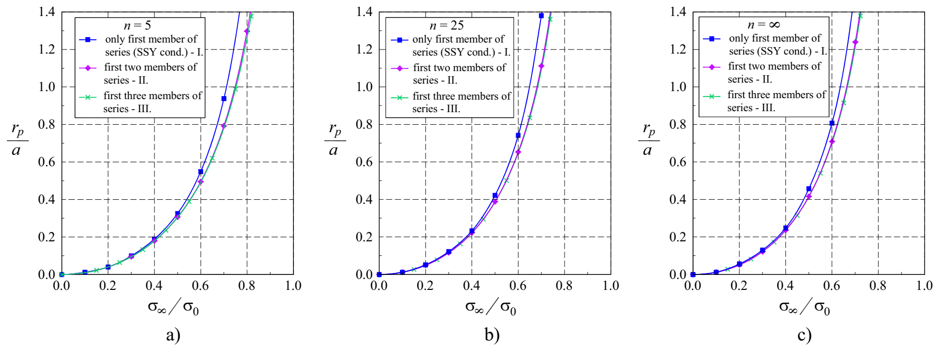

Figure 2. Comparison of the solutions $r_{\mathrm{p}}/a=f(\sigma_\infty/\sigma_0)$ obtained by using the commercial code *Mathematica* in a case if one, two and three members of the series expansion (7) are taken a) for n =5, b) for n =25 and c) for n = ∞

Hence, the pair of the complex conjugates roots and one real root are obtained. In the solutions (17) the mark „i“ stands for the imaginary unit, i.e. $\mathrm{i}=\sqrt{-1}$. The pair of the complex conjugates roots, as a solution, is not physical acceptable. Only a real solution is physical acceptable and that is the genuine solution. The solution is *unambiguous*, because one specified value of the external load $\sigma_{yy}^{\infty} = \sigma_{\infty}$ corresponds only one, exactly determined magnitude of the plastic zone r_{p}.

If we preserve the value of a *strain hardening exponent* n = 5, at the equation (16), but we change the magnitude of external load σ_{∞}/σ_0 from 0 to 1, we will get a monotonously increasing and smooth curve of a dependence $r_{\mathrm{p}}/a = f\left(\sigma_{\infty}/\sigma_0\right)$. Two by two curves regarding to the three different values of the strain hardening exponent n = 5, 25 and ∞, are shown at the Figs. 2a,b and c.

Is the solution unambiguous if the first three members of a series expansion (7) or more them are taken in consideration? Let us take the first three members of the series expansion (7), in which we have developed hypergeometric function ${}_2F_1\left(\alpha,\beta;\gamma;z\right)$. After arranging it is obtained

$$ {}_2F_1\left(\frac{1}{2},\frac{1}{2};\frac{3}{2}-\frac{1}{n+1};\frac{r_{\mathrm{p}}}{2b}\right) = 1+\frac{1}{4}\cdot\frac{n+1}{3n+1}\cdot\frac{r_{\mathrm{p}}}{b}+\frac{9}{32}\cdot\frac{\left(n+1\right)^2}{\left(3n+1\right)\cdot\left(5n+3\right)}\cdot\frac{r_{\mathrm{p}}^2}{b^2}. \tag{18} $$

Then the stress intensity factor of the cohesive stresses, within the tip of a fictitious elastic crack, according to the expression (6), submits

$$ K_{\mathrm{coh}}\left(b\right) = \sqrt{\frac{2}{\pi}\cdot r_{\mathrm{p}}}\cdot\sigma_0\cdot\sqrt{\pi}\cdot\frac{\Gamma\left(\dfrac{n}{n+1}\right)}{\Gamma\left(\dfrac{1}{2}+\dfrac{n}{n+1}\right)}\cdot\left(1+\frac{1}{4}\frac{n+1}{3n+1}\cdot\frac{r_{\mathrm{p}}}{b}+\frac{9}{32}\cdot\frac{\left(n+1\right)^2}{\left(3n+1\right)\left(5n+3\right)}\cdot\frac{r_{\mathrm{p}}^2}{b^2}\right). \tag{19} $$

If in addition to the existing designations p and q, according to (13), is introduced one new mark s

$$ s = \frac{9}{32}\cdot\sqrt{\pi}\cdot\frac{n+1}{3n+1}\cdot\frac{n+1}{5n+3}\cdot\Gamma\left(\frac{n}{n+1}\right)\Big/\Gamma\left(\frac{1}{2}+\frac{n}{n+1}\right) = \frac{9}{8}\cdot\frac{n+1}{5n+3}\cdot q\,, \tag{20} $$

the analytical expression (19) is possible to write compactly in a following way

$$ K_{\mathrm{coh}}\left(b\right) = \sqrt{\frac{2}{\pi}\cdot r_{\mathrm{p}}}\cdot\sigma_0\cdot\left(p+q\cdot\frac{r_{\mathrm{p}}}{b}+s\cdot\frac{r_{\mathrm{p}}^2}{b^2}\right). \tag{21} $$

In accordance with the condition (1), let us equate the right sides of the expressions (10) and (21)

$$ \sigma_{\infty}\cdot\sqrt{\pi\cdot b} = \sqrt{\frac{2}{\pi}\cdot r_{\mathrm{p}}}\cdot\sigma_0\cdot\left(p+q\cdot\frac{r_{\mathrm{p}}}{b}+s\cdot\frac{r_{\mathrm{p}}^2}{b^2}\right). \tag{22} $$

By expanding that expression, by putting in the introduced designations p, q and s, according to the expressions (13) and (20), the two unknown quantities will appear seemingly at it, i.e. r_{p} and b. But, there is a connection among them, so that is $b = a + r_{\mathrm{p}}$. Consequently, the substitution must be done, so that the one quantity is replaced with another. At this place, we have decided that the plastic zone magnitude r_{p} we replace with the length b of a fictitious elastic crack. By arranging it, it is obtained *the equation of fifth order* for determining of the length b. That equation looks like

$$\left\{\frac{m}{2}-(p+q)^2-s\cdot\left[2\cdot(p+q)+s\right]\right\}\cdot b^5+\left[(p+q)\cdot(p+3q)+s\cdot(6p+8q+5s)\right]\cdot a\cdot b^4-$$
$$-\left[q\cdot(2p+3q)+2s\cdot(3p+6q+5s)\right]\cdot a^2\cdot b^3+\left[q^2+2s\cdot(p+4q+5s)\right]\cdot a^3\cdot b^2- \tag{23}$$
$$-s\cdot(2q+5s)\cdot a^4\cdot b+s^2\cdot a^5=0$$

We have solved the equation (23) by means of the commercial software „*Mathematica*" beside the discreet values of the parameters $a=10$ mm, $\sigma_\infty/\sigma_0=0.5$ and $n=5$. The roots of that equation are

$$\begin{aligned} &b_1=0{,}497563-1{,}29372\ \mathrm{i}\quad[\mathrm{mm}], && b_3=0{,}74755-1{,}21458\ \mathrm{i}\quad[\mathrm{mm}],\\ &b_2=0{,}497563+1{,}29372\ \mathrm{i}\quad[\mathrm{mm}], && b_4=0{,}74755+1{,}21458\ \mathrm{i}\quad[\mathrm{mm}],\\ &b_5=13{,}0478\quad[\mathrm{mm}], && \left(r_\mathrm{p}=b_5-a=3{,}0478\quad[\mathrm{mm}]\right). \end{aligned} \tag{24}$$

As it is seen, the two pairs of the complex conjugates roots and one real root are obtained. The complex conjugates roots, as the solutions, aren't physically acceptable. Only the real solution is physically acceptable and it is the genuine solution. It is possible to declare that the solution is *unambiguous* for the given parameters of geometry, loading and material. The second and the third curve $r_\mathrm{p}/a=f(\sigma_\infty/\sigma_0)$ are coincided, i.e. there is no more difference in a magnitude of r_p regardless if we take into consideration two, or three members of a series expansion (7), or even more them.

Review on the obtained results

The aim of these investigations was to establish in what manner the isotropic strain hardening of a material influences on the magnitude of a plastic zone around the crack tip r_p. In order to model the different levels of strain hardening of a material, *the strain hardening exponent n* was varied, so it has taken the values n = 3, 5, 7, 10, 25 and ∞. The diagram, presented at the Fig. 1, has been drawn according to the analytical expression (11) and within itself contains the assumption about small plastic zone around the crack tip. By analyzing the diagram it is possible to conclude:

- isotropic strain hardening of a material will considerably influence on the magnitude of a plastic zone r_p,
- that the magnitude of a plastic zone r_p will be as *bigger* as the strain hardening of a material is *smaller*, for the same level of external load. Therefore, bigger r_p for bigger n,
- the magnitude of a plastic zone r_p, for certain level of external load σ_∞, will be the largest by the *elastic perfectly plastic* material.

Furthermore, particularly in this article, our aim was to investigate *the unambiguity of the solution* by determining the magnitude of a plastic zone r_p. By analyzing the results presented at the Figs. 2a, b and c, it was established that:

- the difference in the magnitude of a plastic zone r_p exists, depending on taking into consideration only the first member of the series expansion (7), or the first two members. If only the first member of the series expansion is taken, the obtained solution will be unambiguous and it is obtained by solving *the linear equation*,
- there is no difference in the magnitude of a plastic zone r_p in a case when the first two, three, four members of the series expansion (7) are taken into consideration, or more them. The curves are then coincided.

Conclusion

In the analysis of the parameters of Elastic Plastic Fracture Mechanics (EPFM), in this article, the analytical methods and the commercial software „*Mathematica*“ were used. The stress intensity factor of the cohesive stresses $K_{\text{coh}}(b)$ was determined by means of the Green's functions (3). The determination of that coefficient is based on the *known* distribution of the cohesive stresses within a plastic zone. Exactly that is the biggest unknown. In this paper we have assumed that the cohesive stresses are distributed according to the analytical expression (2). In the paper [7], the authors have determined a distribution of the cohesive stresses by numerical way, by means of the finite element method and by using the commercial software „*Abaqus*“. The obtained results have exceptionally good agreed with the once obtained by means of the analytical expression (2).
In realizing an exact analytical solution, the integral (5) was solved by means of commercial software „*Mathematica*“. Ours suggestions at a research, an analysis and at an application of those solutions, (6) and (7), would be next:

- only the first member of a series expansion (7) is necessary to take into consideration,
- the experimental determination (measurement) of magnitude of the plastic zone r_{p} is necessary to perform and to compare it with an analytical solution, in order to establish how precise and reliable, are the results, we have obtained by analytical way,
- in a case of disagreement among analytical solution with experimental obtained quantity r_{p}, the first two members of a series expansion (7) have to be included into calculation and so compute the magnitude of a plastic zone r_{p}, by using the software „*Mathematica*“.

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