THE USE OF ASYMPTOTIC SOLUTIONS IN FRETTING FATIGUE

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Abstract

A range of asymptotes or local solutions are introduced and described, permitting the process zone in a fretting fatigue test to simulate rigorously that in a range of prototypes.

Introduction

Fretting fatigue is, in some ways, well understood: the mechanics of contact is a thoroughly studied subject with many closed form expressions for contact pressure and internal state of stress, whether in partial slip or sliding, but usually based on half-plane theory. Extensions beyond this idealisation normally require the use of finite element programmes, but again the basic algorithms defining the characteristics of contacts are well-known, even if they are not always easy to implement effectively and with precision. Similarly, the propagation of cracks initiated under fretting conditions is fundamentally no different from plain fatigue cracks. Contact stress fields tend to induce rather steep stress gradients, but the use of the crack tip stress intensity factor as a correlator of crack propagation rates is equally applicable for each. The remaining problem, requiring further study, is that of the mechanics of crack nucleation; the boundary between no nucleation and finite nucleation time has to be established, and, in the latter case, the number of cycles to give a true crack quantified. Present methods of treating this revolve around critical plane methods (with or without volume averaging) [1], short crack procedures [2], and “crack-like” notch analogue approaches [3]. All of these developments throw some light on the problem, but none seems to provide a completely compelling physical description of what is happening in the nucleation stage. It is true that the micromechanics of nucleation vary greatly from metal to metal (alloy to alloy), but the use of calibration experiments in which the character of the state of stress differ from the prototypical fretting problem will give rise to some difficulties. An alternative asymptotic approach, first suggested by Giannokopoulos and others [4-5], based on the use of a crack and notch analogue, has something recommend it, because of the potential use of the familiar crack tip stress intensity factor in determining the local contact corner state of stress. Giannakopoulos’ technique provided the seed for a more extensive development of asymptotic methods, not relying on the assumption of adhesion, and which provide a powerful means of correlating laboratory experiment and prototype.

The idea is simple enough: it is to encapsulate the local state of stress in the neighbourhood of crack nucleation in a simple, scalable form, and to use the (dimensional) scaling factor as a means of (a) choosing the laboratory test conditions so as to match the prototype with precision and fidelity, and (b) determining both the nucleation/no-nucleation boundary and the time to nucleation in terms of these factors. It should be noted that some provisional re-processing of earlier test data has been attempted using this approach, and initial indications are very promising - it provides a means of explaining, for example, the “size effect” observed in Hertzian test specimens. In this paper we shall review progress to date on developing both the fundamental solutions needed, and show that a relatively few parameters may be used to encapsulate the local nucleation environment very effectively. We shall also show how second order asymptotes may be used to add detail to approximate solutions to numerically solved contact problems so that, for example, the effect of small local rounding on a notionally complete contact may be determined from a "purely complete" (and indeed adhered) numerical solution.

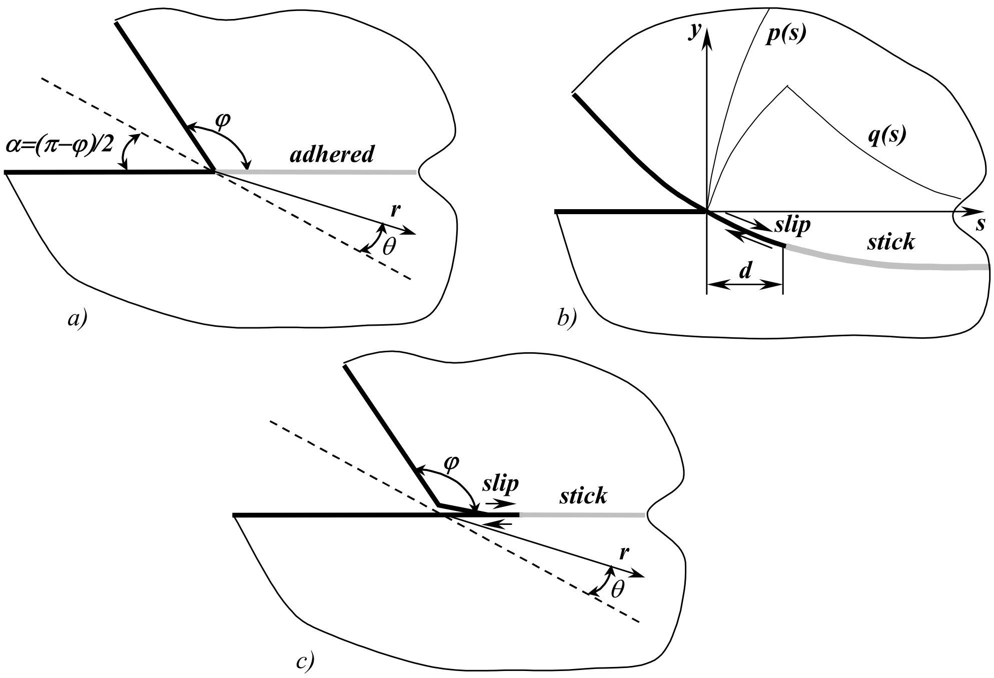

FIGURE 1. Schematic of first order asymptotes: (a) complete; (b) incomplete and (c) receding contacts.

First order asymptotes

It is a remarkable fact that the contact pressure adjacent to the edge of every contact can, to a first approximation, be written in the form $p(s) \sim s^{\lambda-1}$, where s is a coordinate measured from the contact edge, and λ is a constant, to be determined, which may explicitly be found from the geometry and nature of the contact together, under some conditions to be specified, with the elastic constants, and coefficient of interfacial friction, f. An explanation of this result varies from one class of problem to another, and these will now be reviewed.

Complete contacts

Complete contacts are those with an abrupt change in surface profile delineating the edge of the contact. Although truly complete contacts do not arise widely there are examples, such as those between the involute teeth of splines between split shafts in gas turbines, and indeed it was a study of these which gave the initial impetus for the study. The edges of complete contacts always exhibit a notional elastic singularity, and this may be found by "zooming in on the corner" so that all other geometric details pass outside the field of view. Under these conditions the local problem may be studied by solving the problem of contact between a half-plane and semi-infinite wedge, of angle φ chosen to match the prototype, as shown in Fig. 1(a). Various possibilities may be identified. The first is when the two bodies are elastically similar, and the coefficient of friction is sufficiently high for the region to be adhered. It is now clear that, within the limits of the idealisation, the interface plays no rôle, and the problem may be considered equivalent to a monolithic wedge of internal angle $\pi+\varphi$, as studied half a century ago by Williams [6]. It is not possible to derive the complete solution here, but it should be noted that there are two values of λ which render the wedge faces are traction-free, and which are in the interval ($0<\lambda<1$). The smaller of these, and therefore the one giving the stronger singularity, is given by [7]

$$\sin(\lambda q\pi) + \lambda \sin(q\pi) = 0 \tag{1}$$

where

$$q = \frac{2\pi - 2\alpha}{\pi} \tag{2}$$

Furthermore, a complete local distribution of stresses in the neighbourhood of the corner may be found. It is given by

$$\begin{Bmatrix} \sigma_{\theta\theta} \\ \sigma_{rr} \\ \sigma_{r\theta} \end{Bmatrix} = \frac{K_I r^{\lambda-1}}{(\lambda+1)+\Gamma(1-\lambda)} \left\{ \begin{bmatrix} (\lambda+1)\cos[(1-\lambda)\theta] \\ (3-\lambda)\cos[(1-\lambda)\theta] \\ (1-\lambda)\sin[(1-\lambda)\theta] \end{bmatrix} + \Gamma(1-\lambda) \begin{bmatrix} \cos[(1+\lambda)\theta] \\ -\cos[(1+\lambda)\theta] \\ \sin[(1+\lambda)\theta] \end{bmatrix} \right\} \tag{3}$$

where

$$\Gamma = -\frac{\sin[(1-\lambda)q\pi/2]}{\sin[(1+\lambda)q\pi/2]}, \tag{4}$$

and the scaling factor, K_I is analogous to a crack tip stress intensity factor, and corresponds to a solution which is symmetrical with respect to the centreline shown, and from which θ is measured.

If the bodies are adhered but elastically dissimilar the related solution for adhered, elastically dissimilar bonded wedges, due to Bogy [8] may be used, and the general form of the solution is the same, although it is more complicated. Under some conditions the contact corner singularity is oscillatory.

Clearly, when the contact is adhered there is no difference between the contact and a notched monolithic component [9], and, save for the effects of interfacial asperities, the interface would appear to play only a minor function in the problem and which, in the absence of differential motion, is not, strictly, fretting anyway. If the coefficient of friction is insufficient to sustain adhesion [7], an alternative asymptotic solution, initially derived by Theocaris and Gdoutos [10], and developed by Comninou [11] may be used. It is capable of handling both elastically similar and mismatched components. In this case the general form of the solution remains as before, although it now clearly depends on two variables (φ, f), and, in fact, there is only one value of λ in the singular range [8].

Incomplete contacts

Incomplete contacts are those where the contacting bodies have a smooth, convex relative profile, so that the size of the contact increases with applied load, and the contact pressure falls continuously to zero at the contact edge. They may normally be modelled using half-plane theory [12], and standard results from Riemann-Hilbert theory indicate that, again $p(s)\sim s^{\lambda}$, where $\lambda=1/2$ if the bodies are elastically similar, or, if they are mismatched

$$\tan(\lambda\pi) = \frac{1}{f\beta} \tag{5}$$

where β is Dundurs' constant. If we restrict attention to the elastically similar case, we may therefore write the local contact pressure in the form

$$p(s) = K_N \sqrt{s} \tag{6}$$

where K_N has dimensions of $[FL^{-5/2}]$. Clearly, this problem (Fig. 1(b)) is fundamentally different from the complete contact, as there will (save for some very special loading trajectories), always be a partial slip regime, and also the state of stress diverges away from the point of interest. These two facts mean that there is only a point in developing the solution under partial slip conditions, as the stick-slip interface provides a natural barrier to the region in which the crack may form. In fact it is straightforward to develop an asymptotic form for the effects of shearing traction [13]. This corresponds to the condition where, within the stick region the shearing traction, $q(s)$, must be such as to give rise to a constant tangential displacement, and, this means that, remote from the slip region, whose extent we will denote by d, $q(s) \sim 1/\sqrt{s}$. The full solution is given by

$$q(s) = \begin{cases} \dfrac{2K_T}{d}\sqrt{s} & 0 < s < d \\ \dfrac{2K_T}{d}\left(\sqrt{s} - \sqrt{s-d}\right) & s > d \\ 0 & s < 0 \end{cases} \tag{7}$$

where the generalised stress intensity factors are related by

$$\frac{K_T}{K_N} = \frac{fd}{2}. \tag{8}$$

It is important, when using this solution, to bear in mind that the local state of stress falls continuously to zero at the contact edge, where there is only one non-zero component, σ_{ss}. Therefore, the presence of an underlying tension has a first order effect on the characteristics of a bounded contact, whereas it does not in the case of a complete contact. It is tempting to think of it as being analogous to the T-stress in a crack problem, but it has two consequences - one is simply to add to the local stress field, and the other is to modify the partial-slip problem. The two are linked, and may be taken into account by abstracting, from a numerical model of the prototype, the length of the slip zone, d. This will no longer be directly connected to the asymptotic forms, but the complete local stress field may still be assembled, and details are given in [14].

Receding contacts

Receding contacts are normally thought of as arising rather rarely, but, in fact, complete contacts under certain conditions may generate “lift” of the edges quite easily, and this give rise to receding behaviour. A complete description is not possible here, but the important point to note is that the lifted part of the contact, Fig. 1(c) resembles very closely a crack which is part open, part closed, and therefore the characteristics of such problems carry over to the contact. The state of stress at the open-close transition point is precisely the same as that arising at the closure point of a crack. It is therefore bounded, varying like $\sim\sqrt{s}$.

Quantifying damage

The general thesis being expounded is that, if an analysis of the prototypical problem in which there is potential fretting damage is analysed, the generalised stress intensity factors evaluated as a function of time, and then a simple laboratory test employed in which the history of the stress intensity factors is reproduced, the fretting fatigue performance of the two arrangements will be the same. In practice, all that is required is a standard fretting test machine, and which may be fitted with either cylindrical pads if an incomplete contact is being considered or angular pads if a complete contact is being considered [15]. There are, however, some additional general observations which may be made.

In the case of a Hertzian contact, the asymptotes describe fully the state of stress during a loading cycle in a completely internally consistent way: indeed, it is possible to track out the evolving stress state and slip displacement pattern within the asymptote itself.

Turning to a complete contact, the problem becomes slightly more complex because, when the contact is slipping, the order of the exponent (λ-1) varies according to the slipping direction. This need does not, however, necessarily complicate the issue, as there are three possible responses, which can arise;

1. The contact can adhere, regardless of the direction of the slipping force. In this case the fretting damage is not relevant, and the problem becomes one of plain fatigue from a notch.
2. The contact may adhere when the local shear force is in one direction, but slip when the force is reversed. This is clearly physically impossible, as ideas of continuity are violated. The explanation is that the contact edge must shake down (in a frictional sense), to one of complete adhesion, and the resultant steady state condition is no different from (1).
3. The contact may slip in each direction.

This part of the work is currently being refined, and the region in which there is continuing, reversing slip is being explored, with a view to inferring which part of the loading cycle will produce most fretting fatigue damage.

Second order asymptotes

The asymptotes described in the previous section are ideal for quantifying the nature of the process zone, but there are practical cases where further refinement of the local solution is extremely useful. In particular, there is a number of cases where a contact appears to be complete, but there is just a small amount of local rounding. Indeed, it could be argued that all real, notionally complete contacts fall into this category. The behaviour of this class of contact merits further comment. First, the relative size of the process zone is an all-important quantity: if the material is extremely strong, so that the process zone is small compared with that part of the contact patch which lies in the radiused edge, then the field it experiences is precisely that of an incomplete contact. On the other hand, if the radius is extremely small and the material relatively weak, the process zone may extend significantly into the flat portion of the contact, so that it experiences what amounts to a complete contact stress field. If neither of these circumstances is true, and the process zone front lies in an intermediate region, the problem is more complex.

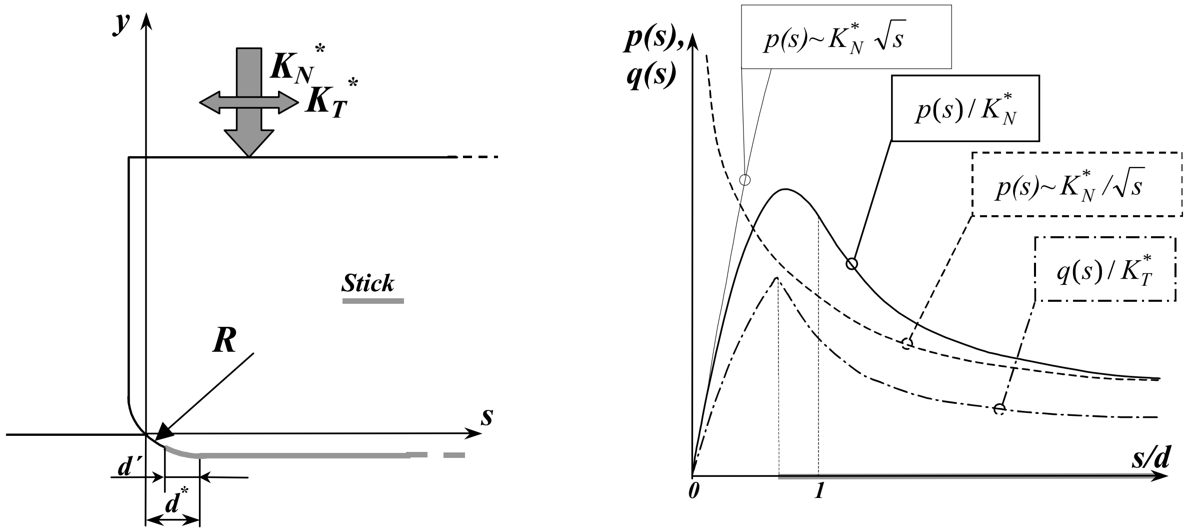

FIGURE 2. Second order asymptote: (a) geometry and (b) tractions.

An asymptote has been devised which is extremely simple in form, and which enables these conditions to be evaluated. At the same time, it enables the solution for an adhered, complete, arbitrary contact to be used to develop the solution for a punch which is similar, save for the presence of some small edge radius. It is of "semi-infinite flat and rounded profile" and it is shown in Fig. 2(a).

The solution was developed from that for a finite, flat and rounded punch, which has received significant attention recently [16], by letting the contact become infinitely large, but keeping the length of the contact in the radiused portion of the contact, d^*, finite. The resulting pressure distribution is therefore square root bounded at the extreme contact edge, it is square root singular remote from the edge, and in between the contact pressure is also explicitly known (Fig. 2(b)). In fact, if we define the scaling factor on the outermost part of the solution, i.e. the singular part, by setting $p(s)=K_N^*/\sqrt{s}\ \ (s \gg d^*)$, the other parts of the asymptote are defined by

$$p(s)=\begin{cases}\dfrac{3K_N^*}{d^*}\sqrt{s} & 0<s<<d^* \\ \dfrac{3K_N^*}{4\sqrt{d^{*3}}}\left[2\sqrt{sd^*}+\left(s-d^*\right)\ln\left|\dfrac{\sqrt{d^*}-\sqrt{s}}{\sqrt{d^*}+\sqrt{s}}\right|\right] & s>0\end{cases} \tag{9}$$

Thus, by taking the solution for any, sharp-edged punch, and finding the value of K_N^*, the asymptote may be matched locally, in the region $s \gg d^*$. The solution may be further refined by considering the effect of shear. A second scaling factor for the shearing action has been introduced permitting a partial slip condition for the semi-infinite flat and rounded punch [17] to be found (see also Fig. 2(b)).

Further considerations and Conclusions

The previous sections have highlighted the range of asymptotic solutions available to improve our understanding of fretting fatigue, and the nature of the state of stress in the relevant region. A very greatly reduced parameter range - at most three quantities, being representative of the effects of normal load, shear load, and underlying tension - are sufficient to quantify the state of stress, both in magnitude and spatially, completely. These quantities are used to establish the state of stress rigorously in a laboratory test, and hence to design a truly equivalent specimen and test piece. A corollary is that the life itself may be defined in terms of these parameters, and therefore conditions for both threshold for nucleation (as against infinite life), and, if the life is finite, the number of cycles to cause nucleation, can both be expressed as a function of these quantities. Work is currently in progress to re-evaluate some of the published fretting fatigue data (such as those found under Hertzian conditions by Nowell [18], Farris [19] and others).

In this paper we have discussed work setting out a framework for both fundamentally quantifying the state of stress within all kinds of contact undergoing contact fatigue, in a narrow region adjacent to the contact edge. It provides a means of achieving the following:

1. Design of laboratory fretting fatigue experiments which replicate very closely indeed the conditions present in a wide range of prototypical geometries.
2. Quantification of the nucleation conditions within a very small set of parameters; at most three, which physically may be thought of as corresponding to normal load, shear load and underlying tension.
3. A means of quantifying size effects implicitly (whether they relate to the macroscopic size of the contact, or emerge from "critical plane" considerations).
4. A means of obtaining a great deal of analytical rigour when dealing with the effects of a very small radius on the edge of an almost sharp contact. This means that detail relating to the contact pressure distribution and shearing traction distribution may be added to an approximate numerical solution.
5. A means of dealing seamlessly with the problem of deciding when "nucleation ends" and "propagation starts": in practice the asymptotic fields extend into the region in which propagation certainly prevails.
6. A rigorous means of testing whether a contact which may appear, prima facie, to be complete, but in fact has a small edge radius, is controlled by complete or incomplete considerations.

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