A regression model for statistical assessment of fatigue strain- and stress lifetimes

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Abstract. In this work, a novel Weibull regression model, based on a former $S-N$ model developed by the authors, is proposed for the probabilistic definition of the $\varepsilon-N$ field as an alternative to the conventional Basquin-Coffin-Manson approach. The model arises from sound statistical and physical assumptions and, integrating the elastic and the plastic local strain, provides an analytical probabilistic definition of the whole strain-life field as quantile curves both in the low cycle and high cycle fatigue regions. The five parameters of the model can be estimated using different well established methods proposed in the fatigue literature, in particular, the maximum likelihood and the two-stage methods providing adequate estimates based on the set of experiments. Runouts, as a frequent case in fatigue testing, can be also dealt with. The model can be applied for probabilistic lifetime prediction using damage accumulation. Finally, the proposed $\varepsilon-N$ model is also used to derive the corresponding stress life curves, i.e. the probabilistic $S-N$ field that in this case presents the typical change in curvature in the low-cycle fatigue region.

Introduction

The strain-based approach considers the magnitude of the varying strain as an alternative to the more popular stress-based approach when local plasticity effects are present. It uses strain-life curves along with the cyclic strain-stress diagram obtained from unnotched specimens as basic material characteristics proving to be a suitable method for estimating the fatigue life of mechanical and structural elements in both low- and high-cycle fatigue regions [1].

Usually, the relation between the total strain amplitude ε_a and the fatigue life measured in cycles N_f is given as strain-life curves based on the former proposals of Basquin [2] for the elastic strain-life and Coffin-Manson [3,4,5] for the plastic strain-life:

$$\varepsilon_a = \varepsilon_a^e + \varepsilon_a^p = \frac{\sigma'_f}{E}\left(\frac{2N_f}{N_0}\right)^b + \varepsilon'_f\left(\frac{2N_f}{N_0}\right)^c . \qquad (1)$$

where the superscripts e and p are used for the *elastic* and *plastic* strain, respectively. N_f is the number of cycles, σ'_f the fatigue strength coefficient, b the fatigue strength exponent, ε'_f the fatigue ductility coefficient, c the fatigue ductility exponent, E the Young modulus, and N_0 a reference number of cycles to render Eq.(1) dimensionless. The estimation of the four model parameters results by fitting two regression lines corresponding to the elastic and plastic components, respectively, the summation of which gives the total strain, see Fig. 1. The transition fatigue life point, where the magnitudes of the elastic and plastic strain amplitudes coincide, must be determined in advance to proceed to data evaluation [6].

Some of the limitations implied in the model given by Eq. (1) are:

- The strain-life fatigue equation is based on arbitrary though reasonable assumptions of power strain-lifetime laws for both elastic and plastic strain components.
- The assumed linear form in the high cycle region of the strain-life on a log-log plot (see Fig. 1) prevents the existence of a fatigue limit.
- The parameter derivation requires prior determination of the transition fatigue life in order to discriminate between the elastic and plastic strain dominant regions in which the regression analysis will be sequentially applied.
- Run-outs cannot be used for the estimation.
- The model reproducing the variability of the predictions is not statistically justified, in particular, the assumed normal assumption implied in the regression analysis.
- Dividing the total strain amplitude ε_a as a sum of elastic and plastic strain, see Eq. (1), unnecessarily complicates calculations for varying load.

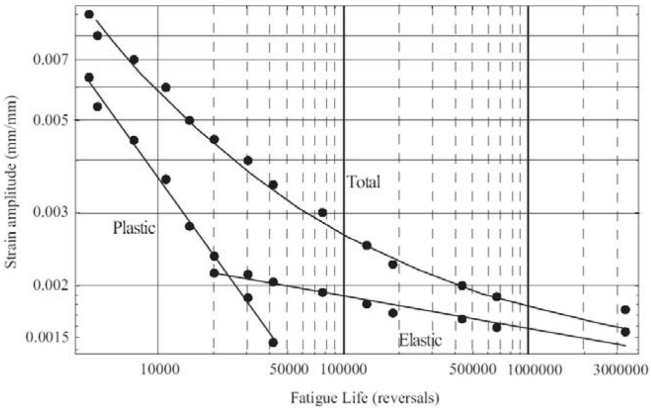

Figure1. Morrow’s model with total strain separated into the elastic and plastic components (from [6]).

In the following Sections an alternative novel Weibull regression model, based on a former $S-N$ model developed by the authors [7,8], is presented and used for the probabilistic definition of the $\varepsilon - N$ field without the need of separating the total strain in its elastic and plastic components. The model arises from sound statistical and physical assumptions and, integrating the elastic and the plastic local strain, provides an analytical probabilistic definition of the whole strain-life field as quantile curves both in the low cycle and high cycle fatigue regions allowing the consideration of runouts. The five parameters of the model can be estimated using different well established methods proposed in the fatigue literature [9]. In particular, the maximum likelihood and the two-stage methods provide adequate estimates based on the set of experiments, whereas runouts can be also dealt with, a frequent case in fatigue testing. The model can be applied for probabilistic lifetime prediction using damage accumulation [7]. Finally, the proposed $\varepsilon - N$ model will be also used to derive the corresponding stress life curves, i.e., the probabilistic $S-N$ field that in this case presents the typical change in curvature in the low-cycle fatigue region.

Note that the new model is only required for the parameter estimation and statistical definition of strain-life curves and its subsequent application to the current strain-based approach. No objection is made to the remaining procedures implied in the conventional strain-based approach.

Strain-life fatigue model

The application of the compatibility condition to the distributions of lifetime (number of cycles) for given stress range (or stress amplitudes) or of the stress amplitude for given lifetime in the $S-N$ field (see Fig.2a), together with other statistical and physical considerations such as weakest link principle, stability, limit behavior and limited range of the variables, leads to a functional equation the solution of which [10] results in a fatigue model that provides a probabilistic description of the whole $S-N$ field [7,8]. Since the basic assumptions used to derive the former Weibull $S-N$ model also apply in the case of the $\varepsilon-N$ field, due to a parallelism in both problems being dealt with, the authors propose a Weibull regression model for the strain-life case assuming that the fatigue life N_f and the total strain amplitude ε_a (the stress range $\Delta\varepsilon$ could also be considered) are random variables. Proceeding with non-dimensional variables, the same considerations as those used in the derivation of the $S-N$ model, i.e., compatibility, physical and statistical conditions, are employed. A more detailed description is given in [11], leading to the following model:

$$p = F(N_f^*;\varepsilon_a^*) = 1 - exp\left\{-\left[\frac{log(N_f/N_0)log(\varepsilon_a/\varepsilon_{a0})-\lambda}{\delta}\right]^{\beta}\right\}; \qquad log(N_f/N_0)log(\varepsilon_a/\varepsilon_{a0}) \geq \lambda, \tag{2}$$

where p is the probability of failure, N_f^* the normalized life time, ε_a^* the normalized total strain amplitude, N_0 the threshold value of lifetime, ε_{a0} the endurance limit of ε_a, and $\lambda, \delta,$ and β the non-dimensional location, scale and shape Weibull model parameters, respectively (see Fig. 2b):

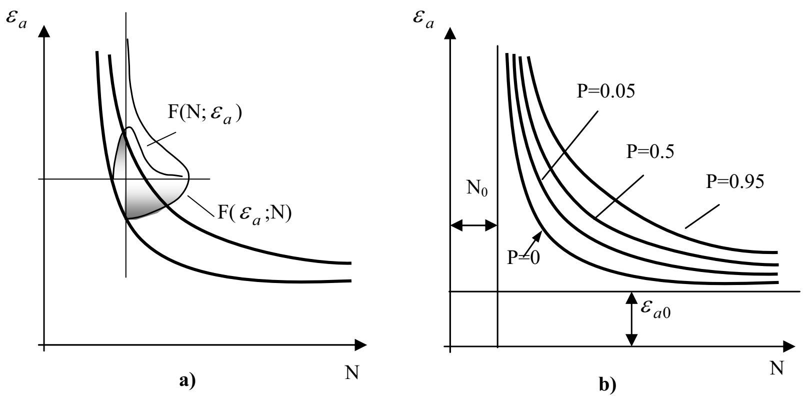

Figure 2. a) Compatibility between probability distributions in the $\varepsilon-N$ field (schematically) and b) $\varepsilon-N$ field with percentile curves.

Notice that Eq. (2) has a dimensionless form, and reveals that the probability of failure p depends only on the product $N_f^* \varepsilon_a^*$, where $N_f^* = log(N_f / N_0)$ and $\varepsilon_a^* = log(\varepsilon_a / \varepsilon_{a0})$. Accordingly, $N_f^* \varepsilon_a^*$ has a Weibull distribution:

$$N_f^* \varepsilon_a^* \sim W(\lambda, \delta, \beta). \tag{3}$$

The parameters $N_0, \varepsilon_{a0}, \lambda, \delta$ and β of the model given by Eq. (2) can be estimated using different well established methods proposed in the fatigue literature, see [7,8,9]. In particular, the maximum likelihood and the two-stage methods provide adequate estimates based on the set of experiments, whereas runouts can be also dealt with, a frequent case in fatigue testing. Accordingly, the proposed method is an adequate alternative candidate to the traditional model in Eq. (1), to represent the $\varepsilon - N$ field. The experimental data for the parameter evaluation can be obtained using the current testing strategy, primarily conditioned by model (1), which implies two linear regression analyses corresponding to the assumed elastic and plastic components of the strain life curve [6]. Other alternatives to the testing program can be envisaged taking into account the greater flexibility of the proposed model, which permits considering groups of several data at least for three different strain amplitudes as already found for the $S - N$ model, see [7,8].

Conversion of the strain-life into stress-life curves

Unlike the stress-life curves, which validity in the low cycle fatigue region must be questioned due to the existence of noticeable plastic effects associated to the hardening zone of the cyclic $\sigma - \varepsilon$ diagram, the proposed strain-life model is reasonably supported by the relatively high upper limit of the strain in the low cycle fatigue region. Consequently, this $\varepsilon - N$ model is applicable to both low- and high-cycle fatigue regions in contrast to the conventional $S - N$ model [7,8] in which the change of curvature in the low-cycle fatigue region causes troubles in a direct parameter estimation of the stress-life curve, and points out its validity limits. Up to present, the expected correspondence between the strain-life and the stress-life curves has not been studied, probably due to the incompatibility or inconsistency between the basic assumptions supporting both models in the approaches being currently in use. Nevertheless, the analytical expression for the probabilistic strain-life field developed in this paper suggests finding initially the $\varepsilon - N$ field to derive a valid $S - N$ field in both low- and high cycle fatigue regions using the cyclic $\sigma - \varepsilon$ diagram as [12].

In the following, an unnotched isotropic material with a cyclic stress-strain curve represented by a Ramberg-Osgood model:

$$\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{H'}\right)^{1/n'} \tag{4}$$

is assumed. Despite some materials exhibit cyclic hardening and others softening, the hysteresis loop accepted for the $\sigma - \varepsilon$ relation of this material is supposed to get stabilized in σ_a for given ε. This allows rewriting Eq. (4) now as a function of the strain and stress amplitudes, σ_a and ε_a:

$$\varepsilon_a = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{H'}\right)^{1/n'} \tag{5}$$

with the same parameter meanings as above.

After replacing Eq. (5) into Eq. (2), the stress-life field for this stabilized stress-strain hysteresis loops becomes:

$$p = F(N_f^*;\sigma_a^*) = 1 - exp\left\{ -\left[\frac{log(N_f / N_0)log\left(\frac{1}{\varepsilon_{a0}}\left(\frac{\sigma_a}{E} + \left(\frac{\sigma_a}{H'}\right)^{1/n'}\right)\right) - \lambda}{\delta} \right]^{\beta} \right\}; \tag{6}$$

$$log(N_f / N_0)log\left(\frac{1}{\varepsilon_{a0}}\left(\frac{\sigma_a}{E} + \left(\frac{\sigma_a}{H'}\right)^{1/n'}\right)\right) \geq \lambda \; .$$

This expression can be applied for the derivation of the $S-N$ field of any material once the parameters of model (1) and those of the Ramberg-Osgood equation have been derived.

Practical example

In this Section, the proposed approach is illustrated through its application to the experimental data obtained in [6] for a SAE 1137 carbon steel, with Ramberg-Osgood's parameters H' = 1230 MPa and n' = 0.1608. The model given by Eq. (2) was fitted for the strain amplitudes data corresponding to the total strain. The following parameter were estimated: B = 3.596; C = -7.713; λ = 13.716; δ = 0.852; β = 1.796 leading to the 0, 0.05, 0.50, 0.95 and 0.99 percentiles curves in Fig. 3, where the data points are included along with the expected run-out value. The model shows a good fit to the data proving to have a small scatter.

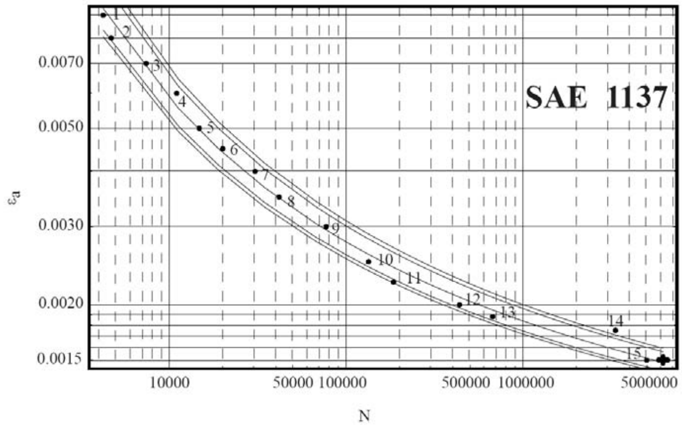

Figure 3. $\varepsilon - N$ field for the data results for a SAE 1137 carbon steel from [6] according to model (2) showing representative percentile curves. The expected value of the runout appears as a cross.

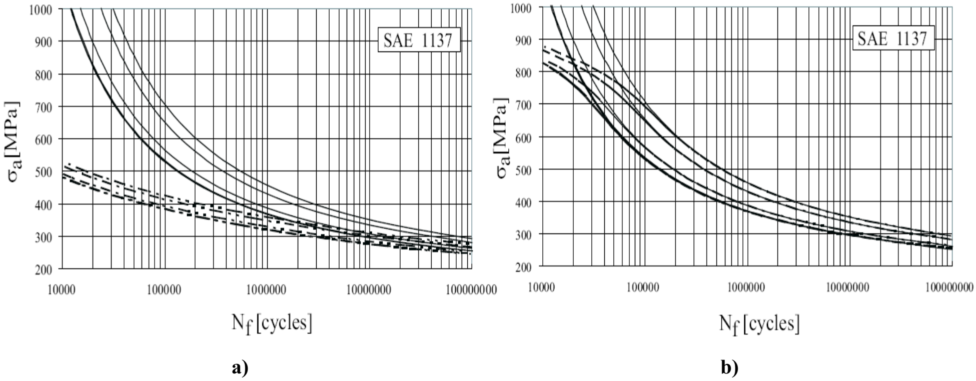

Figure 4. $S-N$ fields resulting from a perfect elastic material (continuous lines) and for Ramberg-Osgood (dashed lines) when a) H' = 1230 MPa and n' = 0.1608 and b) H' = 1230 MPa and n' = 0.06, respectively (from [12]).

After replacing the H' and n' values into Eq. (4), the $S-N$ resulting curves (dashed lines) have been obtained, see Fig. 4. For comparison, the $S-N$ curves assuming a perfect elastic material are also included (continuous lines). In order to illustrate the influence of the Ramberg-Osgood parameters on the shape of the $S-N$ field, an additional hypothetical carbon steel with H' = 1230 MPa and n' = 0.06, corresponding to a cyclic $\sigma-\varepsilon$ diagram approaching to the ideal elastic-plastic case, is shown in Figure 4b. The diversion between the $S-N$ fields in the low cycle region becomes apparent. Other cases, as for instance the ferritic steel SAE D4512 from [6], can be consequently handled, see [12].

Conclusions

The main conclusions derived from this paper are the following:

- A Weibull regression model for statistical analysis of strain life data is proposed. The model is based on the same fundamental assumptions as those derived for a former stress-lifetime model developed by the authors.
- The model permits considering the total strain directly without the need to separate the total strain in its elastic and plastic components. It provides a complete analytical description of the statistical properties of the physical problem been dealt with, including the quantile curves. Run-out data can be incorporated into the analysis.
- Contrary to the current strain-life proposal, in which the calculation of the number of cycles given the range of strain must be solved iteratively, the approach proposed allows us to proceed to a direct calculation of the damage accumulation due to the existence of a closed expression for the $\varepsilon-N$ curve. This implies a considerable advantage in the estimation of the fatigue life.
- The strain-lifetime model can be used for the derivation of the real stress-lifetime field, including the quantile curves, both in the low cycle and high cycle fatigue regions.
- The procedure allows us to recognize the lifetime region in which the plasticity effects on the fatigue behavior play a significant influence causing the $S-N$ curve trend diverting

from that in the long-life region. Thus, the validity limits in the application of stress-based approach for constant or varying loads can be recognized.

- The Ramberg-Osgood parameters have, as expected, a remarkable influence in the way the $S-N$ curve curvature changes in the low cycle fatigue region.
- The experimental analysis of possible discrepancies between the $S-N$ curve directly obtained from experimentation and that from the strain-lifetime curve would contribute to understand the plastic deformation behavior of the material.

Acknowledgements

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