3-D Local Hybrid Method Based on Surface Displacement Measurement for Structural Health Monitoring

Kenji Machida

Department of Mechanical Engineering, Tokyo University of Science

2641 Yamazaki, Noda-shi, Chiba, 2788510, Japan

mac@rs.noda.tus.ac.jp

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Abstract. Real structure is opaque and it is very difficult to evaluate the stress field inside a structure. Therefore, it is very important to evaluate the stress field inside structure from the information on the surface. Then, it waits for development of the method of evaluating the stress field inside structure from the easy measurement on a free surface for structural health monitoring. The stress analyses of large structure are possible by the 3-D finite element method. However, great time and a great labor, and great expenses are needed for modeling and calculation. To evaluate the 3-D stress field inside structure from surface displacement measurement, the 3-D local hybrid method based on inverse analysis was developed. Then, we investigated whether this method could be applied to through-thickness or surface cracked specimens subjected to uniform, mixed-mode and bending loading problems. The good result was obtained on the bending problem as well as the uniform loading problem. The 3-D local model hybrid method is a practically very useful approach which the stress field inside structure can be analyzed by measuring only the displacement field near a crack or a hole without analyzing entire structure by the 3-D finite element method.

Introduction

It is indispensable to structural integrity assessment, maintenance service, life prediction, etc. to get to know the mechanical behavior of a structure, and it is important to get to know the stress field inside the structure which has a surface crack. Although the various methods of measuring the condition of a structure are devised now, most of the measurement by experiment is about the structure surface and it is dramatically difficult to measure the stress state inside it. The stress analyses of large structure are possible by the 3-D finite element method (FEM). However, great time and a great labor, and great expenses are needed for modeling and calculation. Then, we developed the 3-D local hybrid method which analyzes only the local model of a crack part and can evaluate the stress field inside a body from structure surface data with easy measurement [1-12]. In previous studies, the good result was obtained with the surface cracked specimen subjected to uniform tension [6-8]. In this study, we examined whether the 3-D local hybrid method could apply to the surface cracked specimen subjected to bending. Hence, it is necessary to use the suitable local model size for analyses, so that the analyses which changed broadly the width and thickness of the full model and the local model and the aspect ratio of a surface crack were conducted. The digital image correlation (DIC) method [13-17] was adopted as measurement of the surface displacement data needed for the analyses by the 3-D local hybrid method. However, although the digital image correlation method could measure at least the in-plane displacement with sufficient accuracy, it turned out that it cannot measure correctly the in-plane displacement accompanied by the out-of-plane displacement. On the other hand, a position is not constant as for the structure subjected to bending such as a cantilever, it is predicted that the distance to a camera changes before and after loading. Then, the influence of out-of- plane displacement on the measurement accuracy of the digital image correlation method was evaluated first, and how to eliminate the influence was examined. Finally, the surface displacement

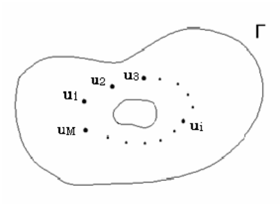


Fig.1 Measurement of u_i.

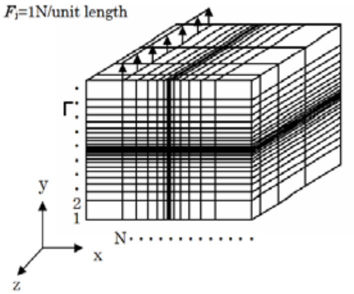


Fig.2 Application of unit uniform load in thickness direction.

was measured from the experiment of the cantilever subjected to bending, and it was examined whether the 3-D local hybrid method could estimate an internal stress state.

Theoretical Background

Analysis Method on Uniform Loading Problem. The 3-D local hybrid method was built according to the approach of Murakami and Yoshimura [1]. Now let Γ denote the measurement area of the displacement at the surface of the 3-D body as shown in Fig. 1. The nodal displacement of 8-nodes isoparametric elements in Γ region is supposed to be measured by the experiment. Let M and $\boldsymbol{u}$ denote the number of nodes and displacement vector respectively. It is considered that the analysis of the 3-D stress field is possible, if the nodal forces on boundary Γ are determined from the information of $\boldsymbol{u}$. Let us consider the 3-D mesh which extended the 2-D mesh to the thickness direction as shown in Fig. 2. Γ is the boundary at the surface, several points or a narrow region is constrained, and unit uniform load is applied to one line in thickness direction including a principal node on Γ. The number of lines is N. Let $\boldsymbol{u}_{ij}$* denote the displacement vector of point i due to this unit uniform load. The displacement and the nodal force vector have two components of the x and y directions, respectively. Let $\boldsymbol{F}_j$ (j=1~2N) denote the nodal force applied to j point in the x and y directions. If $\boldsymbol{F}_j$ is the appropriate value, the error $\boldsymbol{e}_i$ shown by the following equation should become zero.

$$\boldsymbol{e}_i = \boldsymbol{u}_i - \sum_{j=1}^{N} \boldsymbol{u}_{ij}^{*} \boldsymbol{F}_j \text{ , } i{=}1{\sim}M, j{=}1{\sim}2N. \tag{1}$$

A least squares method is applied to determine the unknown value, $\boldsymbol{F}_j$. Here, S is defined by the following equation as the sum square error.

$$S = \sum_{i=1}^{M} \left| \mathbf{e}_i \right|^2 . \tag{2}$$

It is considered that $\boldsymbol{F}_j$ considered appropriate satisfies the following equation.

$$\frac{\partial S}{\partial \boldsymbol{F}_j} = 0 . \tag{3}$$

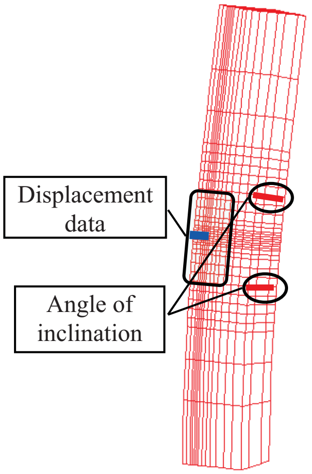


Fig.3 Extraction of virtual experimental data.

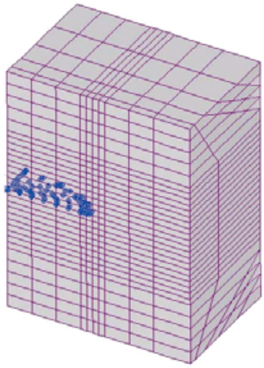


Fig.4 Local hybrid model.

$\boldsymbol{F_j}$ can be determined by solving 2N simultaneous equations of Eq. 3. If $\boldsymbol{F_j}$ is obtained, we can determine the stress and strain of the arbitrary points inside the 3-D model shown in Fig. 2 by FEM with the boundary condition of $\boldsymbol{F_j}$.

Analysis Method on Bending Problem. On a bending problem, the inverse-analysis approach of the preceding paragraph is inapplicable. Then, the analyses were carried out according to the following procedure.

1. The rectangle column with a surface crack as shown in Fig. 3 is fixed at the lower part and subjected to bending load at the upper part. And the 3-D full model FEM analyses are conducted.
2. The displacement data of hybrid analysis region is extracted from the surface displacement data of the 3-D full model as the virtual experimental data.
3. The inclined displacement is obtained from the displacement information of the node at the right end of the 3-D full model corresponding to the location of the upper and lower sides of the 3-D local model as shown in Fig. 4.
4. Assuming that the cross sections keep plane, the inclined displacement is given to the top and bottom planes of the 3-D local model on the basis of experimental data. And FEM analyses are conducted.
5. Assessment of accuracy is performed by comparing the 3-D full model FEM and the 3-D local hybrid method.

Compensation of Influence of Out-of-Plane Displacement in DIC. Although an actual camera lens is a complicated configuration, it is modeled in the form simple as a pinhole here as shown in Fig. 5. Magnification *M* of the lens is expressed with *a/b* where *a* is the distance from an object to a lens and b is the distance from the lens to screen. When distance *a* changes, the size of the image projected on the camera screen changes. When the object moves only Δa, the apparent strain generated can be expressed by Eq. 4. Moreover, supposing the point of coordinates (x, y) moves (x', y') on the camera screen depending on the out-of-plane displacement, the relation between them can be expressed by Eq. 5.

$$\varepsilon = -\frac{\Delta a}{a} \tag{4}$$

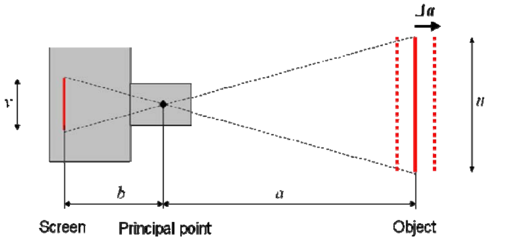


Fig.5 Relationship between screen and object position.

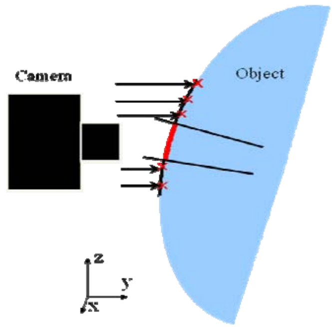


Fig.6 Schematic of deflection curve and inclination measurement.

$$x = \left(1 + \frac{\Delta a}{a}\right)x', \quad y = \left(1 + \frac{\Delta a}{a}\right)y' \tag{5}$$

Therefore, if the out-of-plane displacement is measurable, it is thought that it is possible to compensate the displacement field in consideration of the influence of the out-of-plane displacement. **Measurement Method of Out-of-Plane Displacement and Inclination.** In order to compensate the out-of-plane displacement in the digital image correlation method, it is necessary to measure the out-of-plane displacement. In this study, the out-of-plane displacement was measured using the displacement sensor. Change of the distance between a beam and a camera was measured by fixing five displacement sensors and a camera to a jig as shown in Fig. 6. The out-of-plane displacement Y of a measurement domain is expressed with the function of z like the following equation.

$$Y(z) = \beta_0 + \beta_1 z + \beta_2 z^2 + \beta_3 z^3 \tag{6}$$

From the displacement obtained by sensors, the deflection curve of the object surface was estimated by a least squares method. The out-of-plane displacement in the digital image was calculated from this deflection curve. Moreover, the inclination information inside the object was acquired from the normal of this deflection curve.

Model for Decision of the Optimal Local Model Size in Bending Problem

The schematic diagram of a specimen with a surface crack is shown in Fig. 7. *2b*, *2h* and *t* are width, height, and thickness of the specimen, respectively. *2c* and *a* are crack length at the surface, and crack depth at the crack bottom, respectively. Figure 8 shows the model for the decision of the optimal local model size in bending problem. From the symmetry with respect to the longitudinal centerline, the right half of the specimen was modeled for the full model and the local model. Here *2b* is 60 mm, *2h* is 200 mm, and *t* is 30 mm. The aluminum alloy was assumed as a material of the specimen. A Young's modulus and a Poisson's ratio are 67.3 GPa and 0.33, respectively. For the full model, displacement constraint of the base of the model was carried out in X, Y, and Z directions. For a surface crack, the size of the full model and the size of a crack are independently. Then, the decision method of the optimal local model size was considered. The hybrid analyses were carried out by changing the aspect ratio (a/c) with 0.4, 0.6 and 0.8, and by changing the crack length (c) with 5 mm, 6 mm, 7 mm, and 8 mm. The number of elements and nodes of the full model is 3094 and 14890, respectively.

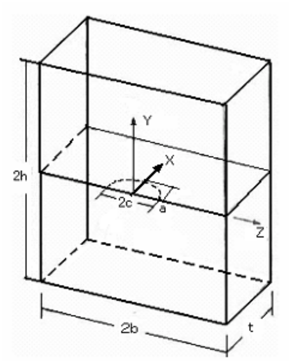
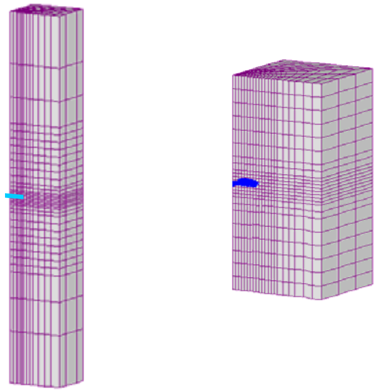


Fig.7 Schematic of surface cracked specimen.



(b) Full model (b) Local hybrid model
Fig.8 Model for decision of optimal local model size in bending problem.

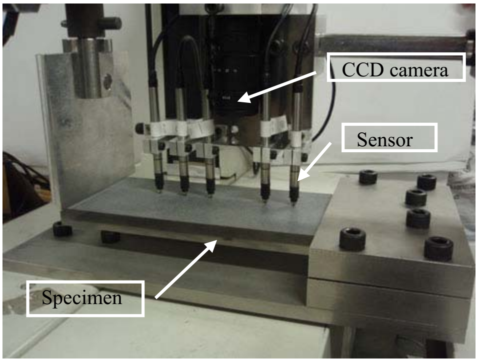
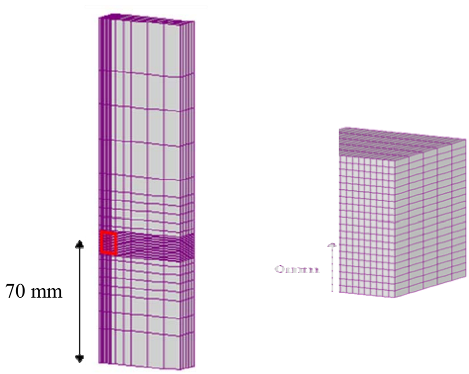


Fig. 9 Setup of the bending test.



(a) Full model (b) Local hybrid model
Fig.10 Model for bending test.

To investigate the relation between the accuracy of 3-D local hybrid method and hybrid local model size, the width, the thickness and the height of the local model were changed widely. Height was changed from 12 mm to 200 mm. Width was changed from 12 mm to 30 mm. Thickness was changed from 5 mm to 30 mm.

Displacement Measurement and Analyses of a Cantilever

Figure 9 shows the setup for the bending test. The sensor used is AT-005V contact type displacement sensor made by KEYENCE CORPORATION of which measuring range is ±2.5 mm and resolving power is 0.1 μm. The five sensors and a camera were put in order in series, it fixed with the jig, and the deflection and the out-of-plane displacement of a cantilever were measured. The length, width and thickness of the beam are 260 mm, 100 mm and 15 mm, respectively. The material of the specimen used for the bending test is an acrylic resin. A Young's modulus and a Poisson's ratio are 3.2 GPa and 0.38, respectively. The speckle pattern was created on the surface by the spray so that it might be easy to take correlation of the image. The measurement domain of displacement by the digital image correlation method is 700×700 pixels, and subset size is 60×60 pixels. The

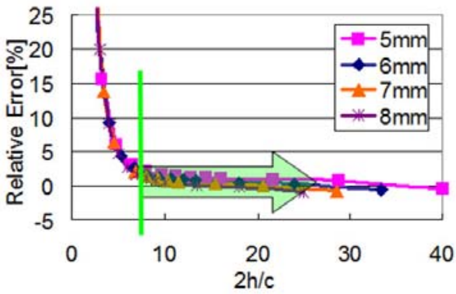


Fig.11 Variation of the relative error of J integral of the aspect ratio 0.8 with the ratio of 2h/c.

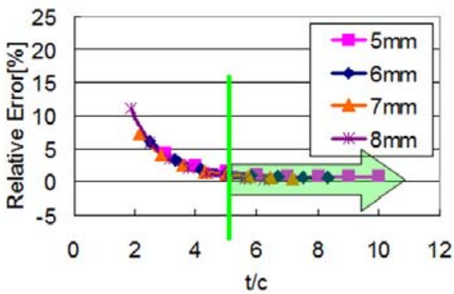


Fig.12 Variation of the relative error of J integral of the aspect ratio 0.8 with the ratio of t/c.

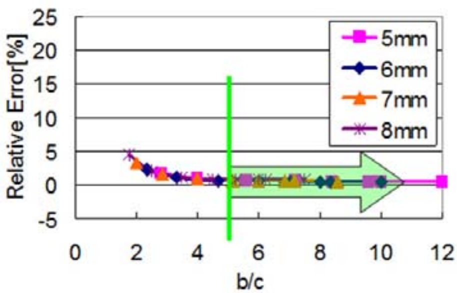
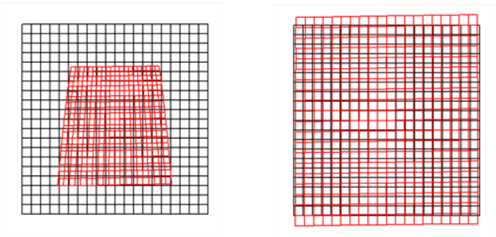


Fig.13 Variation of the relative error of J integral of the aspect ratio 0.8 with the ratio of b/c.

magnification is 20.83 μm/pixel. Figure 10 shows the model for the bending test. The lower end was fixed and displacement was measured at 70 mm by a bottom end. The mesh was set to 20×20 elements. Moreover, the local model used for analyses was the 3-D model with five layers extended the 2-D mesh used in the digital image correlation method. The number of elements and nodes was 2000 and 9891, respectively. Although a surface crack is not putting in this time, if this measurement is possible, it is expected that the good result would be obtained also with a surface cracked specimen.

Results and Discussions

Optimal Height of Local Hybrid Model. First, the effect of the height of the local model used for analyses was considered. When not changing the width and thickness of the local model and changing only height, the variation of the relative error of the J integral of the full model and the local model decreases and approaches nearly zero with an increase in the height of the local model. Moreover, the error becomes large as the aspect ratio becomes large. Therefore, if the size of the local model is determined supposing the case where the aspect ratio is large, even when the aspect ratio is small, it will be thought that it is sufficient size. Then, what summarized the result of the aspect ratio 0.8 is shown in Fig. 11. An axis of ordinate is a relative error of the J integral, an axis of abscissa is a ratio of the height to crack length of the local model. The results of each crack length overlap mutually. This shows that optimal local model height can be determined to the crack length. If the height ($2h$) of the local model is increased about 8 times of c, it can be said that sufficient analytic accuracy is acquired.



(a) Before compensation (b) After compensation

Fig.14 Deformation diagram of a beam actually obtained by the digital image correlation method.

Table 1 Relative error of strain of the front surface at different load.

Load [N]	ε [10^{-6}] FEM	HYB	Relative error [%]
40	419.5	436.0	3.93
60	637.4	645.7	1.29
80	862.9	844.1	2.17
100	1090.3	1061.8	2.61

Table 2 Relative error of strain of the front surface at 80 N.

Number of experiment	Strain [10^{-6}]	Relative error [%]
1	844.1	2.17
2	850.4	1.44
3	862.4	0.06
4	890.6	3.21
5	893.5	3.55
6	888.2	2.94
7	829.1	3.92
8	888.0	2.92
9	835.0	3.23
10	845.1	2.06
Average	862.7	2.55

Table 3 Relative error of strain of the back surface at 80 N.

Number of experiment	Strain [10^{-6}]	Relative error [%]
1	-828.5	7.42
2	-802.6	4.06
3	-875.6	13.52
4	-791.8	2.67
5	-731.7	5.13
6	-712.3	7.65
7	-798.5	3.53
8	-752.8	2.39
9	-768.9	0.30
10	-793.6	2.90
Average	-828.5	7.42

Optimal Thickness of Local Hybrid Model. Like the case of height, the relative error decreases and approaches nearly zero with an increase in the thickness of the local model. Moreover, when specimen thickness is the same, an error also becomes large as an aspect ratio becomes large. What summarized the result in case the aspect ratio of each crack length is 0.8 is Fig. 12. Since the lines of each crack length overlap mutually, even if the crack length changes, it is thought that the ratio with crack length can determine the specimen thickness of the local model. From these results, if the specimen thickness of the local model is 6 or more times of *c*, it can be said that accurate analyses are possible.

Optimal Width of Local Hybrid Model. Also in the case of specimen width, the same tendency in height and thickness was seen. Figure 13 shows the variation of the relative error of J integral of the aspect ratio 0.8 with the ratio of *b*/*c*. If specimen thickness of the local model is made into 6 or more times of crack length, it can be said that accurate analyses can be conducted. As mentioned above, the method of determining height, thickness and width of the optimum local model used for analyses was found out on the basis of crack length.

Strain Measurement of a Cantilever by DIC. Figure 14 shows the deformation diagram of a cantilever actually obtained by the digital image correlation method. Under the influence of the out-of -plane displacement, it seems to reduce to the whole. Since the free edge of the cantilever separates more distantly from a camera, it changes in the shape of a trapezoid as a load becomes large. Figure 14(b) shows what was compensated in consideration of the out-of -plane displacement. Although the displacement diagram leans slightly, it is considered since the camera leaned slightly to the beam. However, the appropriate displacement diagram without turbulence was obtained by compensation was obtained. The stress analysis was conducted with the 3-D local hybrid method based on these surface displacement data. Table 1 shows the relative error of strain of the front surface in comparison with the 3-D FEM and the 3-D local hybrid method at different load. The result of 40-100 N shows that the similar result is obtained on each loading condition. Next, Table 2 shows the result of ten times repetition experiments at the load of 80 N. The strain of the front surface obtained in FEM analyses is 862.9×10^{-6}, when it compares on the basis of FEM, an average error is about 3%, and a standard deviation is 27.0×10^{-6}. It is thought that the good result is obtained about the front surface of the beam. Table 3 is compared with the strain of the back surface of the beam which has not carried out direct measurement and it of FEM. The average error is about 5% and the standard deviation is 50×10^{-6}, and those are approximately twice the front surface. As compared with the result of the front surface, the variation in the value is large. That is, since the error of inclination is added to the error of the front surface displacement obtained by the experiment, it is thought that an error becomes large compared with the front surface.

Summary

1. The accurate result can be obtained by making the width and thickness of a local model about 6 times the length of the crack and making height of the local model about 8 times of the length of the crack.
2. It is possible to measure the surface displacement of the body subjected to bending accompanied by the out-of-plane displacement with the digital image correlation method.
3. The strain of the beam subjected to bending can be evaluated at about 50×10^{-6} accuracy by using both the digital image correlation method and the 3-D local hybrid method.
4. The 3-D local model hybrid method combined with the digital image correlation method is practically very useful approach which the stress field inside structure can be analyzed by measuring only the displacement field near a crack or a hole, so that an application to the structural health monitoring is expected.

References

[1] Y. Murakami and M. Yoshimura: Trans. JSME., Vol. 61 (1995), p. 2482.
[2] K. Machida and K. Usui: Trans. JSME, Vol.68-676A (2002), p. 1794.
[3] K. Machida: Adv. Comp. & Exp. Eng. & Sci., (2003), CD-ROM Ver.
[4] K. Machida and G. Mizukami: Proc. 2004 SEM X (2004), CD ROM Ver.
[5] K. Machida: Adv. in Exp. Mech., Ed. Carmine Pappalettere, EURASEM (2004), CD ROM Ver.
[6] K. Machida and G. Mizukami: Trans. JSME, Vol.71-709A (2005), p. 1220.
[7] K. Machida and G. Mizukami: Trans. JSEM, Vol.5-2 (2005), p. 54.
[8] K. Machida: Key Eng. Mat., Vols. 306-308 (2006), p. 523.
[9] K. Machida, G. Mizukami and H. Oyama: Key Eng. Mat., Vols. 321-323 (2006), pp. 28.
[10] K. Machida, T. Ueno and H. Oyama: Key. Eng. Mat., Vols. 326-328 (2006), p. 991.
[11] K. Machida and T. Ueno: Key. Eng. Mat., Vol. 345-346 (2007), p. 469.
[12] K. Machida and T. Ueno: Adv. Mat. Res., Vol. 33-37 (2008), p. 73.
[13] K. Machida and H. Yamada: Key Eng. Mat., Vol. 270-273 (2004), p. 800.
[14] K. Machida: App. Mech. Mat., Vol. 3-4 (2005), p. 39.
[15] K. Machida and Y. Suzuki: Key Eng. Mat., Vol. 321-323 (2006), p. 32.
[16] K. Machida, K. and K. Okazaki: Key Eng. Mat., Vol. 345-346 (2007), p. 1307.
[17] K. Machida and K. Okazaki: Key Eng. Mat., Vol. 353-358 (2007), p. 3096.