THREE DIMENSIONAL FINITE ELEMENT CALCULATIONS OF CRACK TIP PLASTIC ZONES AND K_{IC} SPECIMEN SIZE REQUIREMENTS

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Abstract

Three dimensional finite element calculations are performed of crack tip plastic zones in CT-specimens of the aluminum alloy Al 7075. With specimens of in-plane dimensions sufficient for measuring valid fracture toughnesses K_{Ic} the specimen thickness is varied over a large range. Compared to the "dog-bone" model the results show characteristic differences: The crack tip plastic zones near to or at the specimen surface do in general not agree with the usual results of 2D plane stress calculations. Furthermore, for specimen thicknesses well above the usual minimum thickness the crack tip plastic zones have not decayed yet to the size of 2D plane strain calculations. The conventional minimum size specimen requirements for determining valid fracture toughnesses are addressed in view of the found results.

Introduction

For measuring the fracture toughness K_{Ic} of a material validity criteria have to be met. These criteria guarantee that the assumptions of linear elastic or small scale yielding fracture mechanics are fulfilled and that a state of plane strain dominates at the crack tip. According to the conventional concept, the crack tip plastic zone along the crack front across the thickness of the specimen is given by the so called "dog-bone" model which assumes a constant stress intensity factor along the crack front, and a state of plane strain inside the specimen and a state of plane stress at the surface of the specimen. By a consideration of the dimensions of the test specimen in comparison to the size of the crack tip plastic zone the validity criteria result in the well known minimum size requirements for test specimens. In particular, the criterion of a dominating plane strain state of stress requires a specimen thickness (dimension of the specimen in z-direction in Fig. 1) that is sufficiently large so that those parts of the crack front directly at and near the specimen surface (considered to be under a state of plane stress) can be neglected with respect to those parts of the crack front along the middle of the specimen being under a state of plane strain. According to the standards this is the case when $B > 2.5\,(K_{Ic}/\sigma_{YS})^2$. Furthermore, an overall linear elastic behaviour of the specimen results, if the crack tip plastic zone is sufficiently small with respect to the dimensions of the specimen in in-plane directions (x-

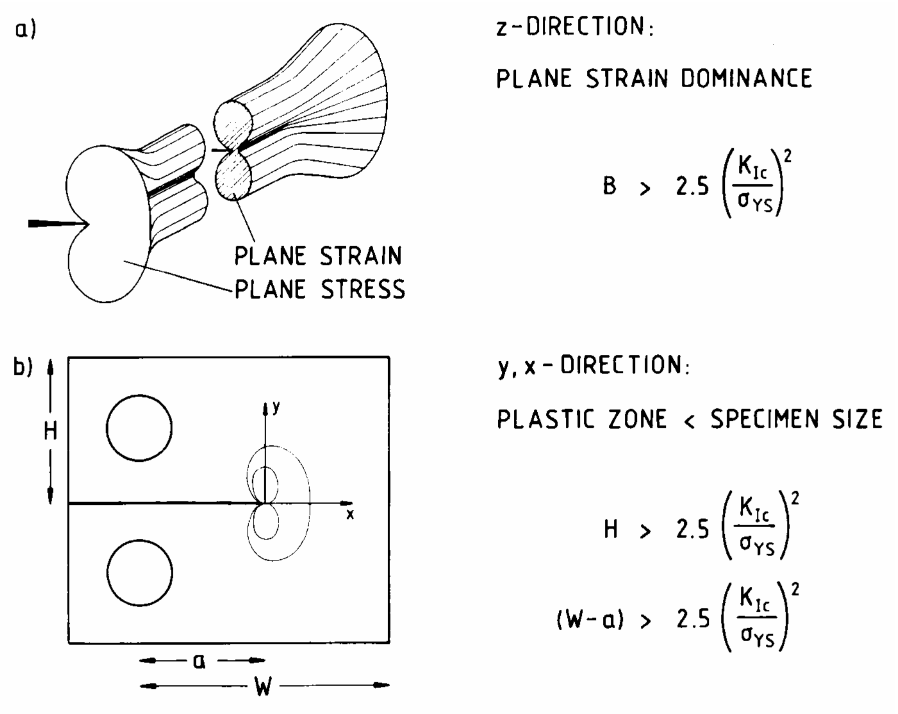

FIGURE 1. Validity criteria and “dog-bone” model with derived minimum size specimen requirements for determining the plane strain fracture toughness K_{Ic}.

and y-direction in Fig. 1), i.e. with respect to the height H and the ligament length (W-a) of the specimen. According to the standards this is the case when $H > 2.5(K_{Ic}/\sigma_{YS})^2$ and $(W\text{-}a) > 2.5(K_{Ic}/\sigma_{YS})^2$. These conditions represent the well-known minimum size specimen requirements according to ASTM E 399 and ESIS P2 for measuring valid plane strain fracture toughnesses K_{Ic} [1,2].

This paper reports on calculations of the actual shape of the crack tip plastic zones along the crack front across the thickness of the specimen (see also [3]). First, the considered type of specimen and the material, the parameters that are varied as well as specifications of the numerical calculations are given. The obtained results reveal differences between the calculated crack tip plastic zones and the conventional “dog-bone” model. Consequences resulting for the determination of valid fracture toughnesses are addressed.

Numerical Calculations

The calculations are performed for a CT-specimen (notations in accordance with ASTM E 399) made of the aluminum alloy Al 7075. In a previous study [4] the following material properties have been measured for this alloy: fracture toughness $K_{Ic}^{LT} = 30$ MPa m$^{1/2}$, yield strength $\sigma_{GY} = 533$ MPa. According to the resulting minimum specimen thickness $B_{min} =$ 7.9 mm, the calculations have been performed for CT-specimens of width $W = 50$ mm with a related thickness being sufficiently large. For specimens with these in-plane dimensions kept constant the actual thickness B of the specimen was varied in eight steps over a very large range from far below ($B = 0.1$ mm) to far above ($B = 50$ mm) the minimum specimen thickness B_{min} (or also the thickness $B_{W=50\text{mm}}$). The calculations were performed for specimens loaded by a stress intensity factor $K_I = K_{Ic}$, with the actual loads specified according to the considered specimen thickness.

The finite element code ABAQUS is used in the simplifying linear elastic approach. Standardized three dimensional non-singular 8-knot-elements are chosen (also in the near crack front region). The crack tip plastic zones are determined using von Mises equivalent stresses equal to the measured yield strength σ_{GY} of the material.

Discretization of the specimen is performed automatically by means of the I-DEAS mesh generation code. Near to the crack tip quadratic elements of 0.05 mm mesh size are used, whereas in areas remote from the crack tip the mesh size is increased up to 5 mm (see Fig. 2). In thickness direction of the specimen, successively sheets of elements are added for modelling increasing specimen thicknesses. For specimens with medium thicknesses, thus, systems with 100,000 degrees of freedom are obtained. The suitability of the used finite element nets has been studied and verified in previous work [5,6]. Load input to the specimen was modelled by means of stiff linear elements in the load transfer area with the boundary conditions specified according to the conditions given by the loading fixture.

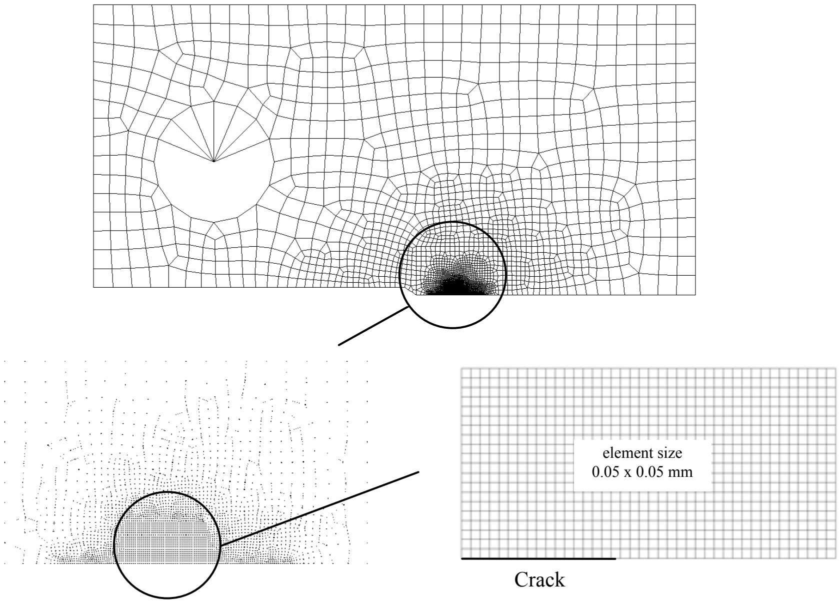

FIGURE 2. CT-specimen with finite element mesh.

Figure 3 gives the numerical results obtained by the finite element analysis for different specimen thicknesses. Shown are the crack tip plastic zones at the surface and in the middle plane of the specimen and, additionally, for an interim plane underneath the specimen surface (definition given later). Furthermore, the results of corresponding 2D-calculations for the states of plane stress and plane strain are given. Figure 4 gives a three dimensional view of the plastic crack tip zone calculated for a specimen of thickness $B = 2.5$ mm.

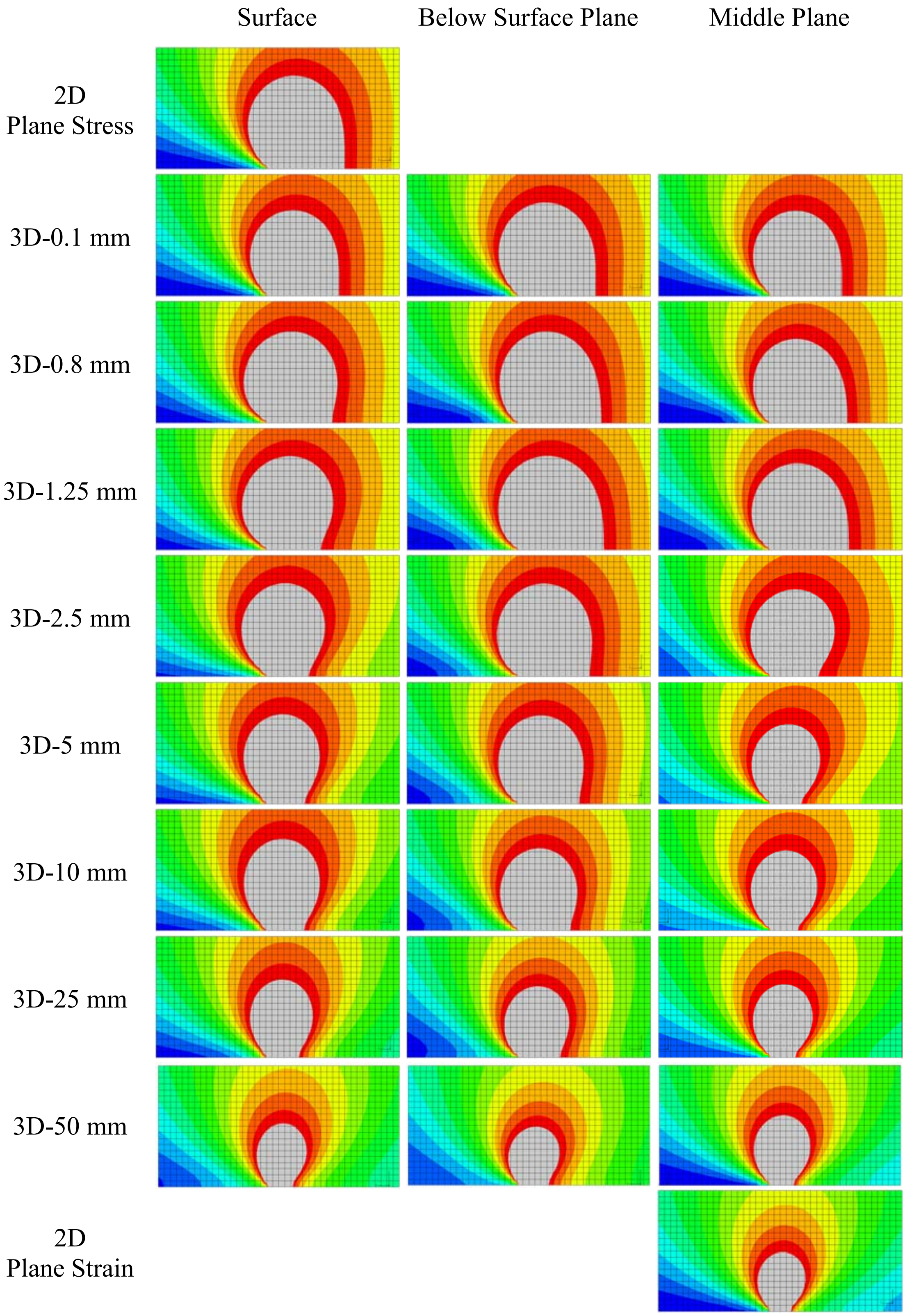

FIGURE 3. Crack tip plastic zones at the surface (left column), in a plane below the surface (middle column), and in the middle plane of the specimen (right column).

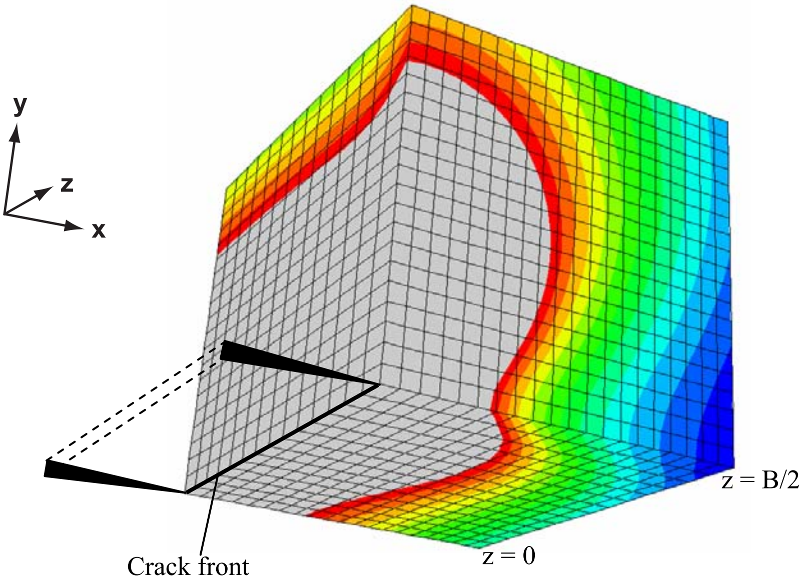

FIGURE 4. Three-dimensional view of the crack tip plastic zone (B = 2.5 mm).

Figure 5, additionally, shows the stress intensity factor along the crack front obtained by extrapolation techniques of the σ_y-stresses along the ligament for a specimen of $B = 25$ mm.

The results found by the linear elastic approach have exemplarily been verified by calculations using an elastic-plastic model and the actually measured stress strain curve of the material: The three dimensional views in Fig. 6 of the crack tip plastic zones that resulted from the linear elastic and the elastic-plastic calculations (for a specimen of 10 mm thickness) show a very similar behaviour.

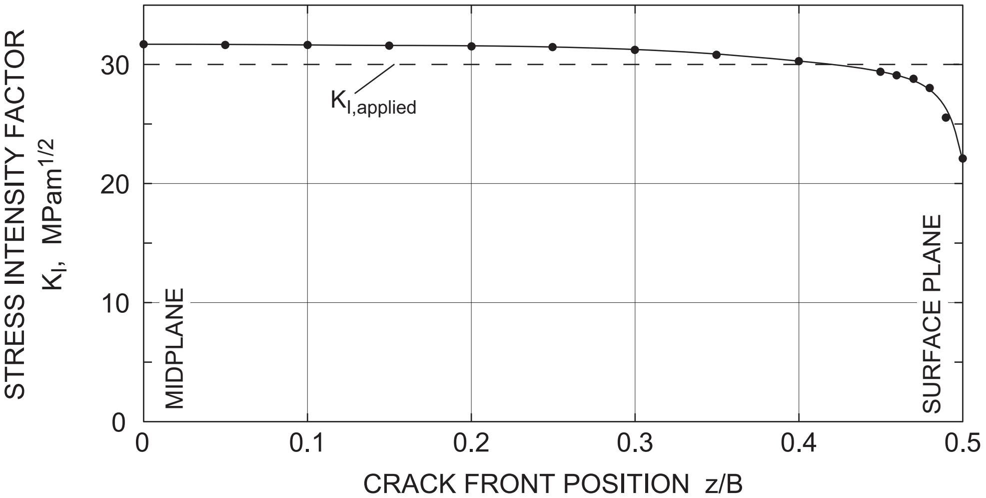

FIGURE 5. Stress intensity factor K_I along the crack front across the thickness of the specimen.

Discussion of the Results

A comparison of the numerically calculated crack tip plastic zones (Fig. 3) with the "dog-bone" model (Fig. 1a) reveals characteristic differences: First, it is recognized that the 2D-results for the considered CT-specimen made of Al 7075 for the states of plane stress and plane strain agree well with the limit states of the "dog-bone" model. A consideration of the obtained 3D-results of the plastic zones for specimens of various thicknesses, however, shows the following discrepancies in comparison to these two limit states:

At the surface of very thin specimens (see e.g. data for 0.1 mm thickness) crack tip plastic zones result which are practically identical to those for the state of plane stress. For these specimens of very thin thickness, the crack tip plastic zones in the middle of the specimen agree with those at the specimen surface for the condition of plane stress – as expected.

For specimens of larger thicknesses, the crack tip plastic zones at the surface, however, do not stay the same; compared to the zone for very thin specimens, the size as well as the shape change: The crack tip plastic zone becomes smaller and the shape becomes similar to that for a state of plane strain. One may be tempted to speculate that the smaller size of the crack tip plastic zones at the surface is due to the smaller stress intensity factor at the

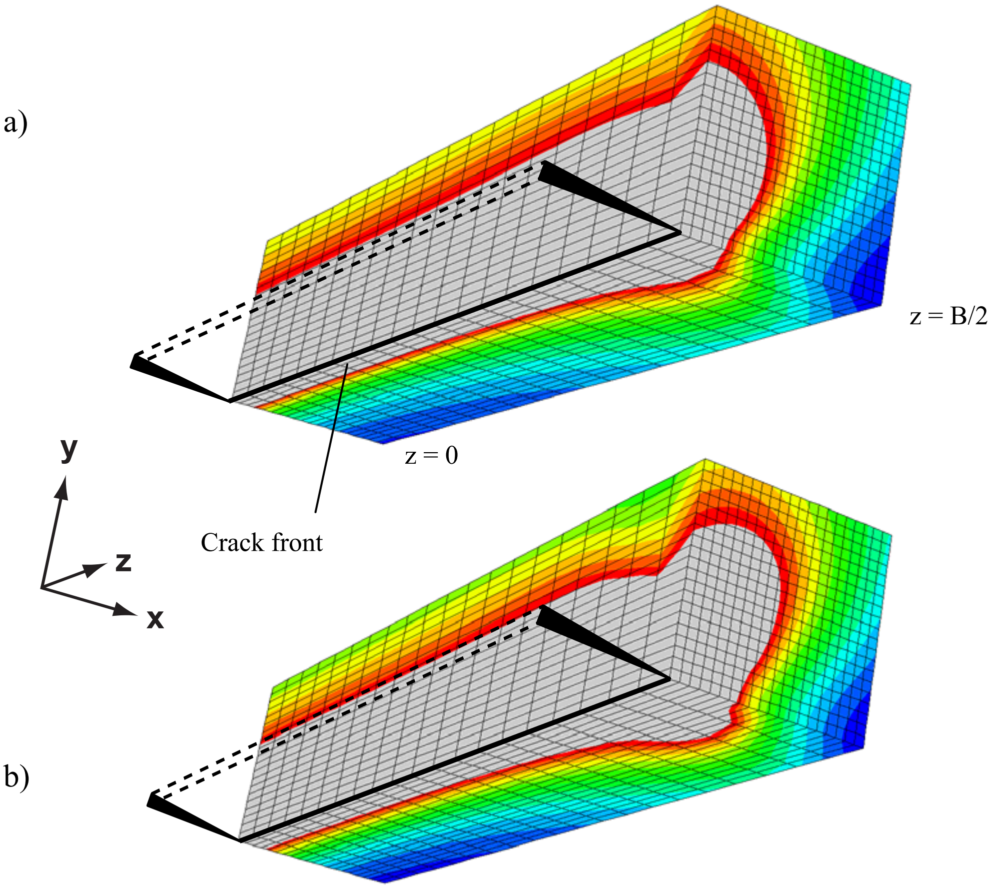

FIGURE 6. Comparison of the crack tip plastic zones determined by (a) a linear elastic and (b) an elastic-plastic model (B = 10 mm).

specimen surface, as it is shown in Fig. 5. This, however, would not give a consistent view since it is not only the size of the zone which becomes smaller but it is also the shape of the crack tip plastic zone that changes. The two observations in total, therefore, obviously indicate that the actual state of stress at the free surface of the specimen gets changed and is not represented by a state of a plane stress anymore. Astonishingly, hints in accordance with this finding have been reported a long time ago in 1971 by G.C. Sih. In [7] it is stated, that the plane stress state of stress does not represent a limit case of a three dimensional solution and that the state of plane stress violates the three dimensional compatibility conditions. This remark obviously has not been given adequate attention.

The crack tip plastic zones in the middle of the specimen, when considering increasing thicknesses of the specimen, show that the shape and the size of the crack tip plastic zones change in such a way that both, shape and size, become more and more similar to those for a state of plane strain – as expected: When the crack tip plastic zones for very large thicknesses ($B = 25$ mm, 50 mm, i.e. thicknesses far above the minimum specimen thickness $B_{min} = 7.9$ mm) are compared to the crack tip plastic zones for the limit case of plane strain, however, it becomes evident that the crack tip plastic zones - despite of these large specimen thicknesses - have not decayed yet to that size that would apply for a plane strain condition. On the basis of this finding it is questionable, therefore, whether valid plane strain fracture toughnesses can be measured with specimens for which the minimum sizes have been fixed according to the current standards. It has to be investigated to what extend fracture toughness data would be influenced by these effects and in how far the minimum specimen thickness should, eventually, be increased that conservative fracture toughness values can be determined.

The three dimensional views (Fig. 4) of the crack tip plastic zone along the crack front across the thickness of the specimen show a complex behaviour in the region underneath the specimen surface: When approaching the specimen surface from inside the specimen, the plastic zone (considered in the direction of the ligament, i.e. in the x-z-plain) first, shows a gradual increase in size, which then, with the formation of a “hump” is followed by a strong decrease in size, very different from the behaviour predicted by the “dog-bone” model for the direction of the ligament. This behaviour indicates, that the stresses - when

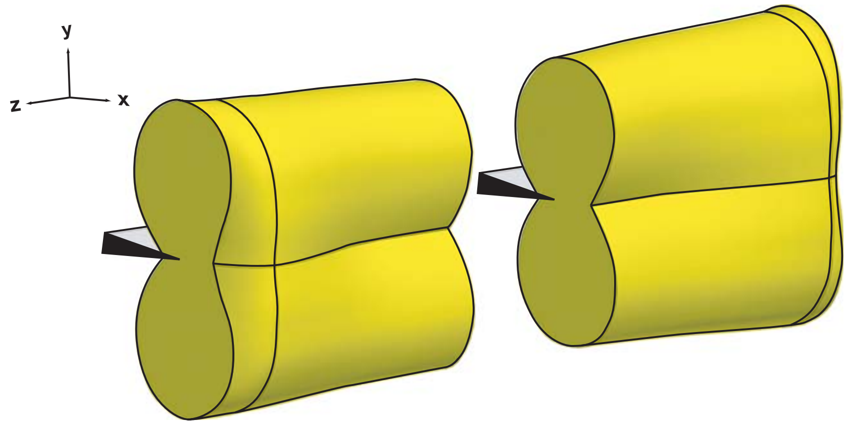

FIGURE 7. Calculated crack tip plastic zone along the crack front across the thickness of the specimen, schematically.

the specimen surface is approached - seem to arrange in such a way that a state of plane stress is tried to be built up, but, since such a state of plane stress obviously does not exist at the surface, the tendency of the results turns around in the opposite direction. The "hump" of the crack tip plastic zone in the x-z-plain corresponds to a "neck" in the y-z-plain. The crack tip plastic zones directly underneath the specimen surface, specifically at that point for which the "hump" assumes its maximum value, are additionally given in Fig. 3 (in the column: below surface plane). A somewhat schematic three dimensional view of the distribution of the crack tip plastic zones along the crack front across the thickness of the specimen is given in Fig. 7. Differences with respect to the conventional "dog-bone" model (see Fig. 1a) are clearly visible.

Taking for the moment a somewhat simplified approach and, on purpose, not considering the previously discussed formation of the "hump" and the "neck" in the shape, one recognizes that the crack tip plastic zones for a specific specimen thickness do not vary very much in shape and size along the crack front across the thickness of the specimen, but remain practically unchanged, thereby assuming an almost cylindrical shape; whereby for specimens of small thicknesses shape and size of the cylindrical cross section of the plastic crack tip zone resemble those for a state of plane stress, whereas for specimens of larger thicknesses shape and size of the cylindrical cross section of the crack tip plastic zone resemble those for a plane strain state of stress assuming a smaller size. An illustrative picture of such a simplified view of the crack tip plastic zones is given in Fig. 8. For interim specimen thicknesses a corresponding cylindrical shape of the crack tip plastic zone results with an interim shape of the cross section.

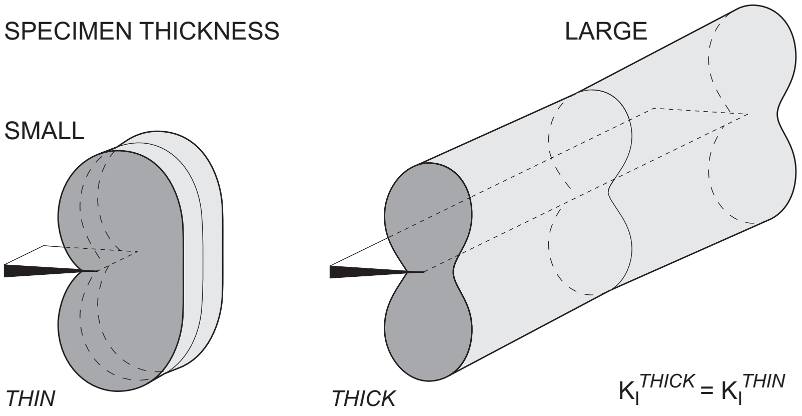

FIGURE 8. Simplified representation of the crack tip plastic zone along the crack front for specimens of small and large thickness.

Several authors [8,9,10] when investigating the dependence of various fracture mechanics parameters, such as the stress intensity factor (a behaviour as reported in Fig. 5 is found), the strain energy release rate, the local mode of fracture, or the stress singularity exponent along the crack front, in particular near or directly at the free surface of the specimen, report on characteristic peculiarities and discrepancies with respect to the usual behaviour. The observations of these authors, which obviously have their origin in the same stress situations that cause the peculiarities of the crack tip plastic zones as reported in this work, often are unjustifiedly considered as of less importance and being negligible for practical situations. It is evident and must be recognized, however, that the influences of the crack tip plastic zones reported in this paper can have severe consequences in practice.

Summary and Conclusions

For CT-specimens made of the aluminium alloy Al 7075 crack tip plastic zones have been calculated by the finite element program ABAQUS in a linear elastic approximation using von Mises comparative stresses. The measurements of the specimens in in-plane directions have been chosen according to the criteria for measuring valid fracture toughnesses K_{Ic}, the actual thicknesses of the specimens, however, were varied from values far below to much above the minimum specimen thickness B_{min}. Different from the “dog-bone” model the following behaviour was found:

- For specimens of larger thicknesses the crack tip plastic zones at the specimen surface do not agree with those for a state of plane stress. In the regime directly underneath the specimen surface a complex behaviour of the crack tip plastic zones with opposing tendencies is observed.

- By taking a somewhat simplified view, the crack tip plastic zone assumes a cylindrical shape along the crack front with a cross sectional area which for specimens of small thicknesses resembles the crack tip plastic zone for a state of plane stress, whereas for specimens of larger thicknesses the cross sectional area resembles that for a plane strain state of stress.

- For specimens far above the minimum specimen thickness B_{min} the plastic crack tip zone in the middle of the specimen has not decayed yet to that size which is determined for a state of plane strain.

Whilst most of the usual validity criteria and requirements for determining fracture toughnesses K_{Ic} can be interpreted in the same form in view of the found results, eventual modifications with respect to the specification of minimum specimen sizes for determining true valid values of the plane strain fracture toughness K_{Ic} may be necessary.

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